Conditional versus Unconditional Utility as Welfare Criterion: Two Examples*

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Abstract

This paper provides two illustrative examples on how a choice of social welfare criterion (conditional vs. unconditional utility) can generate different welfare implications. The first example is based on the standard linear-quadratic permanent income model, and the other example uses a simple two-country DSGE model under autarky and under complete markets. When the conditional welfare criterion used–with the social discount factor set at the private discount factor–we obtain the well-known results that the government should not intervene when there are no market imperfections and that complete markets generate risk sharing gains over autarky. In contrast, using unconditional welfare criterion–which effectively implies that the social discount factor is set to unity–can generate unconventional welfare results.

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1 Introduction

During the last few decades, it has become more common to use some type of normative criteria such as utility-based welfare in evaluating government policies and analyzing the effects of changes in certain economic environments such as financial market structure or exchange rate regime. Traditionally—due to its simplicity of calculation and ease of comparison—many economists have used unconditional (or the long-run level of) utility in evaluating policies; early examples include Taylor (1979) and Rotemberg and Woodford (1997, 1999).[1]

However, the idea of using unconditional utility as a welfare criterion is sometimes criticized inappropriate because it may neglect the welfare effects during the transition period; in most cases, the objective function of private agents involves discounting of the future outcomes and is conditional on certain initial states. For example, King and Wolman (1999) and Woodford (2002) discuss the differences between unconditional and conditional welfare criterion in calculating welfare effects of monetary policies and argue in favor of using the conditional welfare.[2]

The choice between unconditional and conditional welfare is effectively related to the discussion on how to pin down the social discount factor—as is evident in comparing the golden rule and the modified golden rule. The golden rule can be understood as a special case of the modified golden rule when the social discount factor governing the societal objective function approaches unity. That is, maximization of unconditional welfare can be achieved by maximizing conditional welfare by setting the social discount factor to unity.[3] Furthermore, in analyzing certain social policy issues, the social discount factor is in itself a parameter to be determined independently of the private discount factor—especially in discussing long-horizon issues such as climate change (Goulder and Williams, 2012, and Pindyck, 2013) and environmental protection (Barro, 2013).

This note provides two simple and illustrative examples that show how a choice between conditional versus unconditional utility as a welfare criterion–that is, a choice of the social discount factor–affects the model dynamics and the welfare implications. The first example is a simple modification of the linear-quadratic (LQ) permanent income model, and the other deals with risk sharing in complete markets.

[1] Other examples of using unconditional welfare in evaluating policies include Clarida et al. (1999), Rotemberg and Woodford (1999), Erceg, Henderson and Levin (2000), Sutherland (2002), Kollmann (2002), and Kim and Henderson (2005).

[2] One criticism for using conditional welfare is that the results depend on the specific values for the initial states in an optimizing model where some constraints involve future expectations of endogenous variables. In such an environment of time inconsistency due to forward-looking constraints, Jensen and McCallum (2010) argue that "it is preferable for policy makers to commit to implementing the time-invariant policy that maximizes the unconditional expected value of their objective, rather than the timeless-perspective policy." In contrast, the timeless perspective is based on the maximization of the discounted sum of the expected utility conditional on certain initial states. This paper deals with models without any forward-looking constraints and therefore is not subject to such criticism.

[3] See, for example, Damjanovic et al. (2008).

2 Example I: LQ permanent income model with proportional subsidy

In this section, we construct a simple LQ model to find optimal government subsidy. The household maximizes

$$\max \mathrm{E_t}\left[\sum_{s=0}^{\infty}\beta^{s}\left(-C_{t+s}^{2}\right)\right] \tag{1}$$

subject to the budget constraint

$$A_{s+1}=\varphi R\left(A_s-C_s-T_s\right)+Y_{s+1},\ \text{for all } s=t,t+1,..., \tag{2}$$

where $\varphi(\geq 1)$ represents government policy on subsidy, A_s is the beginning-of-period asset position with constant gross interest rate R, T_s is lump-sum tax payment, and Y_{s+1} is the beginning-of-period endowment in the economy.[4] That is, Y_{s+1} is realized after households make consumption decision at time s.

Government collects tax T and return it back to households in the form of government subsidy in the same period. Therefore, the government budget constraint is:

$$T_s=\frac{\varphi-1}{\varphi}\left(A_s-C_s\right),\ \text{for all } s=t,t+1,... \tag{3}$$

Combining (2) and (3), we can derive the following social budget constraint:

$$A_{s+1}=R\left(A_s-C_s\right)+Y_{s+1},\ \text{for all } s=t,t+1,... \tag{4}$$

Combining the first-order conditions w.r.t. C_t and A_t at time t produces the following Euler equation:

$$C_t=\beta R\varphi \mathrm{E_t}\left(C_{t+1}\right). \tag{5}$$

Solving the model using the Euler equation and budget constraint provides us the following equations that govern the behavior of this economy:

$$C_t = \left[1-\left(\beta R^2\varphi\right)^{-1}\right]A_t, \tag{6}$$

$$A_{t+1} = \left(\beta R\varphi\right)^{-1}A_t+Y_{t+1}, \tag{7}$$

which implies that the regularity condition we need is $\beta R^2\varphi>1$.

Assuming that Y_s follows an i.i.d. process with variance σ_Y^2, the law of motion for second moments can be easily calculated from these solutions:

$$\mathrm{E_t}\left[A_{t+1}^2\right] = \left(\beta R\varphi\right)^{-2}A_t^2+\sigma_Y^2, \tag{8}$$

$$C_t^2 = \left[1-\left(\beta R^2\varphi\right)^{-1}\right]^2 A_t^2, \tag{9}$$

[4] For this problem to be well-defined, we assume that there is a condition that rules out a Ponzi scheme under which the consumer keeps consuming by accumulating debt without limit.

and in general for a certain period s,

$$\mathrm{E_t}\left[A_{t+s}^2\right] = (\beta R\varphi)^{-2s} A_t^2 + \frac{1-(\beta R\varphi)^{-2s}}{1-(\beta R\varphi)^{-2}}\sigma_Y^2. \tag{10}$$

With positive subsidy ($\varphi \geq 1$), the return on savings become more volatile and the households increase consumption-wealth ratio in order to reduce the volatility of future wealth. Alternatively, the consumption Euler equation implies a convergent path for consumption.

Assuming $\beta R = 1$ for simplicity of algebra, we can calculate the conditional welfare by taking discounted sum of expected utility as follows:

$$\begin{aligned}
\mathrm{E_t}\left[\sum_{s=0}^{\infty}\beta^s\left(-C_{t+s}^2\right)\right] &= -\left(1-\beta\varphi^{-1}\right)^2\sum_{s=0}^{\infty}\beta^s\left[\varphi^{-2s}A_t^2 + \frac{1-\varphi^{-2s}}{1-\varphi^{-2}}\sigma_Y^2\right] \\
&= -\left(1-\beta\varphi^{-1}\right)^2\left[\left(\frac{1}{1-\beta\varphi^{-2}}\right)A_t^2 + \left(\frac{\frac{1}{1-\beta}-\frac{1}{1-\beta\varphi^{-2}}}{1-\varphi^{-2}}\right)\sigma_Y^2\right] \\
&= -\frac{\left(1-\beta\varphi^{-1}\right)^2}{\left(1-\beta\varphi^{-2}\right)}\left[A_t^2 + \left(\frac{\beta}{1-\beta}\right)\sigma_Y^2\right]. \qquad (11)
\end{aligned}$$

The welfare is maximized when $\varphi = 1$ (no subsidy) as it should be. Since there are no distortions in the economy, it would be optimal not to intervene with a government policy.

However, the solution for optimal subsidy becomes different if we take unconditional welfare. We derive the unconditional welfare by calculating the average expected utility from the following maximization problem. Suppose the policy maker uses the discount rate $\delta\,(\leq 1)$, then the average expected utility under an arbitrary subsidy would be

$$\begin{aligned}
&(1-\delta)\,\mathrm{E_t}\left[\sum_{s=0}^{\infty}\delta^s\left(-C_{t+s}^2\right)\right] \\
&= -(1-\delta)\left(1-\beta\varphi^{-1}\right)^2\sum_{s=0}^{\infty}\delta^s\left[\varphi^{-2s}A_t^2 + \frac{1-\varphi^{-2s}}{1-\varphi^{-2}}\sigma_Y^2\right] \\
&= -(1-\delta)\left(1-\beta\varphi^{-1}\right)^2\left[\left(\frac{1}{1-\delta\varphi^{-2}}\right)A_t^2 + \left(\frac{\frac{1}{1-\delta}-\frac{1}{1-\delta\varphi^{-2}}}{1-\varphi^{-2}}\right)\sigma_Y^2\right] \\
&= -\frac{\left(1-\beta\varphi^{-1}\right)^2}{\left(1-\delta\varphi^{-2}\right)}\left[(1-\delta)A_t^2 + \delta\sigma_Y^2\right]. \qquad (12)
\end{aligned}$$

Now the maximum is obtained when

$$\varphi = \frac{\delta}{\beta}.$$

In the limit when the social discount rate $\delta \to 1$, the optimal subsidy becomes

$$\varphi = \beta^{-1} > 1.$$

This solution in the limit is equivalent to the optimal subsidy that maximizes unconditional welfare. The unconditional second moments would be

$$\mathrm{E_t}\left[A_\infty^2\right] = \frac{1}{1-\varphi^{-2}}\sigma_Y^2, \tag{13}$$

$$\mathrm{E_t}\left[C_\infty^2\right] = \frac{\left(1-\beta\varphi^{-1}\right)^2}{\left(1-\varphi^{-2}\right)}\mathrm{E_t}\left[A_\infty^2\right]. \tag{14}$$

It is easy to see that the unconditional second moment diverges as subsidy approaches zero ($\varphi \to 1$). Therefore, the policy of no subsidy—that would be optimal under the conditional welfare criterion—would produce the worst outcome under the criterion of unconditional welfare. While the optimal policy under the conditional welfare criterion involves–as expected–no subsidy, it would be optimal to provide a positive subsidy under the welfare criterion of unconditional utility.[5]

3 Example II: Two-country production economy with capital

The second example is based on Kim and Kim (2003). The paper constructs a two-country DSGE model and computes risk-sharing gains when two countries move from autarky to the complete-markets economy, using a second-order approximation method. Welfare is defined as conditional welfare with discounting, and the results show that there are positive welfare gains from risk sharing, moving from autarky to the complete-markets economy. However, if we use unconditional welfare with no discounting, the welfare calculation can produce a paradoxical result that autarky generates a higher level of welfare than the complete markets.

In autarky, the representative agent in country i solves the following dynamic problem:

$$\max \mathrm{E_t}\left[\sum_{s=0}^{\infty}\beta^s\left(\frac{C_{it+s}^{1-\gamma}-1}{1-\gamma}\right)\right] \tag{15}$$

subject to

$$C_{is} + K_{i,s+1} - (1-\delta)K_{is} = A_{is}K_{is}^{\alpha}, \text{ for all } s = t, t+1, ..., \tag{16}$$

where A_{is} is productivity shock, $0 < \delta \leq 1$ and $0 < \alpha < 1$. The Euler equation for this economy at time t is

$$C_{it}^{-\gamma} = \beta\mathrm{E_t}\left[C_{i,t+1}^{-\gamma}\left(\alpha A_{i,t+1}K_{i,t+1}^{\alpha-1} + 1 - \delta\right)\right]. \tag{17}$$

[5] As $\beta \to 1$, the optimal positive subsidy derived from the unconditional welfare criterion would yield the level of conditional welfare at $\left(-\frac{4}{3}\sigma_Y^2\right)$ while the policy with no subsidy would amount to the level of conditional welfare at $\left(-\sigma_Y^2\right)$.

We solve the complete-markets economy as a social planner's problem. The objective function is to maximize the sum of two countries' utilities:

$$\max \mathrm{E_t} \left[\frac{1}{2} \sum_{i=1}^{2} \sum_{s=0}^{\infty} \beta^s \left(\frac{C_{it+s}^{1-\gamma} - 1}{1-\gamma} \right) \right] \tag{18}$$

subject to the capital accumulation equations for $i = 1, 2$:

$$K_{i,s+1} = I_{is} + (1-\delta) K_{is}, \text{ for all } s = t, t+1, ..., \tag{19}$$

and the world resource constraint

$$C_{1s} + C_{2s} + I_{1s} + I_{2s} = A_{1s} K_{1s}^{\alpha} + A_{2s} K_{2s}^{\alpha}, \text{ for all } s = t, t+1, ... \tag{20}$$

We solve both autarky and the complete markets model using a second-order accurate solution method to guarantee the validity of welfare implications. Risk-sharing gains are defined as percentage gains of certainty equivalent consumption from autarky to the complete-markets economy. We calculate welfare gains using two methods: First, we calculate differences in unconditional welfare (without discounting) of the autarkic economy and the complete markets economy. We derive the closed form solutions for unconditional welfare for both economies and the detailed solution is in the Appendix. Next, we derive conditional welfare (with discounting) gains when the economy moves from autarky to the complete market. We calculate conditional welfare by taking discounted sum of periodic utility.[6]

Table 1 shows that there are positive risk sharing gains using conditional welfare criterion—as expected—but the gains sometimes become negative using unconditional welfare under certain parameter values (when risk aversion parameter is high).

Table 1. Gains from international risk-sharing
(from autarky to the complete markets economy)

γ (risk aversion parameter)	1	2	5	10
% welfare gains (conditional welfare)	0.0960	0.1401	0.2145	0.2688
% welfare gains (unconditional welfare)	0.0095	0.0099	-0.0015	-0.0359

In order to analyze this "spurious" welfare reversal, we plot the periodic expected utility of autarky and the complete markets economy when $\gamma = 5$ for 1000 periods. Figure 1 shows that the welfare reversal under unconditional expectation occurs because it captures only the long-run steady-state changes of expected utility, not changes in expected utility during the transitional period. Conditional welfare captures changes in utility during the transitional period as it is defined as discounted sum of periodic utilities.

[6] Risk-sharing gains with different γ are calculated by endogenizing σ_a^2 to maintain σ_y^2 constant at 0.027^2 under autarky. We use the following conventional parameter values for simulation: $\beta = 0.95, \alpha = 0.8, \delta = 0.1$.

4 Conclusion

In general, economists have used two kinds of welfare criteria in evaluating economic environments or government policies. One is the unconditional welfare which is equivalent to the absence of discounting in a dynamic setting, and the other is the conditional welfare that takes private discounting on board. This note provides two simple and illustrative examples that show how a choice of a normative criterion affects the model dynamics and welfare implications. Note that we do not take any stance on which measure is the more appropriate for welfare evaluation, since the definition of the right measure depends on how an economist rationalizes the criterion function. It would be interesting but beyond the scope of this paper to expand the discussion in this direction.

A Appendix

A.1 Autarky

We assume that log productivity shock follows an i.i.d. process with mean zero and variance σ_a^2, and the lower case represents the log deviation from the deterministic steady state (i.e. $x = \log X - \log \bar{X}$). From the Euler equation (17) and the budget constraint (16), we can calculate the second-order solution as

$$\begin{aligned} k_{t+1} &= \epsilon_{kk} k_t + \epsilon_{ka} a_t + \frac{\epsilon_{kkk}}{2} (k_t)^2 + \epsilon_{kka} k_t a_t + \frac{\epsilon_{kaa}}{2} (a_t)^2 + \frac{\epsilon_{k\sigma\sigma}}{2} \sigma_a^2, \\ c_t &= \epsilon_{ck} k_{t+1} + \frac{\epsilon_{ckk}}{2} (k_{t+1})^2 + \frac{\epsilon_{c\sigma\sigma}}{2} \sigma_a^2, \end{aligned}$$

where the first-order coefficients are

$$\begin{aligned} \epsilon_{kk} &= \frac{1}{2} \left(1 + \chi - \sqrt{(1+\chi)^2 - 4/\beta} \right), \\ \epsilon_{ka} &= \frac{\beta\delta}{s} \epsilon_{kk}, \\ \epsilon_{ck} &= \frac{1 - \beta\epsilon_{kk}}{(1-s)\epsilon_{ka}}, \end{aligned}$$

and

$$\begin{aligned} \Delta &= 1 - \beta + \beta\delta, \\ s &= \frac{\alpha\beta\delta}{\Delta}, \\ \chi &= \frac{\alpha\gamma + (1-\alpha)(1-s)\Delta^2}{\alpha\beta\gamma}. \end{aligned}$$

The second-order coefficients are

$$\begin{aligned} \epsilon_{kkk} &= \frac{\gamma \left(1 - \epsilon_{kk}^2\right) \left[\alpha^2 + \frac{s(1-\delta)}{\delta} - \left[(1-s)\,\epsilon_{ck}^2 + \frac{s}{\delta} \right] \epsilon_{kk}^2 \right] + \Delta (1-\Delta) (1-\alpha)^2 (1-s)\, \epsilon_{kk}^2}{\gamma (1-s)\, \epsilon_{ck} + \gamma \left(1 - \epsilon_{kk}^2\right) \frac{s}{\delta}}, \\ \epsilon_{ckk} &= \frac{-\Delta (1-\Delta) (1-\alpha)^2 \left[(1-s)\,\epsilon_{ck} + \frac{s}{\delta} \right] + \gamma\epsilon_{ck} \left[\alpha^2 + \frac{s(1-\delta)}{\delta} - \left[(1-s)\,\epsilon_{ck}^2 + \frac{s}{\delta} \right] \epsilon_{kk}^2 \right]}{\gamma (1-s)\, \epsilon_{ck} + \gamma \left(1 - \epsilon_{kk}^2\right) \frac{s}{\delta}}, \\ \epsilon_{kka} &= \left[(1-s)\,\epsilon_{ck} + \frac{s}{\delta} \right]^{-1} \left[\alpha - \left[(1-s) \left(\epsilon_{ckk} + \epsilon_{ck}^2 \right) + \frac{s}{\delta} \right] \epsilon_{kk} \epsilon_{ka} \right], \\ \epsilon_{kaa} &= \left[(1-s)\,\epsilon_{ck} + \frac{s}{\delta} \right]^{-1} \left[1 - \left[(1-s) \left(\epsilon_{ckk} + \epsilon_{ck}^2 \right) + \frac{s}{\delta} \right] \epsilon_{ka}^2 \right], \\ \epsilon_{k\sigma\sigma} &= -\epsilon_{kaa} + (\gamma\epsilon_{ck})^{-1} \left[-\gamma\epsilon_{ckk}\epsilon_{ka}^2 + \Delta (1-\Delta) + (\Delta - \gamma\epsilon_{ck}\epsilon_{ka})^2 \right], \\ \epsilon_{c\sigma\sigma} &= - \left[\epsilon_{ck} + \frac{s}{\delta (1-s)} \right] \epsilon_{k\sigma\sigma}. \end{aligned}$$

Taking expectations of these two second-order solutions generate the mean and variance of k_t and c_t.

$$\begin{aligned}
Var\left[k_{t+1}\right] &= \epsilon_{kk}^2 Var\left[k_t\right] + \epsilon_{ka}^2 \sigma_a^2, \\
Var\left[c_t\right] &= \epsilon_{ck}^2 Var\left[k_{t+1}\right], \\
E\left[k_{t+1}\right] &= \epsilon_{kk} E\left[k_t\right] + \frac{\epsilon_{kkk}}{2} Var\left[k_t\right] + \frac{\epsilon_{kaa}}{2}\sigma_a^2 + \frac{\epsilon_{k\sigma\sigma}}{2}\sigma_a^2, \\
E\left[c_t\right] &= \epsilon_{ck} E\left[k_{t+1}\right] + \frac{\epsilon_{ckk}}{2} Var\left[k_{t+1}\right] + \frac{\epsilon_{c\sigma\sigma}}{2}\sigma_a^2.
\end{aligned}$$

Unconditional mean and variance of consumption become

$$\begin{aligned}
\mathrm{E}\left[c_t^U\right] &= \left[\frac{\epsilon_{ck}}{1-\epsilon_{kk}}\left(\frac{\epsilon_{ka}^2}{1-\epsilon_{kk}^2}\epsilon_{kkk} + \epsilon_{kaa} + \epsilon_{k\sigma\sigma}\right) + \epsilon_{ckk}\frac{\epsilon_{ka}^2}{1-\epsilon_{kk}^2} + \epsilon_{c\sigma\sigma}\right]\frac{\sigma_a^2}{2}, \\
\mathrm{Var}\left[c_t^U\right] &= \epsilon_{ck}^2 \frac{\epsilon_{ka}^2\sigma_a^2}{1-\epsilon_{kk}^2},
\end{aligned}$$

where superscript U denotes unconditional. Unconditional welfare and the certainty equivalent consumption can be calculated as follows

$$\begin{aligned}
E\left[U_t^U\right] &= E\left[c_t^U\right] + \frac{1-\gamma}{2} Var\left[c_t^U\right], \\
c_t^{U,CE} &= \left[E\left[U_t^U\right](1-\gamma)+1\right]^{\frac{1}{1-\gamma}}.
\end{aligned}$$

A.2 Complete markets economy

Solution for the complete markets model implies that consumption should be equal across countries: $C_{1s} = C_{2s}$ for all $s \geq t$. Also, due to the independence assumption of the shocks, the social planner allocates the same amount of capital: $K_{1s} = K_{2s}$ for all $s \geq t+1$.

Using the information that the solution is symmetric between the two countries, we compute the second-order solution as follows:

$$\begin{aligned}
k_{1,t+1} &= 2\eta_{kk}k_{1t} + \eta_{ka}\left(a_{1t}+a_{2t}\right) + \frac{\eta_{k\sigma\sigma}}{2}\sigma_a^2 \\
&\quad + \frac{1}{2}\begin{bmatrix} k_{1t} \\ a_{1t} \\ a_{2t} \end{bmatrix}' \begin{bmatrix} 4\eta_{kkk} & 2\eta_{kka} & 2\eta_{kka} \\ 2\eta_{kka} & \eta_{kaa} & \eta_{kaa^*} \\ 2\eta_{kka} & \eta_{kaa^*} & \eta_{kaa} \end{bmatrix} \begin{bmatrix} k_{1t} \\ a_{1t} \\ a_{2t} \end{bmatrix}, \\
c_t &= 2\eta_{ck}k_{1,t+1} + 2\eta_{ckk}k_{1,t+1}^2 + \frac{\eta_{c\sigma\sigma}}{2}\sigma_a^2,
\end{aligned}$$

where the coefficients η's are

$$
\begin{aligned}
\eta_{ckk} &= \frac{\epsilon_{ckk}}{4}, \eta_{kkk} = \frac{\epsilon_{kkk}}{4}, \eta_{kka} = \frac{\epsilon_{kka}}{4}, \\
\eta_{kaa} &= \frac{1}{4}\left(\epsilon_{kaa} + \epsilon_{ka}\right), \eta_{kaa^*} = \frac{1}{4}\left(\epsilon_{kaa} - \epsilon_{ka}\right), \\
\eta_{k\sigma\sigma} &= \frac{1}{2}\left(\epsilon_{k\sigma\sigma} + \frac{\Delta - \gamma\epsilon_{ck}\epsilon_{ka}}{\gamma\epsilon_{ck}}\right), \\
\eta_{c\sigma\sigma} &= -\left(\epsilon_{ck} + \frac{s}{(1-s)\,\delta}\right)\left(\epsilon_{k\sigma\sigma} + \frac{\Delta - \gamma\epsilon_{ck}\epsilon_{ka}}{\gamma\epsilon_{ck}}\right).
\end{aligned}
$$

Mean and variance of k_t and c_t follow:

$$
\begin{aligned}
Var\left[k_{t+1}\right] &= 4\eta_{kk}^2 Var\left[k_t\right] + 2\eta_{ka}^2\sigma_a^2, \\
Var\left[c_t\right] &= 4\eta_{ck}^2 Var\left[k_{t+1}\right], \\
E\left[k_{t+1}\right] &= 2\eta_{kk}E\left[k_t\right] + 2\eta_{kkk}Var\left[k_t\right] + \eta_{kaa}\sigma_a^2 + \frac{\eta_{k\sigma\sigma}}{2}\sigma_a^2, \\
E\left[c_t\right] &= 2\eta_{ck}E\left[k_{t+1}\right] + 2\eta_{ckk}Var\left[k_{t+1}\right] + \frac{\eta_{c\sigma\sigma}}{2}\sigma_a^2.
\end{aligned}
$$

Unconditional mean and variance of consumption become

$$
\begin{aligned}
\mathrm{E}\left[c_t^U\right] &= \left[\frac{\epsilon_{ck}}{1-\epsilon_{kk}}\left(\frac{2\epsilon_{ka}^2}{1-\epsilon_{kk}^2}\eta_{kkk} + 2\eta_{kaa} + \eta_{k\sigma\sigma}\right) + \eta_{ckk}\frac{2\epsilon_{ka}^2}{1-\epsilon_{kk}^2} + \eta_{c\sigma\sigma}\right]\frac{\sigma_a^2}{2}, \\
\mathrm{Var}\left[c_t^U\right] &= 4\eta_{ck}^2\frac{2\eta_{ka}^2\sigma_a^2}{1-4\eta_{kk}^2}.
\end{aligned}
$$

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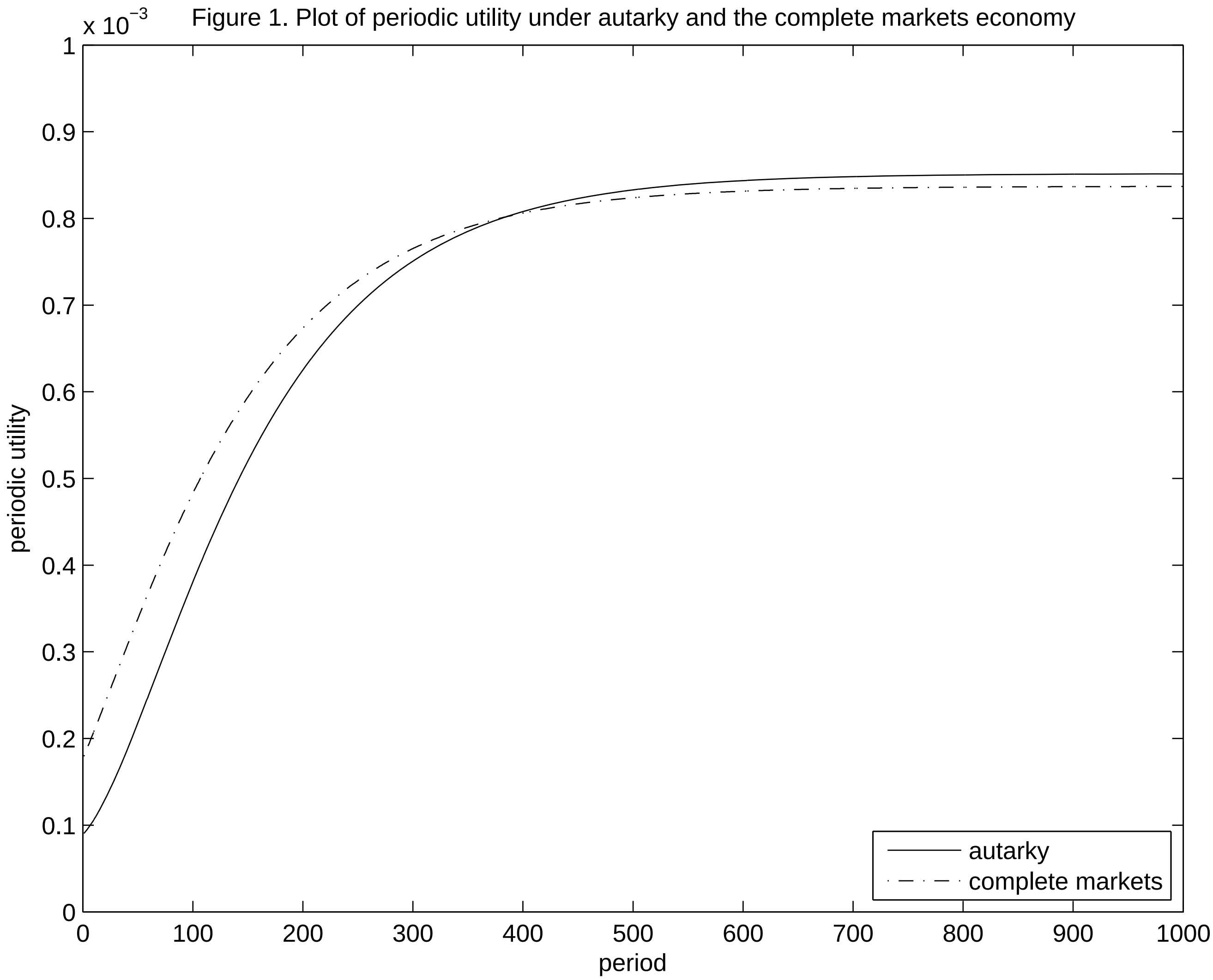

Figure 1. Plot of periodic utility under autarky and the complete markets economy