

# Can Model-Consistent Inflation Expectations Explain the Behavior of Inflation Dynamics?

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## ABSTRACT

Sticky price models associated with firms' forward-looking price-setting behavior play a central role in macroeconomic modeling and monetary policy analysis. Rudd and Whelan (2006), however, reject the empirical relevance of forward-looking behavior in accounting for inflation dynamics, based on the results of inflation expectations obtained as forecasts from a VAR model. This paper shows that their results of rejection of sticky price models with substantial reliance on forward-looking behavior are contingent upon the forecasting models for expected inflation. In order to investigate this, we employ a conventional DSGE model as an alternative forecasting model and find that the model-implied inflation expectations have significant explanatory power for inflation dynamics. In addition, this paper makes the following two points that highlight the importance of forward-looking behavior. First, we show that sticky price models with forward-looking behavior can generate the puzzling negative dependence of changes in inflation on its own lag as documented in Rudd and Whelan (2006). Second, we find that the DSGE model fails to replicate the observed dynamic cross-correlation between output gap and inflation, unless it is associated with both forward- and backward-looking price-setting behavior.

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## 1 INTRODUCTION

What is the importance of forward-looking economic agents on inflation dynamics? The macroeconomic literature has made a convincing argument that forward-looking price-setting behavior of firms plays a significant role in accounting for the dynamics of inflation [Gali and Gertler (1999), Sbordone (2002) and Gali et al. (2005), among others]. The empirical relevance of such behavior has typically been analyzed within the framework of the hybrid new Keynesian Phillips Curve (NKPC) model, which posits that current inflation is affected by expected inflation, lagged inflation, and real marginal cost.

Employing a similar framework, however, Rudd and Whelan (2006) arrive at diametrically opposed conclusions. A distinct feature of their empirical procedure is associated with how the expected inflation term in the hybrid NKPC is treated. In their specification, inflation expectations are approximated by the discounted stream of expected future real marginal costs, forecasted by an estimated vector autoregressive (VAR) model.<sup>1</sup> Based on the model-consistent inflation expectations, they reject the empirical importance of forward-looking behavior on inflation dynamics, and find instead evidence in favor of the backward-looking Phillips curve.

This paper aims to reconcile the two sets of conflicting results by asking whether the finding in Rudd and Whelan (2006) is robust across forecasting models of future real marginal costs. In addition to their VAR approach, we consider a new Keynesian Dynamic Stochastic General Equilibrium (DSGE) model as the alternative specification for forecasting future real marginal costs. The analysis allows examination of how particular forecasting models can produce distinct dynamics of the proxy variable for inflation expectations, which alters the empirical relevance of forward-looking behavior.

Our empirical strategy takes the following steps. First, both forecasting models are estimated with quarterly U.S. data on output gap, inflation, and nominal interest rate, from 1960:Q1 to 2012:Q4. Notice that, in the models, marginal cost is approximated by output gap. The choice of the variable is guided by Rudd and Whelan (2007), who argue that output gap is likely to be a better proxy for marginal cost than labor income share as employed in Gali and Gertler (1999).<sup>2</sup> In order to examine robustness of the result to the choice of output gap, we consider various output gap measures widely used in the existing literature. In a second step, we estimate the reduced-form NKPC equations as in Rudd and Whelan (2006) using the generalized method of moments (GMM; for the VAR-based tests) or Bayesian methods (for the DSGE-based tests). Existing work reveals that the performance of the NKPC depends substantially on the presence of lagged inflation [for example, Fuhrer and Moore (1995) and Christiano et al. (2005), among many others]. Accordingly, in our empirical analysis we examine three variants of the NKPC depending upon the degree of indexation to past inflation—the purely forward-looking NKPC, the hybrid NKPC with partial

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<sup>1</sup>Rudd and Whelan (2006) also evaluate sticky price models based on realized values of future real marginal cost using a GMM estimator. Evaluating sticky price models based on realized values of future marginal cost tends to entail an identification problem, which stems from the lack of valid instrument variables for  $k$ -step ahead future marginal cost when  $k$  is large. As Mavroeidis et al. (2014) put, “inflation is notoriously hard to forecast” and “it is hard to find exogenous (i.e., lagged) economic variables that correlate strongly with expected future inflation.” The weak instrument may become particularly severe for the framework in Rudd and Whelan (2006), which is associated with  $k = 13$ . For this reason, we focus on evaluating sticky price models based on output gap as a proxy for real marginal cost rather than its realized values.

<sup>2</sup>Rudd and Whelan (2007) show that, in contrast to output gap, labor income share typically displays a pattern that would be considered countercyclical, spiking up during postwar U.S. recessions.

indexation, and the hybrid NKPC with full indexation. Finally, the contribution of forward-looking behavior to inflation dynamics is evaluated in terms of two criteria as in Rudd and Whelan (2006): sign and statistical significance of the coefficient on the model-consistent inflation expectations, and measures of goodness of fit.

The main finding is that the importance of forward-looking behavior on inflation hinges critically upon the forecasting models. VAR forecasts of future real marginal costs make almost no contribution in explaining inflation dynamics, which is consistent with the finding in Rudd and Whelan (2006). Embedding the VAR-predicted forward-looking component in the reduced-form NKPC regressions hardly improves the data fit, with decreasing adjusted  $R^2$  values. In addition, the sign and statistical significance of the coefficient on the forward-looking component vary widely across the degrees of indexation and output gap measures. In a sharp contrast to the VAR-based results, DSGE forecasts offer significant explanatory power for inflation dynamics as they increase the regressions goodness of fit. The coefficient on the discounted current and expected future output gap has a positive sign, which underscores the role of forward-looking behavior in inflation determination. We find that the DSGE-based results are robust across the degrees of indexation and output gap measures.

We further investigate the source of the discrepancy and show that the dynamics of the sum of the discounted current and future output gap series are strongly conditional on the forecasting models. The primary difference between the VAR- and DSGE-based series stems from the determinant of the forecasts in the models. Structural shocks to demand are the dominant factor for the discounted sum series in the DSGE model, whereas contemporaneous output gap, the interest rate, and inflation jointly play a critical role in determining the VAR-implied series. In particular, our results indicate that a significant negative dependency of the current interest rate and inflation toward the discounted sum of future output gap in the VAR specification substantially determines the model's forecasts. In addition, the VAR-predicted discounted sum of future output gap varies considerably with output gap measures, which can be a potential source of the dispersed VAR-based results across different output gap measures.

Having established our empirical framework, we conduct two additional analyses in assessing the importance of forward-looking behavior. The analyses make the following points:

- Rudd and Whelan (2006) provide supplementary evidence for the rejection of the hybrid NKPC. They document a negative dependence of changes in inflation on its own lag, and interpret the tendency as “an important feature of inflation dynamics that is absent from the hybrid model.” We demonstrate that the hybrid NKPC, with emphasis on the presence of forward-looking behavior, can generate the negative autocorrelation coefficient when it is associated with more than one lagged inflation term. Our analysis suggests that the negative autocorrelation should not be interpreted as evidence against new Keynesian sticky price models as explanations of inflation.
- We explore how the DSGE model's ability to generate the data-consistent correlation between output gap and inflation depends upon the degree of indexation to lagged inflation. A simulation exercise reveals that the backward-looking Phillips curve is unsuccessful in producing the dynamic correlation pattern observed in the data—output gap is correlated negatively with lagged inflation and positively with future inflation. Abstracting from the expectation term makes supply-side shocks the primary determinant of the cross-correlation, which yields consistently negative correlations for the backward-looking Phillips curve. We

find that the full indexation model, having the forward- and backward-looking components with almost equal weight, best matches the dynamic cross-correlation pattern of the data. These results indicate that the model is in need of both forward- and backward-looking components to generate the data-consistent correlation structure.

The remainder of this paper is as follows. Section 2 describes the small-scale DSGE model employed in this article. Section 3 investigates the importance of forward-looking behavior in determining inflation dynamics. Section 4 discusses the implications of alternative specifications of the Phillips curve in order to examine the robustness of our main findings. Section 5 concludes.

## 2 MODEL

We introduce a standard small-scale DSGE model which consists of households, firms, and the monetary authority. The production sector consists of the representative final goods producer and a continuum of intermediate goods producers. The final good  $Y_t$  is produced by combining a continuum of intermediate goods  $\{Y_{i,t}\}_{i \in [0,1]}$  using a production function given by

$$Y_t = \left( \int_0^1 Y_{i,t}^{(\epsilon^p - 1)/\epsilon^p} di \right)^{\epsilon^p / (\epsilon^p - 1)} \quad (1)$$

where the parameter  $\epsilon^p$  is the elasticity of substitution between differentiated goods. The final goods producer confronts perfect competition in the goods market and takes the price of final goods  $P_t$  as given. The cost minimization problem of the final goods producer yields the demand curve for an intermediate goods producer:

$$Y_{i,t+k} = \left( \frac{\prod_{s=1}^k (\pi)^{1-\kappa} (\bar{\pi}_{t+s-1})^\kappa P_{i,t}}{P_{t+k}} \right)^{-\epsilon^p} Y_{t+k} \quad (2)$$

where  $\pi$  denotes steady-state inflation. Following Calvo (1983) and Yun (1996), we assume that a constant fraction,  $1 - \zeta$ , of intermediate goods producers optimally adjust their prices subject to the demand curve (2). The remaining producers that cannot reset their prices update them mechanically by indexing their prices to a weighted average of past inflation and the steady-state inflation,  $\pi^{1-\kappa} \bar{\pi}_{t-1}^\kappa$ , where  $\bar{\pi}_{t-1} = \pi_{t-1}$ .<sup>3</sup> The aggregate price level evolves according to

$$P_t = \left[ (1 - \zeta) \tilde{P}_t^{1-\epsilon^p} + \zeta (\pi^{1-\kappa} \bar{\pi}_{t-1}^\kappa P_{t-1})^{1-\epsilon^p} \right]^{1/(1-\epsilon^p)} \quad (3)$$

where  $P_t$  and  $\tilde{P}_t$  represent the aggregate price level and optimal price set by the intermediate goods producer, respectively.

Intermediate goods are produced using labor,  $N$ , with the production function  $Y_{i,t} = N_{i,t}^{1-\alpha}$ , where  $\alpha \in [0, 1]$ . Log-linearization of the production function integrated over  $i$  delivers  $y_t = (1 - \alpha)n_t$ . Throughout this paper, lower case letters denote the percentage deviation of a variable from its steady-state value.

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<sup>3</sup>We introduce the notation  $\bar{\pi}_{t-1}$  to consider additional lags of inflation in later parts of the article.

The monopolistically competitive intermediate goods producer  $i$  sets  $\tilde{P}_t$  to maximize its profit which is given by

$$E_t \sum_{k=0}^{\infty} (\beta\zeta)^k \frac{\lambda_{t+k}}{\lambda_t} E_t \left\{ \left[ \prod_{s=1}^k (\pi)^{1-\kappa} (\bar{\pi}_{t+s-1})^\kappa \tilde{P}_t - P_{t+k} \exp(\epsilon_t^p) MC_{t+k} \right] Y_{j,t+k}/P_{t+k} \right\} \quad (4)$$

subject to the demand curve (2) as above.  $\lambda_t$  and  $MC_t$  denote the marginal utility of consumption and real marginal cost, respectively.  $\epsilon_t^p$  is an exogenous cost component that is not taken into account in the model and we interpret it as the cost-push shock. The profit-maximizing condition and (3) jointly yield the hybrid NKPC

$$\pi_t - \kappa \bar{\pi}_{t-1} = \beta E_t (\pi_{t+1} - \kappa \bar{\pi}_t) + \eta (mc_t + \epsilon_t^p) \quad (5)$$

where  $\eta \equiv \frac{(1-\zeta)(1-\beta\zeta)(1-\alpha)}{\zeta(1-\alpha+\alpha\epsilon^p)}$  and  $\epsilon_t^p$  follows an *i.i.d.* process with  $\epsilon_t^p \sim N(0, \sigma_{\epsilon^p}^2)$ . The *i.i.d.* assumption is to maintain parsimony of the model, suitable for the study of the importance of firms' forward-looking behavior. It is worth mentioning that, by assuming transitory cost-push shocks, estimation results are more likely to overemphasize the role of backward-looking behavior in accounting for inflation dynamics. This is because both a serially correlated cost-push shock and lagged inflation are internal propagation mechanisms designed to match the inflation persistence in the data. The missing propagation mechanism associated with the *i.i.d.* assumption of the shock process would allocate more weight to lagged inflation in order to fit the data. In this regard, our modeling choice for the shock process may result in a conservative measure for the importance of forward-looking behavior.

Real marginal cost is proportional to the output gap,  $\tilde{y}_t$ , such as

$$mc_t = \left( \frac{\alpha + \varphi}{1 - \alpha} + \sigma \right) \tilde{y}_t. \quad (6)$$

The representative household maximizes the intertemporal utility function given as

$$E_t \sum_{k=0}^{\infty} \beta^k \left( \frac{C_{t+k}^{1-\sigma}}{1-\sigma} - \Psi \frac{N_{t+k}^{1+\varphi}}{1+\varphi} \right) \quad (7)$$

where  $C_t$  and  $N_t$  denote consumption and the amount of labor, respectively. The objective of the household is to maximize the intertemporal utility function subject to the budget constraint

$$P_t C_t + \frac{B_t}{\exp(-v_t^d)(1+i_t)} = W_t N_t + B_{t-1} + \Pi_t \quad (8)$$

where  $B_t$ ,  $i_t$ ,  $-v_t^d$ ,  $W_t$ , and  $\Pi_t$  represent the amount of riskless bond purchased, interest rate, risk premium shock, nominal wage rate, and firm's profit, respectively.  $v_t^d$  is interpreted as the demand shock and is assumed to follow an AR(1) process,  $v_t^d = \rho_v v_{t-1}^d + \epsilon_t^d$  with  $\epsilon_t^d \sim N(0, \sigma_{\epsilon^d}^2)$ . The household's maximization problem yields the IS curve given as:

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1}) + \frac{1}{\sigma} v_t^d \quad (9)$$

where  $i_t$  denotes the short-term interest rate set by the Federal Reserve. Since technology shocks are abstracted from the model, the output gap is equal to  $y_t$  in this paper.<sup>4</sup>

We close the model by introducing a monetary policy rule, in which the nominal interest rate  $i_t$  responds to its lagged value, the current inflation rate and current output gap, and a difference in the output gap:

$$i_t = \rho i_{t-1} + (1 - \rho)(a_\pi \pi_t + a_y \tilde{y}_t) + a_{\Delta y} \Delta \tilde{y}_t + \epsilon_t^m \quad (10)$$

where  $\epsilon_t^m$  denotes a monetary policy shock, which is assumed to follow an *i.i.d.* process with  $\epsilon_t^m \sim N(0, \sigma_{\epsilon^m}^2)$ .

### 3 EMPIRICAL RESULTS

The hybrid NKPC in (5) displays that current inflation is determined by inflation expectations, lagged inflation, and real marginal cost. If the discount factor,  $\beta$ , is assumed to be unity, the hybrid NKPC can be written as

$$\pi_t = \theta E_t \pi_{t+1} + (1 - \theta) \pi_{t-1} + \gamma (mc_t + \epsilon_t^p) \quad (11)$$

where  $\theta \equiv 1/(1 + \kappa)$  and  $\gamma \equiv \eta/(1 + \kappa) = \eta\theta$ . This equation nests various versions of NKPCs, which are distinguishable based upon the values of  $\kappa$ . When  $\kappa = 0$  and  $\kappa = 1$ , Equation (11) collapses into the forward-looking NKPC and full indexation model employed by Christiano et al. (2005), respectively. The equation also includes the partial indexation model appeared in Smets and Wouters (2007) if  $\kappa \in (0, 1)$ . Using the framework of Rudd and Whelan (2006), this section evaluates the importance of the forward-looking component in accounting for inflation dynamics across the three different specifications of the Phillips curve.

**3.1 CASE 1: NKPC ( $\kappa = 0$ )** Assuming inflation expectations are formulated in a rational, model-consistent way yields the forward-looking NKPC expressed as

$$\pi_t = \gamma \sum_{k=0}^{\infty} E_t mc_{t+k} + \gamma \epsilon_t^p \quad (12)$$

As a consequence, model-consistent inflation expectations are defined as  $E_t \pi_{t+1} = \sum_{k=1}^{\infty} E_t mc_{t+k}$ . Equation (12) makes precise that current inflation in forward-looking models is mainly determined by expected future marginal costs. In contrast, current inflation is primarily driven by past marginal costs in backward-looking models. This fundamental difference between the two modeling approaches has motivated researchers to test the validity of the forward-looking NKPC using (12).

Plugging (6) into (12) yields

$$\pi_t = \gamma X_t^{DSGE} + \gamma \epsilon_t^p \quad (13)$$

where  $X_t^{DSGE} \equiv \left(\frac{\alpha + \varphi}{1 - \alpha} + \sigma\right) \sum_{k=0}^{\infty} E_t^{DSGE} \tilde{y}_{t+k}$  and  $E_t^{DSGE} \tilde{y}_{t+k}$  denotes the predicted value of  $\tilde{y}_{t+k}$  from the DSGE model conditional on date  $t$  information. The regression equation corresponding to VAR forecasts is given by

$$\pi_t = \tilde{\gamma} X_t^{VAR} + \gamma \epsilon_t^p \quad (14)$$

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<sup>4</sup>A similar modeling approach is taken by Giannoni and Woodford (2003) and Dennis (2009).

where  $\tilde{\gamma} \equiv \left(\frac{\alpha+\varphi}{1-\alpha} + \sigma\right) \gamma$  and  $X_t^{VAR} \equiv \sum_{k=0}^{\infty} E_t^{VAR} \tilde{\gamma}_{t+k}$ .  $E_t^{VAR} \tilde{\gamma}_{t+k}$  denotes the expected value of  $\tilde{\gamma}_{t+k}$  by a VAR model in period  $t$ . Following Fuhrer and Moore (1995), Rotemberg and Woodford (1997), Rudd and Whelan (2006), and many others, we employ a trivariate VAR(2) model containing inflation, the short-term nominal interest rate and output gap.

For each forecasting model, either the DSGE or VAR, we use the quarterly U.S. data ranged from 1960:Q1 to 2012:Q4. The short-term interest rate uses the effective Federal Funds rate, while inflation rate is measured by changes in the GDP deflator. Regarding the output gap, we consider four different output gap measures for robustness. They are the Congressional Budget Office (CBO)'s output gap series, quadratic detrended output, HP-filtered (two-sided) output, and Christiano-Fitzgerald (2003, one-sided) filtered output. Hence, the estimation results for the regression equations (13) and (14) unveil whether the forward-looking model provides a good approximation of U.S. inflation dynamics regardless of forecasting models of the output gap.<sup>5</sup>

Two steps are required to assess the validity of the NKPC. First, future output gaps are forecasted based on either the DSGE model or the VAR(2) model in order to compute model-consistent inflation expectations. Then we estimate the parameters of interest in (14) using the generalized method of moments (GMM) technique. Notice that  $\tilde{\gamma}$ , instead of  $\gamma$ , is estimated for the VAR-forecasted regression test, as in Rudd and Whelan (2006). This is because the DSGE model allows for the identification of the structural parameters, while the VAR model does not.

For the second step of the DSGE-forecasted test in (13), it is worthwhile to mention that the procedure can be simplified by embedding an equation governing the evolution of  $X_t^{DSGE}$  as

$$X_t^{DSGE} = E_t X_{t+1}^{DSGE} + \left(\frac{\alpha + \varphi}{1 - \alpha} + \sigma\right) \tilde{\gamma}_t \quad (15)$$

This allows us to directly estimate the parameter  $\gamma$  by calculating  $X_t^{DSGE}$ . Although not presented here, we find that the posterior estimates for the DSGE model are not sensitive to the inclusion of (15) in estimating the model. Therefore, we estimate the DSGE model with an imposition of the equation to gauge the dynamics of  $X_t^{DSGE}$ .<sup>6</sup>

Figure 1 displays the actual time series for the four output gap measures (top panel), as well as the DSGE- (middle panel) and VAR-predicted (bottom panel) discounted sum of expected future output gap. The middle panel shows that the DSGE-implied series tend to comove with the output gap measures, with no significant dependency on the choice of output gap. The dynamics of model-implied forecasts, however, change dramatically under the VAR model. As the bottom panel exhibits, the VAR-implied output gap series depend remarkably on which output gap is used.

<sup>5</sup>Appendix A provides a detailed description of the data.

<sup>6</sup>The prominent benefit of augmenting (15) in the DSGE estimation procedure is that it permits to avoid the difficulty of compounding the infinite number of future output gap terms, associated with the DSGE-based test. There are two GMM-based approaches to estimate the close form solution of the NKPC (12). The first approach is to estimate the closed form solution using realized values of future output gap and the second is to use forecasts of future output gap to calculate  $X_t$ . Then the equation (12) can be estimated using the GMM estimator. In contrast to the latter, the former has a difficulty in dealing with the infinite number of the future output gap terms. If realized values of one to twelve quarter ahead output gaps are used to estimate (12) as in Equation (18) of Rudd and Whelan (2006), the GMM estimates are likely to be subject to weak instrument problems since  $k$ -quarter ahead output gaps and changes in  $(k+1)$ -quarter ahead inflation are weakly correlated with the instrument variables when  $k$  is large. In this regard, the GMM approach based on VAR forecasts, instead of realized values, has some advantages in dealing with the infinite number of the future output gap terms as well as in avoiding weak instrument problems. The DSGE approach also allows us to avoid the difficulty of computing the infinite number of the future output gap terms.

A comparison of the predicted series to the output gap measures reveals that the behavior of the VAR-implied forecasts differs substantially from that of the actual output gap measures.

In order to rationalize the source of the discrepancy, we demonstrate the estimation results on how  $X_t$  is predicted in each modeling approach as follows:

$$\begin{pmatrix} X_t^{DSGE:CBO} \\ X_t^{DSGE:Detrended} \\ X_t^{DSGE:HP} \\ X_t^{DSGE:CF} \end{pmatrix} = \begin{pmatrix} 185.8 & -17.5 & -5.9 & 17.2 & -13.7 & 177.0 \\ 210.6 & -21.3 & -5.7 & 23.9 & -17.5 & 200.6 \\ 223.8 & -13.8 & -4.0 & 11.2 & -10.8 & 223.8 \\ 178.8 & -11.3 & -4.0 & 10.9 & -8.4 & 170.4 \end{pmatrix} \begin{pmatrix} \epsilon_t^d \\ \epsilon_t^i \\ \epsilon_t^p \\ \tilde{y}_{t-1} \\ i_{t-1} \\ v_{t-1}^d \end{pmatrix} \quad (16)$$

$$\begin{pmatrix} X_t^{VAR:CBO} \\ X_t^{VAR:Detrended} \\ X_t^{VAR:HP} \\ X_t^{VAR:CF} \end{pmatrix} = \begin{pmatrix} 10.8 & -5.9 & -7.4 & -1.5 & 1.4 & -3.0 \\ 11.4 & -8.0 & -7.9 & -1.1 & 1.8 & -3.3 \\ 3.6 & -1.7 & -1.3 & -0.5 & -0.2 & -0.6 \\ 7.1 & -0.4 & -0.2 & -6.4 & 0.2 & -0.1 \end{pmatrix} \begin{pmatrix} \tilde{y}_t \\ i_t \\ \pi_t \\ \tilde{y}_{t-1} \\ i_{t-1} \\ \pi_{t-1} \end{pmatrix} \quad (17)$$

As shown in (16), the structural shocks play a key role in determining the DSGE-implied series for  $X_t$ , regardless of the output gap measures. In particular, demand shocks are the most significant driver of the DSGE-implied series. Given that the demand shock substantially governs the output dynamics in the model via the IS curve in (9), this result explains how the DSGE-implied series closely mimic the corresponding output gap measure. In contrast to the DSGE approach, the VAR-predicted  $X_t$ 's are driven solely by the current and lagged values of the key model variables. The results in (17) indicate that  $X_t^{VAR}$ 's are positively related to the current output gap, whereas they are negatively related to the interest rate and inflation. This systematic pattern in the variables has important implications for the evolution of the VAR-implied  $X_t$  series. A notable empirical consequence is that the trajectory of  $X_t^{VAR}$  exhibits a mirror image of the federal funds rate and inflation, as displayed in the bottom panel of Figure 1. Notice that a similar pattern is documented in Rudd and Whelan (2003). Of course, the DSGE-based forecasts for  $X_t$  are also negatively related to the interest rate and inflation. The estimation results in (16), however, indicate that demand shocks play a dominant role in determining the expected future output gap, while the contributions of interest rate and cost-push shocks are relatively minor. The VAR-implied series are governed not only by the output gap, but also by the interest rate and inflation. In a sharp contrast to the DSGE-based results, the importance of the contemporaneous interest rate and inflation on  $X_t$  is somewhat comparable to that of output gap, as displayed in (17).

Table 1 reports the estimates for  $\gamma$  and  $\tilde{\gamma}$  in the regression equations (13) and (14), respectively.<sup>7</sup>  $X_{t-1}^{VAR}$  is used as an instrument for the GMM estimates for  $\tilde{\gamma}$ , whereas the estimation of  $\gamma$  employs Bayesian methods with the prior distribution of  $N(0.001, 0.005)$ .<sup>8</sup> The table makes clear

<sup>7</sup>In order to be consistent with the framework by Rudd and Whelan (2006), we focus on estimating the reduced-form parameters,  $\gamma$  and  $\tilde{\gamma}$ , instead of all the structural parameters in the DSGE model. The companion appendix includes details of the distribution of the posterior estimates for the DSGE model.

<sup>8</sup>We use the random walk Metropolis-Hastings (HM) algorithm to simulate 50,000 posterior draws with the first 25,000 used as a burn-in. This procedure is applied to the DSGE-based regressions throughout the article. Our estimation results based on VAR forecasts remain almost unaltered by the inclusion of lagged output gap and inflation

that the estimated coefficients are conditional on a specific forecasting model for expected future output gap. The results based on the VAR forecasts tend to reject the forward-looking NKPC as the coefficients of the VAR-based regressions are signed negative. This result is insensitive to the measure of output gap. In contrast, the estimates flip sign with the use of the DSGE-implied forecasts. Regardless of the output gap measures, the coefficients on  $X_t$  are positive and statistically significant at 5% level, indicating that the forward-looking NKPC accounts for the dynamics of inflation.

Another perspective on how the presence of forward-looking behavior affects a model's potential fit to data can be gleaned from a comparison between the actual and model-consistent inflation dynamics. The top panel of Figure 2 plots actual inflation and the DSGE model-predicted inflation associated with various output gap measures. In spite of the relatively large deviations in the 1970s and late 1990s, the model-predicted inflation tracks the ups and downs of actual inflation quite closely. By contrast, the bottom panel of Figure 2 makes an explicit comparison between actual inflation and the predicted inflation emerged from a simple regression using only current output gap as a regressor. Compared to the DSGE-consistent predicted values, the alternative specification underperforms in tracing the actual inflation dynamics. The primary difference across the two groups of model-predicted inflation is the presence of forward-looking behavior, so model fit corresponds to the contribution of model-consistent inflation expectations,  $\sum_{k=1}^{\infty} E_t^{DSGE} \tilde{y}_{t+k}$ , in accounting for inflation dynamics. Our results indicate that firms' forward-looking behavior is likely to be a significant determinant of inflation.

**3.2 CASE 2: HYBRID NKPC WITH PARTIAL INDEXATION ( $\kappa \in (0, 1)$ )** A series of papers demonstrates that in new Keynesian models it can be misleading to set aside lagged inflation [for example, Galí and Gertler (1999) and Christiano et al. (2005)]. In this regard, the NKPC is often criticized for the lack of ability to generate data-consistent inflation persistence and the delayed response of inflation to a monetary policy shock. Guided by the well-established argument, we further investigate the empirical importance of forward-looking behavior under the hybrid NKPC.

Following Smets and Wouters (2007), the NKPC is modified to include a lagged inflation term by assuming that a fraction of firms  $\zeta$ , who are unable to optimally adjust their prices, partially index them to past inflation. The resulting inflation dynamics in the partial indexation model (i.e.,  $\kappa \in (0, 1)$ ) take a form of

$$\pi_t = \theta E_t \pi_{t+1} + (1 - \theta) \pi_{t-1} + \gamma (mc_t + \epsilon_t^p) \quad (18)$$

where  $\theta = 1/(1 + \kappa) > 1/2$ . Solving forward (18) iteratively yields

$$(1 - \kappa L) \pi_t = \eta \sum_{k=0}^{\infty} E_t mc_{t+k} + \eta \epsilon_t^p \quad (19)$$

where  $L$  is the lag lag operator (i.e.,  $L^s x_t = x_{t-s}$ ) and  $\eta = \gamma/\theta$ . We then estimate

$$\pi_t - \frac{1 - \theta}{\theta} \pi_{t-1} = \frac{\gamma}{\theta} X_t^{DSGE} + \eta \epsilon_t^p \quad (20)$$

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as additional instruments. The explanatory power of  $X_{t-1}^{VAR}$  for  $X_t^{VAR}$  is quite high: adjusted  $R^2$  associated with the CBO output gap is 0.94, while a similar degree of  $R^2$  is obtained with the other output gap measures.

and

$$\pi_t - \frac{1-\theta}{\theta}\pi_{t-1} = \frac{\tilde{\gamma}}{\theta}X_t^{VAR} + \eta\epsilon_t^p \quad (21)$$

to examine whether the stream of current and expected future values of the output gap has potential explanatory power for U.S. inflation dynamics.

Table 2 reports the estimation results for the regression equations (20) and (21), respectively. The parameter  $\theta$  is estimated to be about 0.53, depending neither on the forecasting methods nor on the output gap measures. This indicates that both forward- and backward-looking components are equally important in capturing inflation dynamics. The estimates for the slope of the hybrid NKPC, however, vary widely across the forecasting models for expected future output gap. The coefficient  $\gamma$  is signed positive and significantly different from zero, when the regression is accompanied by  $X_t^{DSGE}$ . This finding is insensitive to a specific choice of the output gap measure. These results show that the hybrid NKPC with partial indexation assumption is consistent with the data. On the contrary, the estimates for  $\tilde{\gamma}$  are statistically insignificant, confirming the evidence against the hypothesis of rational and model-consistent inflation expectations as documented in Rudd and Whelan (2006). In sum, our main results from the NKPC carry over to the hybrid NKPC with partial indexation:  $X_t^{DSGE}$  conveys relevant information in understanding inflation dynamics, whereas  $X_t^{VAR}$  does not.

**3.3 CASE 3: HYBRID NKPC WITH FULL INDEXATION ( $\kappa = 1$ )** The hybrid NKPC proposed by Fuhrer and Moore (1995) has equal weight on inflation expectations and lagged inflation. A similar full indexation model is employed by Christiano et al. (2005). When  $\kappa = 1$  (i.e.,  $\theta = 1/2$ ), the equation for the model-implied inflation dynamics in (19) collapses into

$$(1-L)\pi_t = \eta \sum_{k=0}^{\infty} E_t m c_{t+k} + \eta\epsilon_t^p \quad (22)$$

where  $\eta = \gamma/\theta = 2\gamma$ . The regression equations emerged from (22) are given as

$$\Delta\pi_t = \frac{\gamma}{1/2}X_t^{DSGE} + \eta\epsilon_t^p. \quad (23)$$

and

$$\Delta\pi_t = \frac{\tilde{\gamma}}{1/2}X_t^{VAR} + \eta\epsilon_t^p \quad (24)$$

Notice that Rudd and Whelan (2006) employ the setup in (24) associated with full instead of partial indexation model as their baseline specification. A comparison between (21) and (24) makes precise the primary advantage of full over partial indexation models in exploring the contribution of the forward-looking component to the dynamics of inflation. In partial indexation models, current inflation  $\pi_t$  is governed not only by the forward-looking term  $X_t$  but also by its lagged value  $\pi_{t-1}$ , which makes it hard to isolate the individual effects of  $X_t$ . In contrast, full indexation models imply that changes in inflation  $\Delta\pi_t$  depend only on the forward-looking component as long as the hybrid NKPC is endowed with the first lagged inflation.<sup>9</sup>

<sup>9</sup>Once the model is modified to allow for more lags, however, changes in inflation are dependent upon its own lags as well.

As summarized in Table 3, the coefficient pattern across the modeling approaches is somewhat different from the previous two cases. A notable finding is that the estimates for  $\tilde{\gamma}$  are now signed positive and statistically significant. However, the VAR-predicted forward-looking component turns out to have no contribution in fitting the data. As shown in the second panel (specification 2) of Table 4, the explanatory power of  $X_t^{VAR}$  for changes in inflation is consistently negative, with the adjusted  $R^2$  ranged from  $-0.003$  to  $-0.037$  across the output gap measures. These results are coherent with the finding of Rudd and Whelan (2006) that “expectations of future output gaps do nothing to improve the equation’s fit.”

When  $X_t^{DSGE}$  is considered, the  $\gamma$  coefficient is estimated to be positively different from zero regardless of the output gap measures. More importantly, the contribution of the forward-looking component for the model’s fit is reversed, as in the second panel of Table 4. Incorporating  $X_t^{DSGE}$  improves the data fit as the DSGE-based forecasts explain about 4.5 to 6 percent of the variability of changes in inflation. Despite the relatively low goodness-of-fit, we find that the DSGE-implied series plays a crucial role for the evolution of inflation behavior.<sup>10</sup> In order to explore the importance of the forward-looking component in accounting for inflation dynamics more formally, the next section provides supplementary evidence by simulation exercises.

### 3.4 THE FORWARD-LOOKING COMPONENT AND DYNAMIC CROSS-CORRELATION BETWEEN THE OUTPUT GAP AND INFLATION

Based on the DSGE model presented in Section 2, this section conducts a simulation exercise to assess how the model’s ability to generate data-consistent dynamic correlations between the output gap and inflation changes across various degrees of indexation  $\theta$ ’s. Since  $\theta$  governs the weight between forward- and backward-looking behavior on inflation, the exercise is designed to highlight the importance of forward-looking price-setting behavior, conditioning on the DSGE model. To this end, we simulate the output and inflation variables by feeding sequences of all the model shock innovations into its equilibrium system, with  $\theta$  values from zero to one. The CBO output gap is used for estimation.<sup>11</sup> We then calculate the dynamic cross-correlations between the model-implied output and inflation for each  $\theta$  value considered.

The left panel of Figure 3 displays the actual dynamic cross-correlations between the output gap measures and inflation. A positive change in the current output gap is associated with a subsequent gradual increase in inflation, as the cross-correlation is peaked at about 4 to 6 lags. In the existing literature, this is often interpreted as evidence supporting the presence of backward-looking behavior, in which current inflation is determined by current and past output gaps.

The right panel of Figure 3 makes clear that the model-implied dynamic cross-correlations hinge significantly on the degree of indexation. When  $\theta$  is zero, the dynamic correlation is negative, indicating that supply-side shocks are the dominant factor determining the cross-correlation. Our result suggests that the hybrid NKPC with heavy emphasis on the backward-looking behavior is hardly supported by the data.

We find that the NKPC ( $\theta = 1$ ) is also unlikely to produce the correlation structure between the output gap and inflation observed in the data. The dynamic correlation is unrealistically high, caused by the expectation channel that demand-side shocks create. In response to a persistent

<sup>10</sup>A prominent explanation for the relatively poor performance of  $X_t^{DSGE}$  on  $\Delta\pi_t$  is their distinctive frequency characteristics. While higher frequency components stand out for  $\Delta\pi_t$ ,  $X_t^{DSGE}$  (or  $\hat{y}_t$ ) displays persistence with more of the spectral power at low frequencies.

<sup>11</sup>Although not presented here, we find that the results are robust across the output gap measures.

positive shock on the demand side, firms expect the output gap to be positive for several periods. The expectational effect produces an additional propagation of the shock so that current inflation rises more than the actual size of the shock because inflation is determined dominantly by current and expected future output gaps when  $\theta$  is close to one.<sup>12</sup> The exceptionally high cross-correlations between the output gap and inflation are likely to be amplified by the effect. For example, the model-implied contemporaneous correlation is around 0.5, whereas the values from the data are around 0.1 regardless of output gap measures.

The model best matches the cross-correlations of the data when  $\theta$  is 0.499, which is in favor of the specifications in Fuhrer and Moore (1995) and Christiano et al. (2005). This indicates that the model is in need of both forward- and backward-looking components to generate the data-consistent correlation structure.

## 4 ROBUSTNESS: AUGMENTING AN ADDITIONAL INFLATION LAG

In this section we discuss the implications of alternative specifications of the Phillips curve in order to judge the robustness of the main results above. In particular, we examine the partial and full indexation models embedded with an additional lagged inflation term.

**4.1 PARTIAL INDEXATION ( $\kappa \in (0, 1)$ )** We assume firms that are unable to optimally adjust their prices automatically index their prices to the weighted average of the steady-state inflation rate and two lags of inflation. Accordingly, we replace  $\bar{\pi}_{t-1} = \pi_{t-1}$  with  $\bar{\pi}_{t-1} = \omega_1\pi_{t-1} + \omega_2\pi_{t-2}$  where  $\omega_1 + \omega_2 = 1$  and  $0 \leq \omega_1, \omega_2 \leq 1$ . The resulting Phillips curve can be written as

$$\pi_t = \theta E_t \pi_{t+1} + \theta_1 \pi_{t-1} + \theta_2 \pi_{t-2} + \gamma (mc_t + \epsilon_t^p) \quad (25)$$

where  $\theta \equiv \frac{1}{1+\kappa\omega_1}$ ,  $\theta_1 \equiv \frac{\kappa}{1+\kappa\omega_1}(2\omega_1 - 1)$ ,  $\theta_2 \equiv \frac{\kappa(1-\omega_1)}{1+\kappa\omega_1}$ , and  $\gamma \equiv \eta\theta$ . Notice that  $\theta + \theta_1 + \theta_2 = 1$  holds. Solving forward (25) iteratively yields

$$\pi_t - (\lambda_1 \pi_{t-1} + \lambda_2 \pi_{t-2}) = \eta X_t + \eta \epsilon_t^p \quad (26)$$

where  $\lambda_1 \equiv \frac{1-\theta}{\theta\omega_1}\omega_1$  and  $\lambda_2 \equiv \frac{1-\theta}{\theta\omega_1}\omega_2$ . By construction, the model collapses to the hybrid NKPC with partial indexation when  $\omega_2 = 0$ . We estimate

$$\pi_t - (\lambda_1 \pi_{t-1} + \lambda_2 \pi_{t-2}) = \eta X_t^{DSGE} + \eta \epsilon_t^p \quad (27)$$

and

$$\pi_t - (\lambda_1 \pi_{t-1} + \lambda_2 \pi_{t-2}) = \tilde{\eta} X_t^{VAR} + \eta \epsilon_t^p \quad (28)$$

where  $\tilde{\eta} \equiv \tilde{\gamma}/\theta$ .

Table 5 reports the estimation results for the regression equations (27) and (28). Overall, the modification of the model has virtually no effect on our main results regarding the DSGE-implied series. The weight on the forward-looking component,  $\theta$ , is estimated to be around 0.6. These estimates are somewhat larger than the values reported in Section 3.2, indicating that forward-looking behavior is given relatively more weight when the additional inflation lag is included in

<sup>12</sup>In contrast, when  $\theta = 0$ , the expectation channel is shut down and demand shocks play a minor role in determining inflation dynamics.

the Phillips curve. Notice that the significantly positive estimates for  $\gamma$  and  $\theta$  jointly ascribe a significant role in inflation fluctuations to the forward-looking price-setting behavior of firms. The posterior mean estimates for  $\omega_1$  are around 0.63, which illustrates that the nonoptimizing firms index their prices to recent inflation more than to older inflation. Finally, the  $\kappa$  values, reconstructed from the estimates for  $\theta$  and  $\omega_1$  by using  $\kappa = \frac{1-\theta}{\theta\omega_1}$ , are around 0.94.

Unlike the model with no additional inflation lag presented in Section 3.2, the VAR-based results are broadly consistent with the DSGE-based counterpart. The estimates for  $\theta$  are around 0.6. The estimates for  $\omega_1$  are ranged around 0.67, which is slightly higher than the DSGE-based results. Interestingly, however, the results on the  $\tilde{\gamma}$  coefficients hinge critically upon the output gap measures. The parameter estimates are positive and statistically significant under the CBO, detrended, and CF-filtered output gap measures, whereas the coefficient is estimated to be positive but insignificant when associated with the HP-filtered series. This finding indicates a potential vulnerability associated with the VAR-based tests that examine the importance of the forward-looking component for inflation dynamics.

**4.2 FULL INDEXATION ( $\kappa = 1$ )** Rudd and Whelan (2006) document a negative dependence of changes in inflation on its own lag. They describe the tendency as “an important feature of inflation dynamics that is absent from the hybrid model,” and regard it as a rationale for the rejection of new Keynesian sticky price models. In this section, we establish a hybrid NKPC with full indexation ( $\kappa = 1$ ) augmented with an additional lag of inflation, in order to investigate whether the model can be reconciled with the negative dependence borne out by the data.

We consider the augmented full indexation ( $\kappa = 1$ ) model given by

$$\pi_t = \theta E_t \pi_{t+1} + \theta_1 \pi_{t-1} + \theta_2 \pi_{t-2} + \gamma (mc_t + \epsilon_t^p) \quad (29)$$

where  $\theta \equiv \frac{1}{1+\omega_1}$ ,  $\theta_1 \equiv \frac{2\omega_1-1}{1+\omega_1}$ ,  $\theta_2 \equiv \frac{1-\omega_1}{1+\omega_1}$ , and  $\gamma \equiv \frac{\eta}{1+\omega_1}$ . Notice that  $\theta + \theta_1 + \theta_2 = 1$  holds. If  $\omega_1 = 1$ , the model collapses into the hybrid NKPC with a lag of inflation. Equation (29) can be derived from

$$\pi_t - \bar{\pi}_{t-1} = E_t(\pi_{t+1} - \bar{\pi}_t) + \eta(mc_t + \epsilon_t^p) \quad (30)$$

where  $\bar{\pi}_{t-1} = \omega_1 \pi_{t-1} + \omega_2 \pi_{t-2}$  and  $\omega_1 + \omega_2 = 1$ . Iterating (30) forward yields

$$\pi_t - \omega_1 \pi_{t-1} - (1 - \omega_1) \pi_{t-2} = \eta X_t + \eta \epsilon_t^p \quad (31)$$

Rearranging (31) delivers the dynamics of the first difference of inflation as follows

$$\Delta \pi_t = (\omega_1 - 1) \Delta \pi_{t-1} + \eta X_t + \eta \epsilon_t^p \quad (32)$$

By using  $\omega_1 = \frac{1-\theta}{\theta}$ , the resulting Phillips curve can be written as

$$\Delta \pi_t = \mu \Delta \pi_{t-1} + \eta X_t + \eta \epsilon_t^p \quad (33)$$

where  $\mu \equiv -(\frac{2\theta-1}{\theta})$ ,  $\eta = \frac{\gamma}{\theta}$ , and  $X_t = \sum_{k=0}^{\infty} E_t mc_{t+k}$ . A notable feature of the Phillips curve in (33) is that the relative importance between forward- and backward-looking behavior is associated with the sign of the parameter  $\mu$ : it is negative if forward-looking behavior is relatively more important than backward-looking behavior in price setting, and vice versa.

We test whether the first difference of inflation negatively depends on its own lag by estimating

$$\Delta \pi_t = \mu \Delta \pi_{t-1} + \eta X_t^{DSGE} + \eta \epsilon_t^p \quad (34)$$

and

$$\Delta\pi_t = \mu\Delta\pi_{t-1} + \tilde{\eta}X_t^{VAR} + \eta\epsilon_t^p \quad (35)$$

where  $\eta = \frac{\gamma}{\theta}$  and  $\tilde{\eta} = \frac{\tilde{\gamma}}{\theta}$ .<sup>13</sup>

Table 6 summarizes the parameter estimates in Equations (34) and (35). We find that the coefficients on  $X_t^{DSGE}$  are statistically different from zero as the 95th percentile intervals for  $\gamma$  do not include zero for all the output gap measures. The mean estimates for  $\theta$  are around 0.6 with the 95th percentile intervals exceeding 0.5. This underscores the importance of forward-looking behavior on the inflation dynamics.<sup>14</sup> Consequently, the estimates for  $\mu$  are consistently negative and statistically different from zero.

Figure 4 details the finding by plotting the prior and posterior distributions of  $\mu$  and  $\eta$  in Equation (34). Above all, the data seems to be informative in identifying the both parameters as the comparison of the prior to posterior distributions reveals. The posterior distributions of  $\mu$  are clearly below zero, rejecting the hybrid NKPC that attributes the dominant driving force of inflation to backward-looking behavior. The figure also makes clear that the posterior distributions of  $\eta$  exceed zero, which tells the same story regarding the empirical importance of forward-looking behavior. Notice that these results are not sensitive to the output gap measures.

Having established the estimation results, our finding demonstrates that the hybrid NKPC, with emphasis on the presence of forward-looking behavior, can generate the negative coefficient when it is associated with more than one lagged inflation term. Thus, the negative autocorrelation should not be interpreted as evidence against new Keynesian sticky price models as explanations of inflation.

Tuning to the VAR-based results, the estimates for  $\tilde{\eta}$  and  $\tilde{\gamma}$  are also positive and statistically significant. The coefficient  $\theta$  is estimated to be around 0.6, implying that the reduced form parameter  $\mu$  is negative. Despite the highly significant and positive coefficient estimates, the explanatory power of the regressions associated with  $X_t^{VAR}$ 's is somewhat inferior to that based on  $X_t^{DSGE}$ 's. The last panel (specification 4) of Table 4 reports the adjusted  $R^2$  for the VAR- and DSGE-based regressions (34) and (35), together with the benchmark in which the forward-looking component is omitted (i.e.,  $\eta = 0$ ). Compared to the regressions with  $\eta = 0$ , augmenting the VAR-predicted  $X_t$  series systematically decreases the adjusted  $R^2$ , indicating that the term  $X_t^{VAR}$  contributes nothing in improving the equation's fit. On a contrary,  $X_t^{DSGE}$  improves the performance of the regression as adjusted  $R^2$  becomes higher than the benchmark case. This suggests that conventional inferences based on VAR approaches can draw erroneous conclusions on the role of the forward-looking component for inflation dynamics.

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<sup>13</sup>It is worth mentioning that there is a primary advantage of estimating the regression equations (34) and (35) in evaluating the relative importance of forward- and backward-looking behaviors. In the existing literature, the relative contribution of forward- and backward-looking behaviors in price setting varies widely [for example, Gali and Gertler (1999), Sbordone (2002), and Rudd and Whelan (2005, 2006)]. Mavroeidis et al. (2014) conduct a meta-study based on the survey of more than 100 existing works on the topic. One of their main finding is that the parameter  $\theta$  is difficult to identify by using the methodologies employed in the existing literature. Based on the sign of the parameter  $\mu$ , instead of the magnitude of the forward-looking parameter  $\theta$ , our test is less likely to suffer from the weak identification problem in accessing the role of the forward-looking component. The negative dependence is always observed regardless of considering the DSGE- and VAR-implied  $X$  series as a determinant of inflation.

<sup>14</sup>To be robust, we extend the lag length up to 4 periods, and find that our results are not susceptible to the number of lags. For example, the parameter  $\theta$  is estimated to be around 0.67 across output gap measures when the hybrid NKPC is associated with 4 lags.

## 5 CONCLUSION

Rudd and Whelan (2006) posit a particular forecasting model for inflation expectations and then report results conditioning on that choice. Alternative forecasting models are not examined. This paper has shown that the results of their regression-based tests are altered significantly under an alternative forecasting tool, a DSGE model. On the contrary to their conclusion, the DSGE-based results highlight the empirical importance of forward-looking behavior in understanding inflation dynamics.

Our simulation exercises show that a plausible degree of forward- and backward-looking behavior is required to explain the observed dynamic correlation between the output gap and inflation. In addition, the puzzling negative dependence of the first difference in inflation on its own lag appears when forward-looking behavior is more important than backward-looking behavior in accounting for inflation dynamics. Overall, our analysis suggests that sticky price models with substantial reliance on forward-looking behavior are compatible with data.

## A DATA

This article uses U.S. quarterly data from 1960:Q1 to 2012:Q4. Detailed data descriptions are as follows:

$$\text{Output} = \log(\text{Real GDP}) \times 100,$$

$$\text{Potential Output} = \log(\text{Real Potential GDP}) \times 100,$$

$$\text{Price Inflation} = \log(\text{GDP Implicit Price Deflator}/\text{GDP Implicit Price Deflator}(-1)) \times 400,$$

$$\text{Nominal Interest Rate} = \log(\text{Effective Federal Funds Rate}),$$

where sources of the original data are:

- Real GDP: Billions of Chained 2009 Dollars, Quarterly, Seasonally Adjusted (Source: Federal Reserve Economic Data–FRED, Series ID “GDPC1”)
- Real Potential GDP: Billions of Chained 2009 Dollars, Quarterly, Not Seasonally Adjusted (Source: Federal Reserve Economic Data–FRED, Series ID “GDPPOT”)
- GDP Implicit Price Deflator: Index 2009=100, Quarterly, Seasonally Adjusted (Source: Federal Reserve Economic Data–FRED, Series ID “GDPDEF”)
- Effective Federal Funds Rate: Quarterly, Not Seasonally Adjusted (Source: Federal Reserve Economic Data–FRED, Series ID “FEDFUNDS”)

## B TABLES

$X_t$	Parameter	Prior	Posterior			
			CBO	Detrended	HP-Filtered	CF-Filtered
DSGE	$\gamma$	$N(0.001, 0.005)$	0.0053 [0.0035, 0.0071]	0.0056 [0.0040, 0.0072]	0.0119 [0.0085, 0.0157]	0.0097 [0.0068, 0.0125]
VAR	$\tilde{\gamma}$		-0.0351 [-0.0490, -0.0212]	-0.0270 [-0.0391, -0.0148]	-0.1654 [-0.2186, -0.1121]	-0.1788 [-0.3456, -0.0121]

Table 1: Regression coefficient estimates for the NKPC ( $\kappa = 0$ ). This table reports the mean and associated [2.5%, 97.5%] intervals (in brackets). The DSGE-based coefficient  $\gamma$  is estimated by Bayesian methods with the prior specifications as in the third column. The VAR-based coefficient  $\tilde{\gamma}$  is estimated by GMM using  $\{X_{t-1}^{VAR}\}$  as an instrument. The 95th percentile intervals for the GMM estimates are constructed using (asymptotic) Newey-West standard errors.

$X_t$	Parameter	Prior	Posterior			
			CBO	Detrended	HP-Filtered	CF-Filtered
DSGE	$\gamma$	$N(0.001, 0.005)$	0.0008 [0.0001, 0.0015]	0.0009 [0.0008, 0.0010]	0.0032 [0.0008, 0.0055]	0.0024 [0.0003, 0.0047]
	$\theta$	$B(0.6, 0.1)$	0.5301 [0.5077, 0.5519]	0.5414 [0.5242, 0.5624]	0.5224 [0.4957, 0.5516]	0.5250 [0.4927, 0.5541]
VAR	$\tilde{\gamma}$		0.0006 [-0.0011, 0.0023]	0.0005 [-0.0005, 0.0016]	-0.0033 [-0.0161, 0.0095]	0.0125 [-0.0007, 0.0257]
	$\theta$		0.5275 [0.5030, 0.5521]	0.5276 [0.5089, 0.5462]	0.5382 [0.5004, 0.5760]	0.5273 [0.5081, 0.5464]

Table 2: Regression coefficient estimates for the hybrid NKPC with partial indexation ( $\kappa \in (0, 1)$ ). This table reports the mean and associated [2.5%, 97.5%] intervals (in brackets). The DSGE-based coefficients  $\gamma$  and  $\theta$  are estimated by Bayesian methods with the prior specifications as in the third column. The VAR-based coefficients  $\tilde{\gamma}$  and  $\theta$  are estimated by GMM using  $\{X_{t-1}^{VAR}, \pi_{t-1}\}$  as instruments. The 95th percentile intervals for the GMM estimates are constructed using (asymptotic) Newey-West standard errors.

$X_t$	Parameter	Prior	Posterior			
			CBO	Detrended	HP-Filtered	CF-Filtered
DSGE	$\gamma$	$N(0.001, 0.005)$	0.0014 [0.0005, 0.0021]	0.0012 [0.0005, 0.0020]	0.0042 [0.0020, 0.0064]	0.0033 [0.0013, 0.0052]
VAR	$\tilde{\gamma}$		0.0027 [0.0014, 0.0039]	0.0021 [0.0013, 0.0030]	0.0101 [0.0055, 0.0147]	0.0235 [0.0137, 0.0333]

Table 3: Regression coefficient estimates for the hybrid NKPC with full indexation ( $\kappa = 1$ ). This table reports the mean and associated [2.5%, 97.5%] intervals (in brackets). The DSGE-based coefficient  $\gamma$  is estimated by Bayesian methods with the prior specifications as in the third column. The VAR-based coefficient  $\tilde{\gamma}$  is estimated by GMM using  $\{X_{t-1}^{VAR}\}$  as an instrument. The 95th percentile intervals for the GMM estimates are constructed using (asymptotic) Newey-West standard errors.

Specification	Output Gap	$\eta = 0$	VAR	DSGE
1. Partial indexation: $\pi_t = \lambda_1 \pi_{t-1} + \eta X_t$	CBO	0.770	0.766	0.774
	Detrended	0.770	0.766	0.772
	HP-filtered	0.770	0.773	0.775
	CF-filtered	0.770	0.766	0.773
2. Full indexation: $\Delta \pi_t = \eta X_t$	CBO	0	-0.037	0.054
	Detrended	0	-0.031	0.056
	HP-filtered	0	-0.025	0.060
	CF-filtered	0	-0.003	0.046
3. Partial indexation with an additional inflation lag: $\pi_t = \lambda_1 \pi_{t-1} + \lambda_2 \pi_{t-2} + \eta X_t$	CBO	0.787	0.773	0.792
	Detrended	0.787	0.776	0.794
	HP-filtered	0.787	0.781	0.794
	CF-filtered	0.787	0.782	0.792
4. Full indexation with an additional inflation lag: $\Delta \pi_t = \mu \Delta \pi_{t-1} + \eta X_t$	CBO	0.105	0.062	0.156
	Detrended	0.105	0.069	0.160
	HP-filtered	0.105	0.088	0.163
	CF-filtered	0.105	0.096	0.155

Table 4: Adjusted  $R^2$  ( $\bar{R}^2$ ) for various model specifications and output gap measures.

$X_t$	Parameter	Prior	Posterior			
			CBO	Detrended	HP-Filtered	CF-Filtered
DSGE	$\gamma$	$N(0.001, 0.005)$	0.0016 [0.0002, 0.0029]	0.0016 [0.0002, 0.0033]	0.0045 [0.0010, 0.0076]	0.0035 [0.0004, 0.0066]
	$\theta$	$B(0.5, 0.1)$	0.6014 [0.5523, 0.6592]	0.5756 [0.4640, 0.6660]	0.5952 [0.5414, 0.6437]	0.5947 [0.5407, 0.6507]
	$\omega_1$	$B(0.5, 0.1)$	0.6302 [0.5501, 0.7168]	0.6388 [0.5535, 0.7174]	0.6322 [0.5469, 0.7120]	0.6399 [0.5569, 0.7309]
VAR	$\tilde{\gamma}$		0.0031 [0.0013, 0.0049]	0.0021 [0.0009, 0.0033]	0.0113 [-0.0017, 0.0242]	0.0262 [0.0073, 0.0450]
	$\theta$		0.5994 [0.5264, 0.6563]	0.6035 [0.5495, 0.6575]	0.6017 [0.5431, 0.6603]	0.6093 [0.5523, 0.6663]
	$\omega_1$		0.6736 [0.5110, 0.8361]	0.6768 [0.5116, 0.8420]	0.6728 [0.5090, 0.8366]	0.6772 [0.4997, 0.8548]

Table 5: **[Robustness]** Regression coefficient estimates for the hybrid NKPC with partial indexation ( $\kappa \in (0, 1)$ ). This table reports the mean and associated [2.5%, 97.5%] intervals (in brackets). The DSGE-based coefficients  $\gamma$ ,  $\theta$ , and  $\omega_1$  are estimated by Bayesian methods with the prior specifications as in the third column. The VAR-based coefficients  $\tilde{\gamma}$ ,  $\theta$ , and  $\omega_1$  are estimated by GMM using  $\{X_{t-1}^{VAR}, \pi_{t-1}, \pi_{t-2}\}$  as instruments. The 95th percentile intervals for the GMM estimates are constructed using (asymptotic) Newey-West standard errors.

$X_t$	Parameter	Prior	Posterior			
			CBO	Detrended	HP-Filtered	CF-Filtered
DSGE	$\eta$	$N(0.001, 0.005)$	0.0020 [0.0006, 0.0033]	0.0021 [0.0007, 0.0034]	0.0052 [0.0019, 0.0084]	0.0043 [0.0010, 0.0072]
	$\mu$	$N(0, 0.5)$	-0.3279 [-0.4389, -0.2133]	-0.3254 [-0.4309, -0.2218]	-0.3163 [-0.4305, -0.2037]	-0.3159 [-0.4261, -0.2047]
	$\gamma$	$N(0.001, 0.005)$	0.0011 [0.0004, 0.0019]	0.0012 [0.0004, 0.0020]	0.0034 [0.0009, 0.0057]	0.0028 [0.0008, 0.0049]
	$\theta$	$B(0.5, 0.1)$	0.6014 [0.5590, 0.6345]	0.5957 [0.5605, 0.6318]	0.5942 [0.5573, 0.6296]	0.5956 [0.5555, 0.6328]
VAR	$\tilde{\eta}$		0.0055 [0.0027, 0.0082]	0.0044 [0.0025, 0.0063]	0.0217 [0.0109, 0.0325]	0.0557 [0.0272, 0.0843]
	$\mu$		-0.3278 [-0.5056, -0.1501]	-0.3286 [-0.5006, -0.1567]	-0.3304 [-0.5149, -0.1460]	-0.3334 [-0.5497, -0.1171]
	$\tilde{\gamma}$		0.0033 [0.0017, 0.0049]	0.0027 [0.0016, 0.0037]	0.0130 [0.0068, 0.0192]	0.0334 [0.0146, 0.0523]
	$\theta$		0.5980 [0.5345, 0.6616]	0.5983 [0.5368, 0.6599]	0.5990 [0.5328, 0.6651]	0.6000 [0.5222, 0.6779]

Table 6: **[Robustness]** Regression coefficient estimates for the hybrid NKPC with full indexation ( $\kappa = 1$ ). This table reports the mean and associated [2.5%, 97.5%] intervals (in brackets). The DSGE-based coefficients  $\eta$ ,  $\mu$ ,  $\gamma$ , and  $\theta$  are estimated by Bayesian methods with the prior specifications as in the third column. The VAR-based coefficients  $\tilde{\eta}$ ,  $\mu$ ,  $\tilde{\gamma}$ , and  $\theta$  are estimated by GMM using  $\{X_{t-1}^{VAR}, \Delta\pi_{t-1}\}$  as instruments. The 95th percentile intervals for the GMM estimates are constructed using (asymptotic) Newey-West standard errors.

C FIGURES

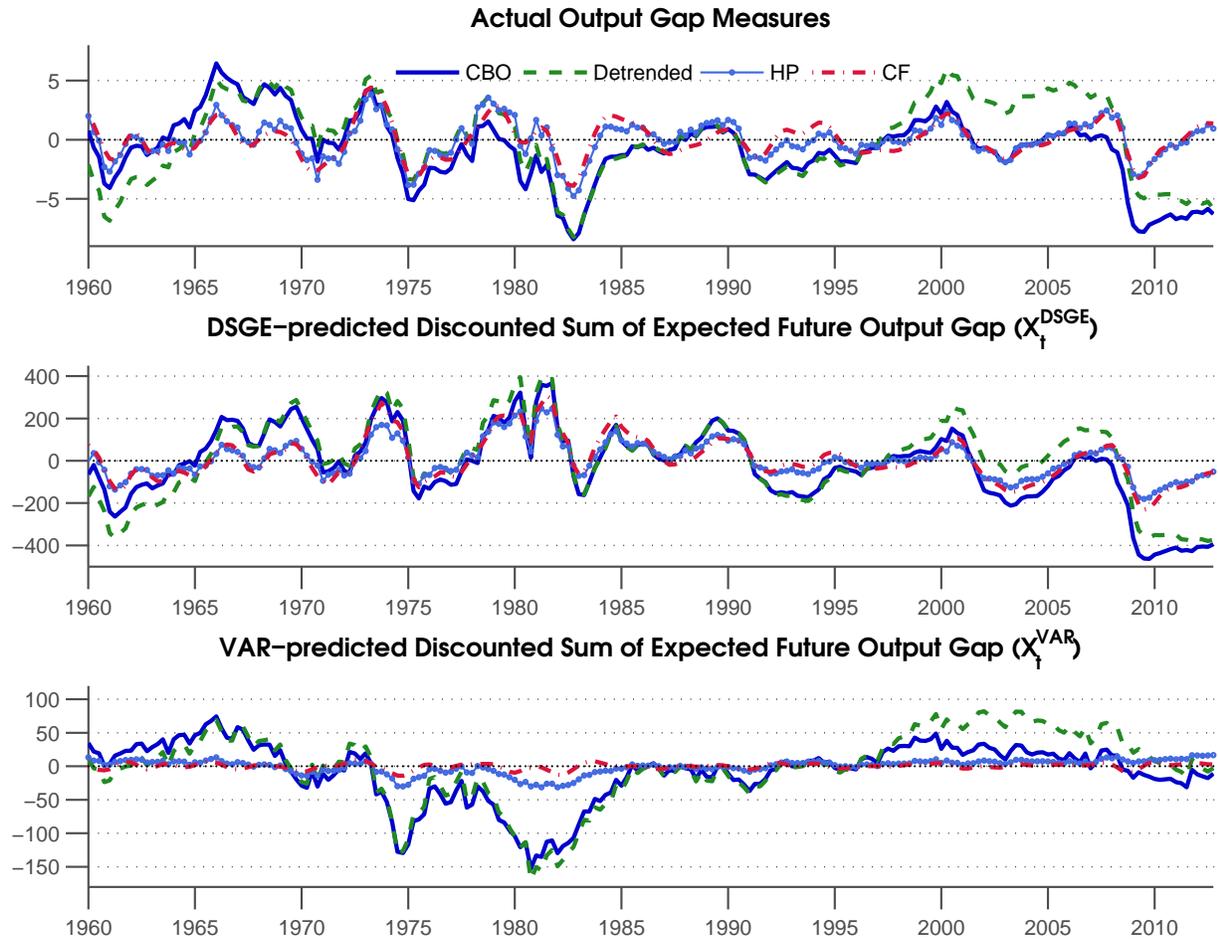


Figure 1: Sum of current and expected future output gaps ( $X_t$ ). [Top left panel] Actual output gap measures. [Middle panel] DSGE-predicted  $X_t$  series ( $X_t^{DSGE}$ ) under the NKPC (i.e.,  $\kappa = 0$ ), evaluated at the mean of posterior parameter estimates. [Bottom panel] VAR-predicted  $X_t$  series ( $X_t^{VAR}$ ).

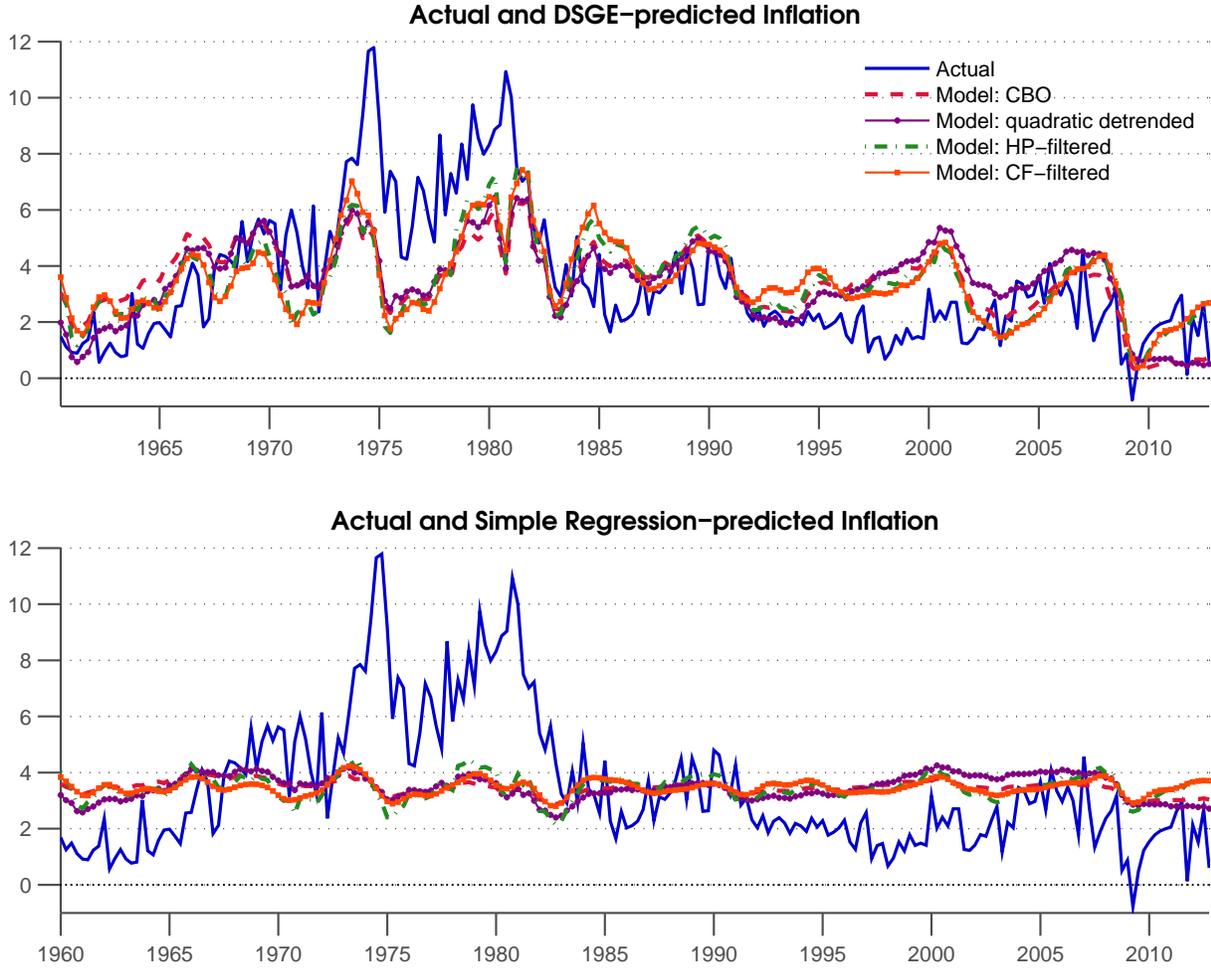


Figure 2: **[Top panel]** Actual inflation and the DSGE model-predicted inflation evaluated at the mean of posterior parameter estimates, associated with various output gap measures. The DSGE model-predicted inflation is obtained by  $\hat{\pi}_t = \hat{\gamma}X_t^{DSGE} + \hat{\gamma}\epsilon_t^p$ . **[Bottom panel]** Actual inflation and the predicted values of inflation by a simple model of inflation,  $\hat{\pi}_t = \text{constant} + \hat{\gamma}y_t + \hat{\gamma}\epsilon_t^p$ , in which the forward-looking component is absent. The regression coefficient is estimated by GMM using  $\{\tilde{y}_{t-1}, \tilde{y}_{t-2}\}$  as instruments.

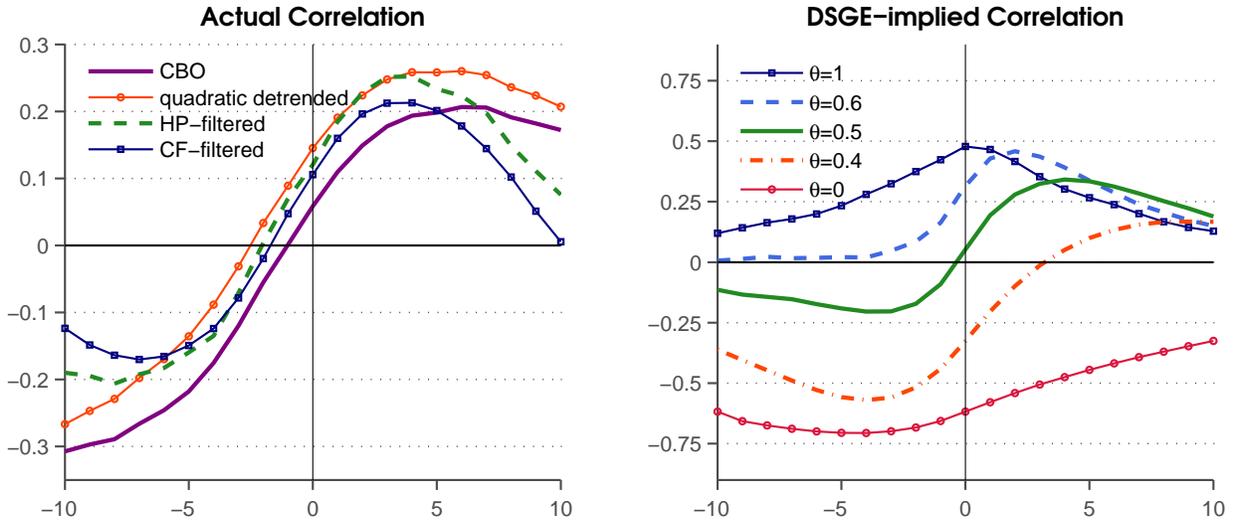


Figure 3: Actual and DSGE-implied dynamic cross-correlation between the output gap and inflation:  $corr(\tilde{y}_t, \pi_{t+k})$  where  $k$  is integer such that  $-10 \leq k \leq 10$ . **[Left panel]** The actual dynamic cross-correlation between the output gap and inflation. Four different output gap measures are used: the Congressional Budget Office (CBO)’s output gap, quadratic detrended output, HP-filtered (two-sided), and CF-filtered output (one-sided). **[Right panel]** The DSGE-implied dynamic cross-correlation between the two variables for various values of  $\theta$ . The CBO output gap is used for the estimation of the DSGE model.

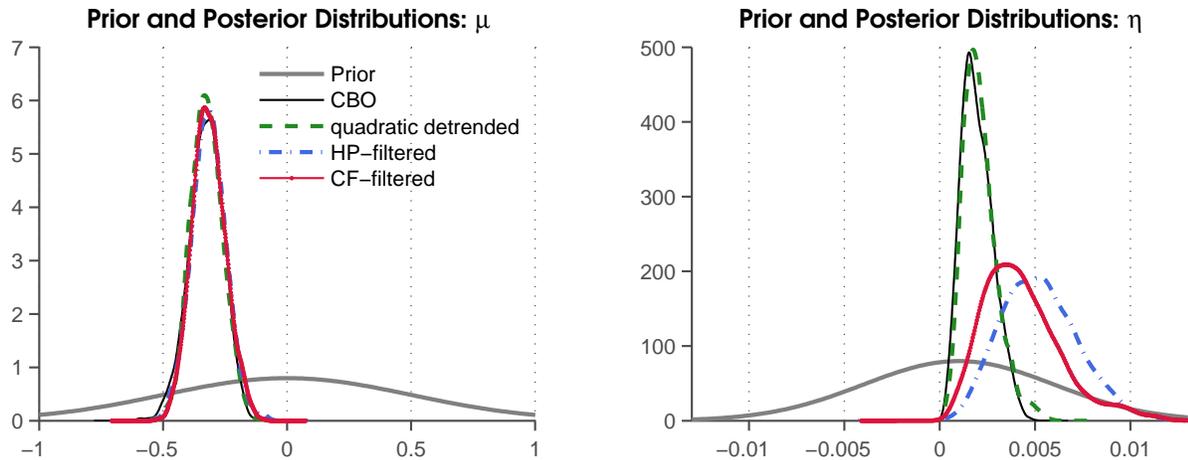


Figure 4: Prior and posterior distributions for  $\mu$  and  $\eta$  from the full indexation model with an additional lagged inflation term.

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