Endogenous Regime Switching Error Correction Model for Stock Price and Dividend

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Abstract

This paper considers a new model for stock price and dividend. The model is an error correction model with two distinct features. First, it adopts the time varying coefficient cointegration to explain the long-run relationship between stock price and dividend. Second, the model allows for regime switching with endogenous autoregressive latent factor. In such an endogenous regime switching model, the future transition between states depends on the current state as well as the realization of the underlying time series. The empirical application on the S&P 500 Index and dividend shows that it fits the data better than existing models. It is shown that the linear cointegration is not suitable to describe the long-run relationship between stock price and dividend and the error correction model with endogenous regime switching is better in explaining the data than one with the conventional Markov switching. Additionally, the latent factor extracted from our model specifically reveals the periods for each regime and the periods of high volatility regime includes the NBER recession periods and some periods with financial crisis.

Keywords and phrases: stock price, dividend, endogenous regime switching, time varying coefficient cointegration, error correction model

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1 Introduction

There has been active research on the relationship between stock price and dividend. Typically, the present value (PV) model has been adopted to explain the relationship between stock price and dividend over the last decades. The PV model, however, is known to be inappropriate to explain the fluctuated stock price. A series of studies done by Shiller (1981a, 1981b) showed evidence against the linear PV model since stock prices are too volatile to accord with the simple present value model. Campbell (1987) also provided the evidence against cointegrating relationship between stock price and dividend.

Many studies of stock return predictability are based on cointegrating relationship between stock price and dividend. While stock return predictability is associated with the cointegrating relationship between two series, the empirical evidence has been inconclusive; see Froot and Obstfeld (1989), Craine (1993) and Balke and Wohar (2002) among others.

In the meanwhile, Fama and French (2001) demonstrated that dividend-price ratio decreased since firms changed the dividend payout policy. Several researchers attempted to investigate stock return predictability under the situation where the proportion of firms with traditional dividend-payout policy falls; see Lettau and Van Nieuwerburgh (2008) and Kim and Park (2013). Accordingly, there have been recent studies to provide nonlinear long-run relationship or fractional cointegration of two series; see Gallagher et al. (2001), Kanas (2003), Bohl and Siklos (2004), Kanas (2005), Esteve and Prats (2008), Chen et al. (2009), Esteve and Prats (2010), and McMillan and Wohar (2010).

We reinvestigate the relationship between stock price and dividend using a recently developed econometric method called regime switching with endogenous autoregressive latent factor by Chang et al. (2014). The important feature of this method is that future transition of states depends on the current state as well as the realization of underlying time series. Such an endogenous regime switching model has not been considered in the related literature. Even if there has been research to allow for regime switching, researchers adopted the conventional Markov switching in which future transition is completely determined by the current state only and does not depend on the realization of underlying time series; see Driffill and Sola (1998), Psaradakis et al. (2004), and Hu and Shin (2014).

We consider the monthly S&P 500 Index and dividend data from January 1974 to December 2014. As the first step, we examine the long-run relationship between stock price and dividend. We show that the linear cointegration is not suitable to describe the long-run relationship between stock price and dividend and they are nonlinearly cointegrated, which corresponds to earlier studies listed above. We adopt the time varying coefficient cointegration proposed by Park and Hahn (1999) and it turns out to be an appropriate way to model the non-linear long-run relationship between stock price and dividend.

Next, using the residual of the time varying coefficient cointegration model, we consider several error correction models. First of all, the usual linear error correction model does not show any meaningful relationship between stock price and dividend. In this case, most coefficients are insignificant. However, when we allow for regime switching in the error correction model, the model exhibits reasonable results. Specifically, when we allow for the endogenous regime switching proposed by Chang et al. (2014), the model fits the data better than one with the conventional Markov switching.

The estimation results of our endogenous regime switching error correction model are following. First, the latent factor extracted from our model specifically reveals the periods for each regime. We classify the regime with higher volatility as the high-regime and one with lower volatility as the low-regime. The high-regime includes about 20% of the data. While the average of stock return is 0.62% for the entire sample period, it is -1.26% in the high-regime and 1.11% in the low-regime. This corresponds to the commonly observed asymmetric relationship between stock return and volatility. It is well known that negative returns are associated with higher volatility than positive returns and this aspect is called the leverage effect; see Black (1976), Pagan and Schwert (1990), Engle and Ng (1993) and Harvey and Shephard (1996) for a rather incomplete list of related studies.

Second, while the error correction coefficient is estimated to be significant in both regimes, it is shown that error correction (or disequilibrium adjustment) is faster in the high-regime than the low-regime. The short-run relationship between stock price and dividend is significant only in the low-regime; stock price decreases by 0.616% when dividend increases by 1%. This result corresponds to earlier studies by Campbell and Beranek (1955), Miller and Modigliani (1961) and Dasilas (2009) among others.

Third, in our model, the transition probability is time varying as explained by Chang et al. (2014) while it is constant in the conventional Markov switching model. When there is a negative event in the stock market, the transition probability from low-regime to high-regime rapidly increases. The fact that future transition depends on the current state as well as the realization of underlying time series is one of the main features of the endogenous regime switching model. Additionally, the revealed regimes by the latent factor of our model show that the high-regime periods more or less coincide with the NBER recession periods and also contain most periods with financial crisis.

The rest of the paper is organized as follows. Section 2 introduces the model and explains the time varying coefficient cointegration and the endogenous regime switching model. Sections 3 provides the data description and main results of the paper. Section 4 concludes the paper.

2 Econometric Methods and the Model

In this section, we will introduce our model for stock price and dividend after explaining two main econometric methods used for our model.

2.1 Cointegrating Regression with Time Varying Coefficients

Park and Hahn (1999) introduced time varying coefficients cointegrating regression, which we employ here to demonstrate that stock price and dividend are cointegrated. The regressions are useful to interpret long-run economic relationships that are exhibited to evolve over time. Furthermore, this approach exploits the available information efficiently to estimate parameters of the model.

The time varying coefficient (TVC) model is given by

$$y_t = \beta_t x_t + u_t \tag{1}$$

where u_t is a latent disequilibrium error sequence assumed to be weakly dependent and β_t the coefficient to be estimated is now allowed to change over time in a smooth way. Specifically, we let

$$\beta_t = \beta\left(\frac{t}{n}\right) \tag{2}$$

where n is the sample size, $t = 1, 2, \cdots, n$ and β is a sufficiently smooth function defined on the unit interval $[0, 1]$ and adopts a flexible Fourier functional form, which decomposes the function into a linear combination of a polynomial and pairs of periodic functions. Thus, we assume that the smooth function β can be approximated by the function $\beta_{p,q}$ defined as

$$\beta_{p,q}(r) = \delta_0 + \sum_{j=1}^{p} \delta_j r^j + \sum_{j=1}^{q} (\delta_{p+2j-1}, \delta_{p+2j})\phi_j(r)$$

where $\phi_j(r) \equiv (\cos 2\pi jr, \sin 2\pi jr)'$ for $r \in [0, 1]$. According to Park and Hahn (1999), the function β given in (2) can be well approximated by $(\beta_{p,q})$ as p and q increase.

In order to attain eifficient esetimators and a valid inferential basis for the parameters in the TVC model, we employ the canonical cointegrating regression (CCR) proposed by Park (1992). Let $w_t = (u_t, \Delta x_t)$, where (u_t) are the stationary errors in the TVC model (1). For the process (ω_t), we further define the long-run covariance matrix $\Omega = \sum_{k=-\infty}^{\infty} \mathbf{E} w_t w'_{t-k}$, the contemporaneous covariance matrix $\Sigma = \mathbf{E} w_0 w'_0$, and the one-sided long-run covariance matrix $\Gamma = \sum_{k=0}^{\infty} \mathbf{E} w_t w'_{t-k}$.

Ω, Σ and Γ are partitioned with the partition of ω_t into cell submatrices Ω_{ij}, Σ_{ij} and Γ_{ij}, for $i,j = 1,2$. By defining $\delta_{p,q} \equiv (\delta_0, \cdots, \delta_{p+2q})'$, $\Psi_{p,q}(r) \equiv (1, r, \cdots, r^p, \phi_1'(r), \cdots, \phi_q'(r))'$ with $r \in [0,1]$, the CCR transformed regression of the TVC cointegrating model is given by

$$y_{pqt} = \delta_{p,q}' x_{pqt} + u_{pqt} \tag{3}$$

whose elements are defined by

$$\begin{aligned}
y_{pqt} &= y_t - \left(\Psi_{p,q}\left(\frac{t}{n}\right) \otimes \begin{bmatrix} \Delta_{12}' & \Delta_{22}' \end{bmatrix} \Sigma^{-1} w_t\right)' \delta_{p,q} - \begin{bmatrix} 0 & \Omega_{12}\Omega_{22}^{-1} \end{bmatrix} w_t \\
x_{pqt} &= \Psi_{p,q}\left(\frac{t}{n}\right)\left(x_t - \begin{bmatrix} \Delta_{12}' & \Delta_{22}' \end{bmatrix} \Sigma^{-1} w_t\right) \\
u_{pqt} &= u_t - \Omega_{12}\Omega_{22}^{-1}\Delta x_t + (\beta - \beta_{p,q})\left(\frac{t}{n}\right) x_t
\end{aligned}$$

Then, OLS estimation of the CCR transformed model (3) can be used since CCR estimation yields efficient and optimal estimators demonstrated by Park (1992).

2.2 Endogenous Regime Switching Model

For the conventional Markov switching model, the Markov chain selecting the state of regime is completely independent from all other parts of the model. In other words, the future transition between states in the Markov switching is completely determined by the current state only and does not depend on the realization of underlying time series. To overcome such a shortcoming in conventional Markov switching, Chang et al. (2014) proposed an endogenous regime switching model where the future transition between states depends on the realization of underlying time series as well as the current state. [1]

[1] Kim et al. (2008) propose the regime switching model allowing for endogeneity as well. One of the primary difference between Chang et al. (2014) and their model is that Kim et al. (2008) postulate the presence of contemporaneous correlation between the state variable and the innovation while the innovation in Chang et al. (2014) is assumed to be correlated with the state variable in the next period. Furthermore, Chang et al. (2014) provide a general class of process by allowing for nonstationary transition whereas Kim et al. (2008) impose stationarity in transition. For a

In this approach, the mean or volatility process is switched between two regimes, depending upon whether the underlying autoregressive latent factor ω_t takes values above or below threshold level τ.

The endogenous regime switching model can be generally expressed as

$$y_t = m(x_t, \omega_t) + \sigma(\omega_t)u_t = m(x_t, s_t) + \sigma(s_t)u_t \tag{4}$$

where mean and volatility functions are denoted by m and σ, respectively, and x_t is a regressor. Let a series (ω_t) follow a first order autoregressive process as below.

$$\omega_t = \alpha\omega_{t-1} + v_t \tag{5}$$

for $t = 1, 2, \cdots$ with parameter $\alpha \in (-1, 1]$ and i.i.d. standard normal innovations (v_t). Given the realized value of the latent factor ω_t and the threshold level τ, we interpret two events $\{\omega_t < \tau\}$ and $\{\omega_t \geq \tau\}$ as two regimes that are switched. The state process (s_t) represents low or high state depending upon whether it takes value 0 or 1;

$$s_t = 1\{\omega_t \geq \tau\}, \tag{6}$$

where $1\{\cdot\}$ is the indicator function. The latent factor (ω_t) is assumed to be correlated with the previous innovation in the model. Specifically, (u_t) and (v_t) are jointly i.i.d. as

$$\begin{pmatrix} u_t \\ v_{t+1} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}\right). \tag{7}$$

For $\rho \neq 0$, since the autoregressive latent factor ω_{t+1} is correlated with the observed time series y_t, the future transition between states is endogenously affected by underlying time series. This is why Chang et al.

more detailed discussion, please see Chang et al. (2014).

(2014) name it as endogenous regime switching. However, if $\rho = 0$, it is an exogenous regime switching model because the future transition between states now does not depend on y_t. Chang et al. (2014) show that if $\rho = 0$ together with $|\alpha| < 1$, the endogenous regime switching model reduces to the conventional Markov switching model. See Section 2.2 in Chang et al. (2014) for details on the relationship between the endogenous regime switching model and the conventional Markov switching model.

The use of the endogenous regime switching model seems desirable for the following reasons: (a) the model allows for endogeneity in regime switching and, therefore, the dynamics of mean and volatility can be better explained; (b) the model becomes observationally equivalent to the conventional Markov switching model when the autoregressive latent factor is exogenous ($\rho = 0$). In other words, the endogenous regime switching model is regarded as an extended Markov switching model; (c) the model allows the transition of the state process to be persistent.

In the endogenous regime switching model, we need to use the modified Markov switching filter developed by Chang et al. (2014) since the state process (s_t) defined in (6) is not a Markov chain unless $\rho = 0$ and, as a result, the conventional Markov switching filter is not applicable.

To develop the modified filter, a newly introduced transition probability is considered accordingly as follows.

$$\mathbb{P}(s_t|s_{t-1}, y_{t-1}) = (1 - s_t)\omega(s_{t-1}, y_{t-1}) + s_t[1 - \omega(s_{t-1}, y_{t-1})] \tag{8}$$

where ω is the transition probability of the endogenous state process (s_t) to low state. The state process (s_t) is defined in (6).If $|\alpha| < 1$ and $|\rho| < 1$, ω is given by

$$\omega(s_{t-1}, y_{t-1}) = \frac{\left[(1 - s_{t-1}) \int_{-\infty}^{\tau\sqrt{1-\alpha^2}} + s_{t-1} \int_{\tau\sqrt{1-\alpha^2}}^{\infty}\right] \Phi\left(\left(\tau - \rho\frac{u_{t-1}}{\sigma_{t-1}} - \frac{\alpha x}{\sqrt{1-\alpha^2}}\right) / \sqrt{1 - \rho^2}\right)\psi(x)dx}{(1 - s_{t-1})\Phi(\tau\sqrt{1 - \alpha^2}) + s_{t-1}[1 - \Phi(\tau\sqrt{1 - \alpha^2})]}$$

As shown in above, the endogeneity of regime switching has an important effect on the performance of transition probabilities. Specifically, a negative correlation, $\rho < 0$, implies that a negative shock to (y_t) in the current period decreases the probability of staying in low regime in the next period whereas a positive realization of u_t increases the probability of having low regime at $t + 1$.

2.3 The Model

Psaradakis et al. (2004) and Hu and Shin (2014) consider error correction models with regime switching for the relationship between stock price and dividend. Their models allow for nonlinear adjustment to equilibrium driven by the conventional Markov switching. It is shown that a regime switching ECM is well suited to situations where variables are unlikely to follow linear adjustment to the long-run equilibrium.

As in Psaradakis et al. (2004) and Hu and Shin (2014), we also adopt an ECM with regime switching. However, the distinct features of our model is that we adopt the TVC model for the long-run relationship between stock price and dividend and, more importantly, the endogenous regime switching in the ECM. Hence, we call our model as the *endogenous regime switching error correction model* (endogenous RS-ECM).

We denote by p_t and d_t stock price and dividend, respectively. Both variables are in logarithm. Our endogenous RS-ECM has the following specification;

$$\Delta p_t = \lambda_0(s_t) + \lambda_1(s_t)\hat{\epsilon}_{t-1} + \gamma(s_t)\Delta d_t + \sigma(s_t)u_t \tag{9}$$

where the state process (s_t) is defined as in (6), $\hat{\epsilon}_{t-1}$ is the lagged residual from the TVC model of p_t and d_t as described in (10) and Δ refers to the differencing operator defined by $\Delta p_t = p_t - p_{t-1}$. The state dependent parameters λ_0, λ_1, γ, and σ are switched between two regimes such that $\lambda_j(s_t) = \underline{\lambda}_j(1 - s_t) + \overline{\lambda}_j s_t$ for $j \in \{0, 1\}$, $\gamma(s_t) = \underline{\gamma}(1 - s_t) + \overline{\gamma}s_t$ and $\sigma(s_t) = \underline{\sigma}(1 - s_t) + \overline{\sigma}s_t$. The latent factor ω_t is defined as in (5). We assume (u_t) and (v_{t+1}) are jointly i.i.d. as in (7) with $\rho \neq 0$. Subsequently, we estimate endogenous switching ECM using the maximum likelihood method given the modified filter entailing suggested transition probabilities. For the detail description, see Chang et al. (2014). As explained in the previous sub-section, this implies that the future transition between states is endogenously affected by underlying time series. For comparison, we also estimate the conventional Markov switching ECM (MS-ECM henceforth).

3 Main Results

3.1 Data Description

For our analysis, we employ the monthly S&P composite stock price and dividend data covering the time period January 1974 through December 2014. We are referring to the data series provided by Robert Shiller. Figure 1 displays a pair of the log of stock price (solid line) and log of dividend (dotted line) in the United States for the sample period. Stock price and dividend tend to move together. However, for some short periods in the sample such as around 1975, the beginning of 2000s and the end of 2000s, the stock price behaved quite differently from the dividend. This means that there are substantial short-run deviations from long-run equilibrium.

Table 1 shows the results of unit root tests for the series. We consider two alternative autoregressive specifications for the series: with and without a linear deterministic trend. The test results strongly support the presence of a unit root in each series. For stock price, the estimated autoregressive coefficients are close to unity, the Augmented Dickey-Fuller (ADF) tests cannot reject the null hypothesis of a unit root and the KPSS tests reject the null hypothesis of stationarity at 1% significance level. The results for dividend are similar as those for stock price except for one ADF test. Overall, the results show that both series can be modeled as unit root processes.

3.2 Nonlinear Cointegration

According to a study by Campbell and Shiller (1988), US stock prices (p_t) and dividends (d_t) have a linear long-run relationship. In contrast, Froot and Obstfeld (1989) find the strong evidence of a nonlinear relationship between stock prices and dividends in the presence of an intrinsic bubble.[2] Therefore, the long-term relationship between two series (p_t) and (d_t) may not necessarily be linear. In order to find cointegrating relationship between two non-stationary variables (p_t) and (d_t), first, we begin with simple regression model of $p_t = \theta_0 + \theta_1 d_t + v_t$. Then we run OLS estimation and obtain residual ($\hat{v}_t$) for unit

[2]Froot and Obstfeld (1989) suggest the model of stock overreaction behavior providing that the bubble is a nonlinear function of dividends.

root test. Table 2 reports that we fail to reject the presence of a unit root. Consequently, we are not able to support the linear cointegrating relationship between two series. This corresponds to earlier studies showing the nonlinear cointegrating relationship between the stock prices and dividends. See Kanas (2003), Bohl and Siklos (2004), Kanas (2005), Chen et al. (2009), Esteve and Prats (2010) and Kim and Park (2013) among others.

To search for a non-linear cointegration relationship between stock prices and dividends, we adopt the time varying coefficient (TVC) model as shown in Park and Hahn (1999) because the TVC model (1) is useful for exploring complicated non-linear interactions between two variables. We let the parameter to evolve over time and accordingly specify the model as

$$p_t = \beta_t d_t + \epsilon_t \tag{10}$$

where β_t is allowed to change overtime in a smooth way.

Hence, β_t can be approximated by a linear combination of polynomial and/or trigonometric functions on $[0, 1]$. To determine the number of the trigonometric pairs and the degree of a polynomial to be used for the estimation, we consider BIC. As shown in Table 3, a constant, a linear time trend and three pairs of trigonometric terms $\{\sin(2j\pi t), \cos(2j\pi t)\}_{j=1,2,3}$ are selected. The resulting time varying coefficient model is estimated by the canonical cointegrating regression (CCR) method developed by Park (1992) and extended by Park and Hahn (1999).

Table 4 reports the unit root test result for the residuals from the TVC cointegrating model in (10). The unit root test shows that the residual of the TVC model is stationary, which implies that two variables (p_t) and (d_t) are cointegrated. While stock price and dividend do not have a linear cointegrating relationship, they have a nonlinear long-term or equilibrium relationship.

3.3 Endogenous RS-ECM

Using the residual of the TVC model $\hat{\epsilon}_t$ in the previous subsection, we estimate several error correction models. First, we estimate the linear ECM given as

$$\Delta p_t = \lambda_0 + \lambda_1 \hat{\epsilon}_{t-1} + \gamma \Delta d_t + \sigma u_t \quad \text{where } u_t \sim \mathcal{N}(0,1). \tag{11}$$

This model does not allow for regime switching. The term $\hat{\epsilon}_{t-1}$ in the model is the lagged equilibrium error, which represents the deviation of stock price from the long-term equilibrium. The parameters of the ECM in (11) are associated with two distinct effects: the short-run effect and the long-run effect. The parameter γ is associated with the short-run effect: how does the stock price (p_t) change immediately in reaction to a contemporary change of the dividend (d_t). The long-run effect is associated with the parameter λ_1, which is commonly called as the error correction coefficient. A constant fraction of λ_1 of the lagged equilibrium error is eliminated each month.

Table 5 reports the estimation results of the linear ECM. The estimation result shows that the linear specification is inappropriate to model the relationship between stock price and dividend. The error correction coefficient λ_1 is not significantly different from zero. This implies that stock prices in the U.S. have not adjusted to any long-run disparity between (p_t) and (d_t) for the period from January 1974 to December 2014. In addition, it appears that the short-run effect of dividend is also insignificant.

Hence, we allow for regime switching in the model. We focus on the endogenous RS-ECM described in Section 2.3 and also compare it with the MS-ECM [3]. We suppose that the stock return with higher volatility belongs to the high-regime ($s_t = 1$) and the return with lower volatility does to the low-regime ($s_t = 0$).

Interpretation of Estimates

Table 6 reports the estimation results of both endogenous RS-ECM and MS-ECM for the sample period. Comparing the log-likelihood value and information criteria, the endogenous RS-ECM is better than the MS-ECM in describing the data. For the endogenous RS-ECM, the log-likelihood value is higher and both AIC and BIC are lower.

[3]The results of the MS-ECM is almost identical to those of the exogenous RS-ECM (with restriction $\rho = 0$).

The error correction coefficient λ_1 gives the rate at which the model re-equilibrates i.e., the speed at which it returns to its equilibrium level. For the endogenous RS-ECM, $\underline{\lambda}_1$ and $\overline{\lambda}_1$ are estimated to be -0.048 and -0.081, respectively. Since they are significantly different from zero, the result supports the existence of an error correction mechanism. In other words, the result depicts stability of the system and convergence towards equilibrium path in case of any disturbance in the system for both regimes. Moreover, the result implies that the high-regime is associated with fast disequilibrium adjustment while the low-regime is associated with slow adjustment. That is, in the low-regime, about 4.8% of any disequilibrium is absorbed in the next month while the correction is around 8.1% in the high-regime. On the other hand, for the MS-ECM, the error correction term is insignificant in the high-regime while it is significant in the low-regime. This means that the adjustment to equilibrium takes place only in the low-regime for the MS-ECM.

The coefficient of Δd_t, $\gamma(s_t)$, captures the short-run relationship between stock price and dividend. For the endogenous RS-ECM, $\underline{\gamma}$ is estimated to be -0.616 in the low-regime and it is significant. This means that, in the low-regime, stock price p_t decreases by 0.616% when dividend d_t increases by 1%. The finding is consistent with the previous studies that demonstrate an inverse performance between stock prices and dividends (Campbell and Beranek (1955), Miller and Modigliani (1961), Dasilas (2009)). Campbell and Beranek (1955) and Dasilas (2009) point out that stock prices decrease on ex-dividend days by an amount that is less than the dividend. On the other hand, $\overline{\gamma}$ is estimated to be insignificant in the high-regime. This implies that short-term effect does not appear in the high-regime in which stock return is more volatile. It is interesting to note that the short-run relationship between stock price and dividend significantly appears only for the low-regime. For the MS-ECM, the results for $\gamma(s_t)$ are similar to those for the endogenous RS-ECM.

In the endogenous regime switching model, the latent factor is assumed to be correlated with the previous innovation in the model. Thus, the correlation coefficient ρ between the current innovation (u_t) and the next period innovation (v_{t+1}) in (7) measures the degree of endogeneity of regime changes. In Table 6, the estimate for the endogeneity parameter ρ is quite substantial, -0.9998, and we reject the null of no endogeneity at 1% significance level. Given the strongly negative estimated value of the correlation, a positive shock to Δp_t at time t in (9) increases the probability of having low regime at time $t+1$. In contrast, a negative shock to Δp_t increases the probability of having high regime at time $t+1$. In volatility part, a negative shock to Δp_t at time t results in an increase in volatility at time $t+1$ whereas a positive shock to Δp_t attempts to decrease volatility in the following period.

Two events $\{\omega_t < \tau\}$ and $\{\omega_t \geq \tau\}$ considered as two regimes are switched by the realized value of the latent factor ω_t and the threshold level τ. We observe that the threshold level τ is significantly different from zero at 1% level. As long as the latent factor stays above the threshold level $\hat{\tau} = 2.326$, the regime is classified as the high-regime. The estimation results show that 20.41% of the data stay in the high-regime. For the entire sample period, the average of stock returns is 0.62%. Dividing it into two regimes by the extracted latent factor, the average of stock returns is -1.26% in the high-regime and it is 1.11% in the low-regime. This corresponds to the commonly observed asymmetric relationship between stock return and volatility. It is well known that volatility is higher when stock return is negative.

The autoregressive coefficient α of the latent factor is estimated to be 0.956 in the endogenous RS-ECM. This shows that the latent factor is persistent and, therefore, the transition of the state process is also persistent for the data.

Transition Probability and Revealed Regimes

Both graphs of Figure 2 and Figure 3 clearly show the difference in the time series plots of the transition probabilities estimated from the endogenous and the MS error correction models. The estimated transition probability by the endogenous RS-ECM (real line) varies over time since the probability depends upon the previous state (s_{t-1}) as well as the realized value of the lagged stock return Δp_{t-1}. These results are consistent with the study by Chang et al. (2014). On the other hand, the transition probability estimated by the MS-ECM (dotted line) is constant for the entire sample period since the future transition between states is completely determined by the current state and does not depend on the realizations of the underlying time series.

Figure 2 presents the transition probability from low-regime at $t-1$ to high-regime at t estimated by the endogenous and the MS error correction models. For the sample period, this low to high transition probability is estimated to be 4.0% by the MS ECM whereas the estimated probability from the endogenous counterpart is time-varying. For the endogenous RS-ECM, the transition probability exhibits spikes when there is a seriously negative event in the market. Therefore, the transition probability from the endogenous

RS-ECM is more realistic and this feature cannot be accommodated by the MS-ECM. The estimated transition probability from low to high state reaches as high as 99.40% on September 2008 when the Lehman Brothers filed for bankruptcy. This result indicates that the maximum estimated transition probability from low to high regime by the endogenous RS-ECM is 24.85(=99.40/4.00) times larger than its counterpart from the MS-ECM. As illustrated in Figure 2, we similarly demonstrate the same point with the estimated transition probabilities from high state at $t-1$ to low state at t by two models.

We extract the latent factor that determines the states from the endogenous RS-ECM and compare it with NBER-defined recession periods.[4] In the first graph of Figure 4, the extracted latent factor is presented and the shaded areas indicate the high-regime where the latent factor is larger than the threshold value $\hat{\tau} = 2.326$. In the second graph, the stock return series is presented while the shaded areas indicate the high-regime. As shown, stock returns are more volatile and largely negative in the high-regime. Finally, in the third graph, the shades now represent the NBER recession periods during the sample period. We can clearly identify that the high-regime defined by the extracted latent factor more or less coincide with NBER recession periods. As shown in the first graph, the shaded areas other than the NBER recession periods are considered as the financial crisis in the U.S. such as Black Monday (Oct.19th 1987), Asian financial crisis (1997), collapse of Long-Term Capital Management (1998), stock market crash (2002) and debt ceiling crisis (2011). Therefore, our extracted latent factor from endogenous RS-ECM can be used for a potential indicator for recession as well as for financial crisis.

4 Conclusion

We have shown that the error correction model with both time varying coefficient cointegration and endogenous regime switching better explains the long-run and short-run relationships between stock price and dividend than the existing models with linear cointegration or conventional Markov switching. Moreover, the latent factor extracted from our model specifically reveals the periods for each regime. It shows that the periods of high-regime (with high volatility) more or less coincide with the NBER recession periods and

[4]NBER recession dates are available online at www.nber.org.

also contain some periods with financial crisis. Our results could be useful and provide a new approach in investigating whether the dividend-price ratio predicts stock return or not. We leave that for future work.

5 Tables and Figures

Table 1: Unit root test results

		With intercept	With intercept and trend
p_t	AR coefficient	0.999	0.991
	ADF test	−0.611	−1.925
	KPSS test	2.729	0.438
d_t	AR coefficient	0.999	0.996
	ADF test	−0.650	−4.060
	KPSS test	2.747	0.324

Notes: p_t and d_t stand for log stock price and log dividend respectively. The critical values for ADF test are $-3.44(1\%)$, $2.87(5\%)$ and $-2.57(10\%)$ with intercept and $-3.98(1\%)$, $-3.42(5\%)$ and $-3.13(10\%)$ with intercept and linear time trend. The KPSS test has critical values $0.739(1\%)$ with intercept and $0.216(1\%)$ with intercept and linear time trend.

Table 2: Test result for linear cointegration

Augmented Dickey-Fuller Test: Model: $\hat{v}_t = \phi \hat{v}_{t-1} + \xi_t$						
$\hat{\phi}$	se	p-value	t-stat	DW stat	AIC	BIC
0.9898	0.0061	0.0906	-1.6654	1.3973	$-1.8023e+03$	$-1.7981e+03$

Notes: Table 2 represents linear cointegration test result using Engle-Granger two-step method. We estimate the cointegrating equation $p_t = \theta_0 + \theta_1 d_t + v_t$ and conduct the unit root test on the residual. DW stat refers to Durbin Watson statistics. Critical values for ADF test with no intercept (constant) nor time trend are $-2.5672(1\%)$ and $-1.9411(5\%)$.

Table 3: Estimation results of TVC Models

	Model 1		**Model 2**		**Model 3**	
	est.	se	est.	se	est.	se
β_0	2.3695	0.0474	2.4346	0.0297	2.4323	0.0243
$\beta_1 : \frac{t}{T}$						
$\beta_2 : \sin\left(2\pi\frac{t}{T}\right)$	0.0102	0.0680	0.1031	0.0424	0.1030	0.0345
$\beta_3 : \cos\left(2\pi\frac{t}{T}\right)$	−0.2268	0.0630	−0.0933	0.0407	−0.0982	0.0341
$\beta_4 : \sin\left(4\pi\frac{t}{T}\right)$			0.1765	0.0414	0.1793	0.0348
$\beta_5 : \cos\left(4\pi\frac{t}{T}\right)$			−0.0700	0.0389	−0.0746	0.0330
$\beta_6 : \sin\left(6\pi\frac{t}{T}\right)$					−0.0161	0.0338
$\beta_7 : \cos\left(6\pi\frac{t}{T}\right)$					−0.0239	0.0317
BIC:	−559.8		−798.6		−834.9	
	Model 4		**Model 5**		**Model 6**	
	est.	se	est.	se	est.	se
β_0	3.1580	0.0281	3.1470	0.0184	3.3350	0.0100
$\beta_1 : \frac{t}{T}$	−1.2795	0.04750	−1.2459	0.0329	−1.6113	0.0184
$\beta_2 : \sin\left(2\pi\frac{t}{T}\right)$	−0.2200	0.0169	−0.2101	0.0115	−0.3357	0.0063
$\beta_3 : \cos\left(2\pi\frac{t}{T}\right)$	0.0767	0.0119	0.0938	0.0064	0.1055	0.0029
$\beta_4 : \sin\left(4\pi\frac{t}{T}\right)$			−0.0603	0.0069	−0.0160	0.0037
$\beta_5 : \cos\left(4\pi\frac{t}{T}\right)$			0.0488	0.0061	0.0702	0.0029
$\beta_6 : \sin\left(6\pi\frac{t}{T}\right)$					−0.0273	0.0029
$\beta_7 : \cos\left(6\pi\frac{t}{T}\right)$					0.1221	0.0028
BIC :	−1263.2		−1381.2		−1930.8	

Notes: Table 3 includes coefficient estimates and standard error for six TVC models in (10) using canonical cointegrating regression (CCR) estimation method. Bayesian information criterion (BIC) is used to determine the degrees of polynomials and pairs of trigonometric terms of the FFF form approximation.

Table 4: Test for Nonlinear Cointegration

Augmented Dickey-Fuller Test: Model : $\hat{\epsilon}_t = \psi\hat{\epsilon}_{t-1} + \zeta_t$ where $\hat{\epsilon}_t = p_t - \hat{\beta}_t d_t$						
$\hat{\psi}$	se	p-value	t-statistic	DW Stat	AIC	BIC
0.9573	0.0133	0.0016	−3.2217	1.3257	$-1.7853e+03$	$-1.7811e+03$

Notes: Table 4 represents nonlinear cointegration test using Engle-Granger two-step method. It shows the unit root test result on the residual ($\hat{\epsilon}_t$) from CCR estimation of the TVC cointegrating model in (10). Same as Table 2.

Table 5: Estimation result of the linear ECM using the TVC model residual

parameters	est.	se
λ_0	0.0064^{***}	0.0022
λ_1	-0.0170	0.0129
γ	-0.0428	0.2803
σ	0.0371^{***}	0.0012
log-likelihood	920.1522	

Notes: Table 5 reports estimated coefficients from regression of the form (11). The residual ($\hat{\epsilon}_t$) of the TVC cointegrating model in (10) is used as the lagged long-run disequilibrium. *** denotes the level of significance at 1%.

Table 6: Estimation results of the endogenous RS-ECM and the MS-ECM using the TVC model residual

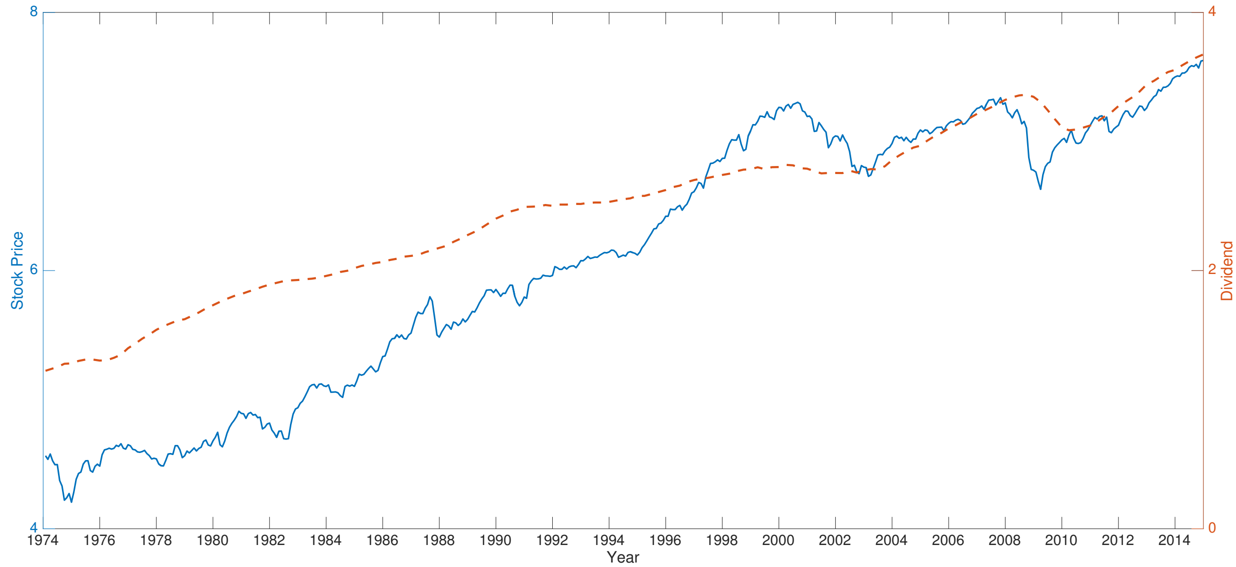
$$\Delta p_t = \lambda_0(s_t) + \lambda_1(s_t)\hat{\epsilon}_{t-1} + \gamma(s_t)\Delta d_t + \sigma(s_t)u_t \text{ where } p_t = \beta_t d_t + \epsilon_t$$

Parameter Estimates of the Regime Switching

	Endogenous RS-ECM		MS-ECM	
parameters	est.	se	est.	se
$\underline{\lambda}_0$	0.0164^{***}	0.0018	0.0178^{***}	0.0021
$\overline{\lambda}_0$	-0.0184^{**}	0.0077	-0.0164^{*}	0.0098
$\underline{\lambda}_1$	-0.0477^{***}	0.0119	-0.0538^{***}	0.0151
$\overline{\lambda}_1$	-0.0814^{**}	0.0327	-0.0263	0.0473
$\underline{\gamma}$	-0.6159^{***}	0.2027	-0.6184^{***}	0.2272
$\overline{\gamma}$	1.1317	1.0199	-0.7438	1.3994
$\underline{\sigma}$	0.0241^{***}	0.0011	0.0244^{***}	0.0001
$\overline{\sigma}$	0.0557^{***}	0.0039	0.0562^{***}	0.0005
α	0.9555^{***}	0.0228	–	–
τ	2.3257^{***}	0.8683	–	–
ρ	-0.9998^{***}	0.0018	–	–
$\mathbb{P}\{s_t = 1 \mid s_{t-1} = 1\}$	time varying		0.85	
$\mathbb{P}\{s_t = 0 \mid s_{t-1} = 0\}$	time varying		0.96	
log-likelihood	990.0463		984.0592	
AIC	$-1.9581e+03$		$-1.9481e+03$	
BIC	$-1.9119e+03$		$-1.9062e+03$	

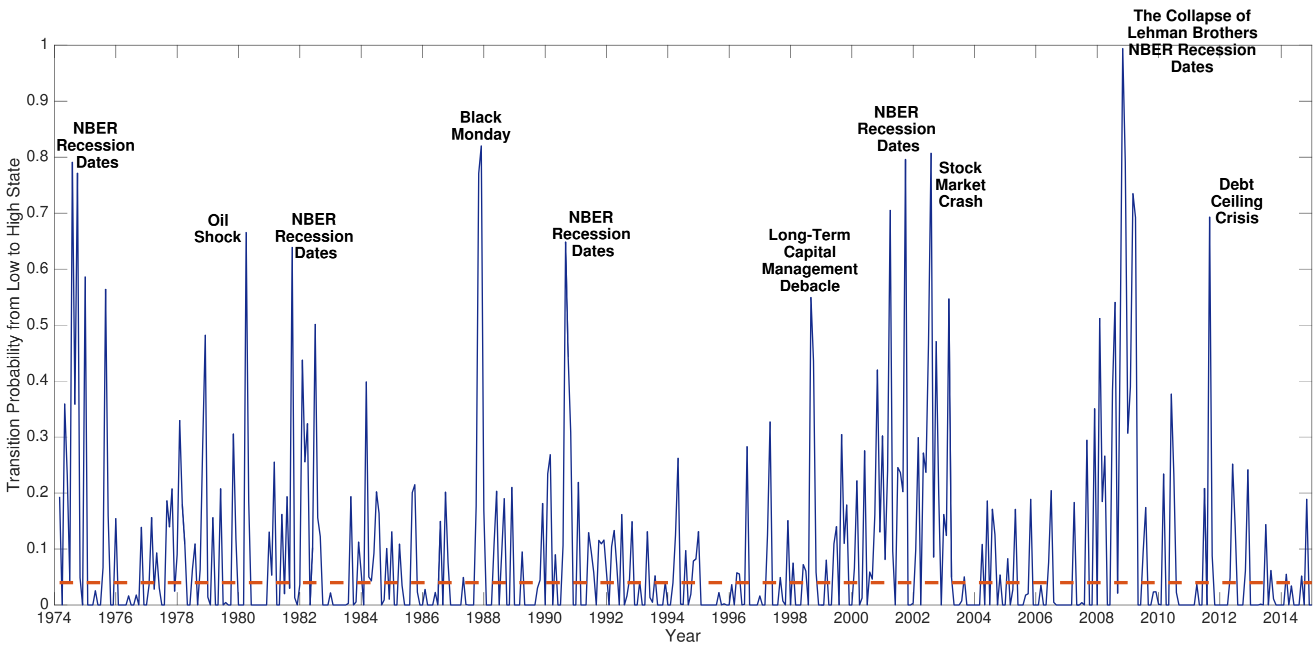
Notes: Table 6 reports the maximum likelihood estimated coefficients for both endogenous RS-ECM and the MS-ECM for the sample period (Jan.1974 - Dec.2014). The residual ($\hat{\epsilon}_t$) of the TVC cointegrating model in (10) is used as the lagged long-run disequilibrium. *** denotes the level of significance at 1%, ** denotes the level of significance at 5% and * denotes the level of significance at 10%.

Figure 1: : Stock Prices and Dividends



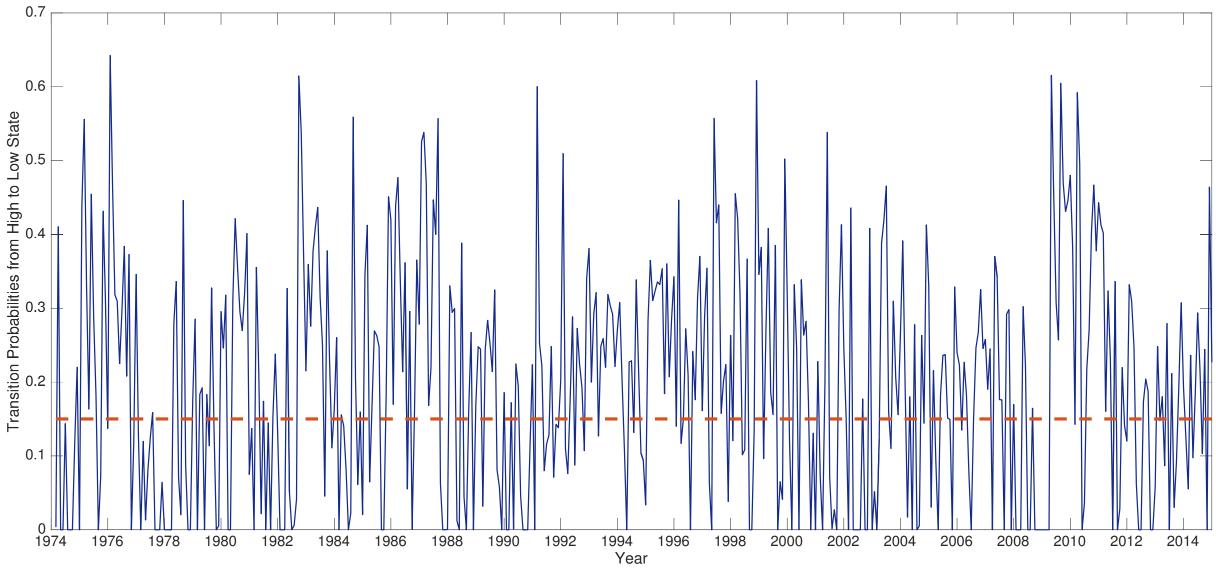
Notes: Figure 1 shows a pair of the log of stock price (solid line) and log of dividend (dashed line) in the United States from January 1974 to December 2014.

Figure 2: : Estimated transition probability from low to high state



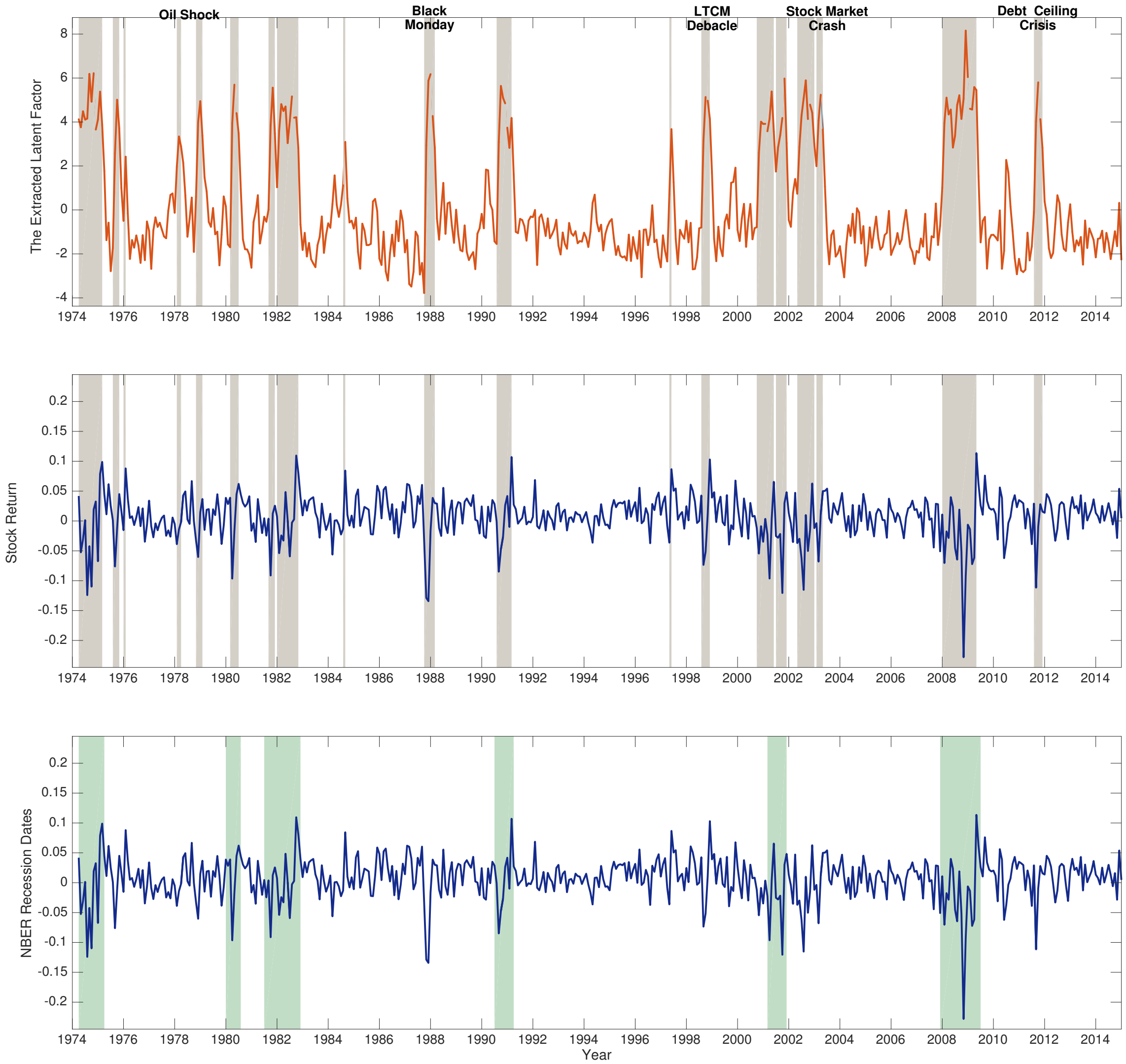
Notes: Figure 2 indicates the transition probability from low state to high state. The solid line refers to $\mathbb{P}(s_t = 1|s_{t-1} = 0)$ in the endogenous RS-ECM while the dashed line represents $\mathbb{P}(s_t = 1|s_{t-1} = 0)$ in the MS-ECM. Several spikes of transition probability from low to high state occur over the sample period. A spike of a large magnitude corresponds to the financial crisis such as oil crises (1973-75, 1979-80), black monday (1987), long-term capital management debacle (1998), stock market crash (2002), debt ceiling crisis (2011) and NBER recession periods.

Figure 3: : Estimated transition probabilities from high to low state



Notes: Figure 3 shows the transition probabilities from high to low state. The solid line indicates $\mathbb{P}(s_t = 1|s_{t-1} = 0)$ in the endogenous RS-ECM, while the dashed line represents $\mathbb{P}(s_t = 1|s_{t-1} = 0)$ in the MS-ECM.

Figure 4: Extracted latent factor



Notes: Figure 4 shows the extracted latent factor, stock return, high regime periods and NBER recession dates during the sample period. In the upper graph, the solid line represents latent factor obtained from endogenous regime switching ECM, while the shaded areas indicate high regime. In the middle graph, the stock return (solid line) and high regime identified from the extracted latent factor are presented. The lower graph of Figure 4 presents the stock return (solid line) and NBER recession dates (shades) during the sample period. Shaded areas (the upper graph) other than the areas corresponding to NBER recession periods (the lower graph) are regarded as the financial crisis including Black Monday (1987), long-term capital management (LTCM) debacle (1998), stock market crash (2002) and debt ceiling crisis (2011).

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