How Much to Share: Welfare Effects of Fiscal Transfers*

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Abstract

Recent sovereign debt crisis has challenged policy makers to explore the possibility of establishing a fiscal-transfer system that could alleviate the negative impact of asymmetric shocks across countries. Using a simple labor-production economy model, we first derive an analytically tractable solution for optimal degree of fiscal transfers. In this economy, fiscal transfers can improve welfare by moving the competitive equilibrium with fiscal transfers closer to the social planner’s solution. We then extend the model to a DSGE setting with capital formation and international borrowing and analyze how implementation of fiscal transfers affects welfare and macroeconomic variables over time. When agents have access to unrestricted international borrowing of riskless bonds, it is possible for fiscal transfers to reduce welfare; however, when borrowing is restricted, fiscal transfers tend to improve welfare. Fiscal transfers work as a substitute to international borrowing for the purpose of risk sharing. We also show that when governments need to raise tax revenue for fiscal transfers, consumption tax is a better option than income taxes.

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• Keywords: fiscal transfers; welfare effects; risk sharing; international borrowing.

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1 Introduction

Recent sovereign debt crisis has forced a number of countries to resort to rescue packages by international organizations and challenged policy makers to explore the possibility of establishing a fiscal-transfer system that can help alleviate the impact of negative shocks across countries or states. Establishing a sustainable international fiscal-transfer system could provide a preventative measure against asymmetric shocks, and such a system is particularly important when independent monetary and exchange rate policies are absent as in the cases of countries in the eurozone and federal states in the US and Canada. Such fiscal federalism has been widely discussed in policy circles. However, few theoretical papers have analyzed the optimal design of fiscal transfer system or the optimal amount of fiscal transfers. This paper tries to shed some lights on this important issue.

Various types of fiscal transfer system already exist in a federal system such as the US and Canada. Part of federal tax is asymmetrically distributed across states in order to lower inequalities among them. Federal disaster relief fund and other federal emergency funds exist to deal with state-specific shocks. However, in Europe and other regional economic unions, there is no such system across countries. Several questions arise in designing an optimal fiscal-transfer system. Can government-led fiscal transfer system replace private sector’s risk sharing mechanism through financial markets? How much money does each country or state need to contribute for fiscal transfers and what should be the optimal distribution rule? When countries adopt a new fiscal-transfer rule, how do welfare and macroeconomic variables respond to the new rule over time? Are responses in the long run different from those in the transitional period? What is the optimal revenue source for fiscal transfers?

In this paper, we first construct a simple two-country labor-production economy model and derive an analytically tractable solution for welfare which depends on the degree of fiscal transfers. In deriving welfare implications, we use a second-order accurate solution method. Using this solution, we examine the channels in which fiscal transfers affect welfare. Optimal level of fiscal transfers depends on the amount of tax revenue, the elasticity of labor supply, and the size of shocks. In this economy, fiscal transfers can improve welfare by moving the competitive equilibrium with fiscal transfers towards the social planner’s solution. In most cases, optimal fiscal transfers require domestic households to receive more transfers from the foreign country than from home, due to positive correlation between labor income and domestic shocks.

Then, we extend our analysis to a DSGE setting with capital formation and bond holdings.
and analyze how implementation of fiscal transfers affects welfare and macroeconomic variables over time based on calibration to the EU data. Simulation results show that when agents have access to unrestricted international borrowing, fiscal transfers may reduce welfare. However, when international borrowing is restricted, fiscal transfers improve welfare in most cases. Fiscal transfers work as a substitute to international borrowing for risk sharing purpose. We also show that when governments need to raise tax revenue for fiscal transfers, consumption tax (VAT) is a better option than income taxes.

This paper consists of five sections. Section 2 describes a static model of a production economy with labor and derive closed-form solution for welfare. Section 3 describes a dynamic model with capital formation and international borrowing and discuss the calibration and our solution method that is based on the second-order perturbation. In Section 4, we evaluate welfare implications of fiscal transfer policy and the role of international borrowing. We also include several sensitivity studies. Finally, section 5 offers conclusion of this paper.

2 Labor Production Economy

2.1 Model

In this section, we consider a two-country static model with labor production. Utility is assumed to be additively separable between consumption and labor. We use a separable utility function in order to derive an analytically tractable expression for welfare. Labor is not mobile across countries. Households in each country \( i (= 1, 2) \) maximize

\[
\max E(U_i) = E \left[ \log C_i + \frac{\varepsilon}{1 + \varepsilon} \left( 1 - \frac{L_i ^{1+\varepsilon}}{1+\varepsilon} \right) \right] \tag{1}
\]

subject to

\[
C_i = (1 - \tau) A_i L_i + TR_i, \tag{2}
\]

where \( C \) is consumption, \( L \) is labor input, \( A \) is i.i.d. normal productivity shock, and \( TR \) is the amount of lump-sum transfer which is taken as exogenous to households in this country.

Parameter \( \varepsilon (> 0) \) represents the elasticity of labor supply in a competitive equilibrium, and \( \tau \) is the tax rate assumed to be between 0 and 1. Note that utility is logarithmic in consumption and that production function is linear in labor.

The optimality condition for country \( i \) is

\[
C_i L_i ^{\frac{1}{\varepsilon}} = (1 - \tau) A_i. \tag{3}
\]

Each country’s government collects tax revenue (fiscal revenue) as follows:

\[
S_i = \tau A_i L_i, \tag{4}
\]

and the revenue is shared with the other country such that the amount of transfer for the first country’s households is

\[
TR_1 = (1 - \kappa) S_1 + \kappa \left( \frac{S_1 + S_2}{2} \right) = \left( 1 - \frac{\kappa}{2} \right) S_1 + \frac{\kappa}{2} S_2. \tag{5}
\]

\(^6\)This form of utility is widely used in macro and labor literature such as in Chang and Kim (2011).

\(^7\)Note that there is only one good in the economy, which makes the model effectively a fixed exchange rate regime model.
The parameter $\kappa$ represents the degree of fiscal revenue sharing in the transfer decision and is dubbed as the degree of “revenue sharing” in this paper. When $\kappa = 0$, there is no fiscal transfers—equivalent to the autarkic economy. The case of $\kappa = 1$ represents full risk sharing of fiscal revenue. That is, the sum of fiscal surplus of the two countries are equally distributed across the two countries. When $\kappa > 1$, domestic country receives more amount of foreign revenue than domestic revenue. In our economy with two countries, the case of $\kappa = 2$ implies that all domestic revenue is taken by the foreign country, and vice versa. The case of $\kappa > 2$ is analogous to short selling.

Combining the equations for $TR_1$ and $TR_2$ with households’ budget constraints, the country resource constraints become

$$C_1 = \left(1 - \frac{\kappa T}{2}\right)A_1 L_1 + \frac{\kappa T}{2}A_2 L_2,$$

$$C_2 = \frac{\kappa T}{2}A_1 L_1 + \left(1 - \frac{\kappa T}{2}\right)A_2 L_2.$$  

(6) (7)

In this consumption equation, $\kappa T$ represent the degree of “output sharing” since how much output to share in the end depends on how much tax to collect and how much tax revenue to share.

The equilibrium consists of the two countries’ resource constraints—(6) and (7)—and the two private-sector optimality conditions. To analyze the optimal behavior of consumption and labor supply, we first derive linearized solutions for the first country:

$$l_1 \approx \frac{\varepsilon \kappa T}{2 (1 + \varepsilon - \varepsilon \kappa T)} (a_1 - a_2),$$

$$c_1 \approx \left[1 - \frac{\kappa T}{2 (1 + \varepsilon - \varepsilon \kappa T)}\right] a_1 + \frac{\kappa T}{2 (1 + \varepsilon - \varepsilon \kappa T)} a_2,$$  

(8) (9)

where lower cases, $l$, $c$, and $a$, denote the log deviations from their deterministic steady states.

In order to properly analyze welfare effects, it is necessary to include the second-order terms in the solution. The linearization method can generate inaccurate results in terms of welfare calculations, especially in open-economy models. A second-order perturbation method is a natural candidate for such an analysis and we apply the “bias correction” approach in Kim and Kim (2003). The second-order solution for the expected utility is:

$$E(U) = (\log C) + \mu_c + \frac{\varepsilon}{1 + \varepsilon} \left[1 - L \frac{1+\varepsilon}{1+\varepsilon} \exp \left(\frac{1+\varepsilon}{\varepsilon} \mu_l + \frac{1}{2} \left(\frac{1+\varepsilon}{\varepsilon} \right)^2 \sigma_l^2\right)\right]$$

$$+ \mu_c + \frac{\varepsilon}{1 + \varepsilon} L \frac{1+\varepsilon}{1+\varepsilon} \left[1 - \exp \left(\frac{1+\varepsilon}{\varepsilon} \mu_l + \frac{1}{2} \left(\frac{1+\varepsilon}{\varepsilon} \right)^2 \sigma_l^2\right)\right]$$  

(10)

---

8 In the case of no revenue sharing, government budget constraint is $TR_i = \tau A_i L_i$, and the equilibrium is $L_i = (1 - \tau) \frac{A_i}{1+\varepsilon}$ and $C_i = (1 - \tau) \frac{A_i}{1+\varepsilon} A_i$.

9 When there are $N$ countries, each country’s share of its own fiscal revenue among its lump-sum transfer is $(1 - \frac{N-1}{N} \kappa)$.

10 Likewise, when there are $N$ countries, the share of each country’s output in its consumption is $(1 - \frac{N-1}{N} \kappa T)$.

11 The second-order sufficient condition for this optimization problem reduces, in the steady state, to $1+\varepsilon - \varepsilon \kappa T > 0$.

12 They show that the conventional linearization is so inaccurate as to generate a paradoxical result of spurious welfare reversal: the level of welfare under autarky is higher than that of the complete markets economy. They also show that the bias correction method produces the same results as the second-order perturbation method when calculating welfare.
where $\bar{C}$ and $\bar{L}$ denote the deterministic steady state, and $\mu_l$, $\mu_c$, $\sigma_l^2$, $\sigma_c^2$ are mean and variance of labor supply and consumption, respectively.\footnote{See Appendix for the analytical expression for mean and variance of consumption and labor supply, and also the detailed derivation of the expected utility.}

### 2.2 Optimal Revenue Sharing Rule

To study the role of fiscal transfers in this economy, we investigate an optimal revenue sharing rule under which both tax rate ($\tau$) and the degree of revenue sharing ($\kappa$) can be chosen by the government.\footnote{We assume that governments can choose the degree of fiscal revenue sharing by coordinating with other governments. Equivalently, the central authority (planner) such as European Union can choose a sharing rule to maximize the overall welfare of member countries.} We first start with an extreme case when labor elasticity is near zero.\footnote{In simulations, we set the value of $\varepsilon$ at $10^{-10}$.} This case can be interpreted as an endowment economy since the amount of labor input does not respond to shocks. When tax rates are zero, the degree of revenue sharing would not influence the outcome since there is nothing to share across countries. However, when tax rates are positive (as illustrated by the case of $\tau = 0.5$ in Figure 1), increasing the degree of revenue sharing from zero will increase the level of welfare as represented by the amount of certainty-equivalent consumption in the vertical axis in Figure 1. However, this relationship between the degree of revenue sharing and the level of welfare is not monotone. When $\tau = 0.5$, the level of welfare increases until $\kappa = 2$ and then starts to decrease. In the case when incomes are (almost) fully taxed ($\tau = 1$), the level of welfare increases until $\kappa = 1$.\footnote{In simulations, we set the value of $\tau$ at $1 - 10^{-10}$.} In these two cases, the optimal policy implies that $\kappa \tau$ has to be set at unity.

In fact, for any degree of taxation, equation (9) implies that when $\kappa \tau = 1$, each country consumes a half of the world output. This full output sharing is optimal in the case of an endowment economy. Optimal tax and revenue sharing policies are attained when the outcome of the competitive equilibrium replicates that of the social planner—who maximizes the world welfare subject to only the world resource constraint (denoted by a red line in Figure 1). When tax rate is low (for example, $\tau = 0.2$), labor income is much larger than the amount of domestic fiscal revenue and in order to achieve full output sharing ($\kappa \tau = 1$), domestic agents should have a large portion of foreign fiscal revenue ($\kappa = 5$). However, an economically meaningful range of $\kappa$ is between 0 and 2, assuming that there is no short selling. Therefore, when tax rates are below 0.5 and short selling is absent, then this economy cannot reach the optimal output sharing and remains suboptimal despite fiscal transfers.

Next, we turn to the case where the amount of labor input is endogenously determined. In our model economy, $\varepsilon$ represents the elasticity of labor supply. In the upper panel of Figure 2, we fix the tax rate $\tau$ at 0.5 and the labor supply elasticity $\varepsilon$ at 1 and plot the level of welfare over different values of $\kappa$. In this case, the optimal degree of revenue sharing $\kappa$ is above two ($\kappa = 2.4$). That is, with endogenous labor supply, optimal revenue sharing implies that domestic agents should hold more foreign revenue compared to the case of an endowment economy (optimal $\kappa = 2$).\footnote{In this case, domestic agents should short-sell domestic fiscal revenue to hold more foreign revenue than currently exists.} In fact, with other values of $\varepsilon > 0$, optimal $\kappa$ is always above 2.

In order to understand why optimal revenue sharing is above 2, we examine the steady-state values in this economy. With nonzero $\varepsilon$, the steady-state values of consumption and labor, $\bar{C} = \bar{L} =$
(1 − τ)^{\frac{1}{1-\sigma}}, depend on tax rate \(\tau\). To investigate the potential effects of changes in these steady-state values on welfare, we set \(\bar{L} = \bar{C} = 1\) to match the steady-state values under exogenous labor supply by assuming some type of government’s subsidy. The lower panel of Figure 2 shows that optimal \(\kappa\) is now at 2 under this steady-state correction. With other values of \(\varepsilon\), the optimal \(\kappa\) is at 2 as well. This confirms that—without the change in the steady state due to the simultaneous presence of positive tax rate and labor supply elasticity—the optimal revenue sharing would be invariant at \(\kappa = 2\), which implies that the optimal output sharing rule is \(\kappa\tau = 1\). Under this rule, the competitive equilibrium of the economy with fiscal transfers becomes equivalent to that of the social planner, as is evident in (6) and (7). Note that \(\varepsilon\) does not affect the social planner’s solution \((\kappa\tau = 1)\) because consumption and production decisions become independent from each other due to the separability of utility function between consumption and labor. When utility function is non-separable between consumption and labor, the optimal output sharing rule may depend on the elasticity of labor supply. In appendix, we analyze the case with non-separable utility.

3 Dynamic Model

In this section, we extend the labor production economy model to a fully-blown DSGE model with capital and international borrowing. We maintain the same assumption that two identical countries have the same preference and production technology. There is a single nondurable tradable good serving as a numeraire. Each country consists of a representative household, a representative firm, and a government. Households decide the level of consumption, leisure, investment, and bond holdings subject to budget constraints. Bond holdings and investment are subject to adjustment costs. We assume that capital is perfectly mobile across countries and the international financial market is incomplete in the sense that agents can trade only non-contingent bonds.

Government is described as a sequence of tax rates on consumption, labor income and capital income, and government spending and lump-sum transfers. Tax rates and government spending are set exogenously and assumed to be constant in order to focus on the role of revenue-sharing rule \((\kappa)\). Any fiscal revenue (or debt) is shared by two countries through a predetermined revenue-sharing rule (defined as fiscal transfers), and households receive fiscal transfers in a lump-sum fashion. Fiscal transfers can be negative in which case they operate as lump-sum taxes. Productivity shocks are the main disturbances in the economy.

3.1 Households and Firms

Households enter the market owning one unit of labor at time \(t\) with predetermined capital and bond holdings. The household receives its wage and rental income from firms, and its interest income from risk-free bonds. Household in each country maximizes the expected lifetime utility given by

\[
E_0 \sum_{t=0}^{\infty} \beta^t U_t, \text{ where } U_t = \left[ C_t^\theta (1 - L_t)^{1-\theta} \right]^{1-\sigma} - 1
\]

Households are subject to the constraint that hours worked plus hours of leisure cannot exceed the time endowment which is normalized to one. Households in both countries have the same discount factor \(\beta\).

\[\text{Note that country subscript is omitted for simplicity and that variables with an asterisk represents foreign country.}\]
The budget constraint of household is given by:

\[
(1 + \tau_{ct})C_t + I_t + B_t + \frac{\zeta}{2} (B_t)^2 = (1 - \tau_{lt})w_t L_t + [(1 - \tau_{kt})r_t + \tau_{kt}\delta] K_t + R_{t-1}B_{t-1} + TR_t,
\]

where \(r_t\) is the rental rate, \(w_t\) is the wage rate, \(\tau_t\) is tax rate (\(\tau_l = \) labor income tax, \(\tau_k = \) capital income tax, and \(\tau_c = \) consumption tax), \(B_t\) denotes the quantity of bonds purchased in period \(t\) maturing in \(t + 1\), \(R_t\) is the gross interest rate on bonds, and \(B_t\) is international bonds and denotes the net quantity purchased irrespective of the issuing country. Bond holding is subject to holding costs \(\frac{\zeta}{2} (B_t)^2\). Note that tax rates are fixed and there is a depreciation allowance, \(\tau_{kt}\delta K_t\). \(TR_t\) is the lump-sum transfer (tax) to the household.

As in Kim (2003), households accumulate capital according to the following equation:

\[
K_{t+1} = \left[ \delta \left( I_t / \delta \right)^{1-\phi} + (1 - \delta) K_t^{1-\phi} \right]^{\frac{1}{1-\phi}}.
\]

A zero \(\phi\) implies no adjustment costs. A positive \(\phi\) implies the presence of adjustment costs and \(\phi = 1\) corresponds to a loglinear capital accumulation equation.\(^{19}\)

The production function follows a Cobb-Douglas form with labor and capital.

\[
Y_t = A_t L_t^\alpha K_t^{1-\alpha}.
\]

Firms’ first order conditions for profit maximization are:

\[
w_t = \frac{\alpha Y_t}{L_t}, \quad r_t = (1 - \alpha) \frac{Y_t}{K_t}.
\]

The first-order conditions for the household are

\[
C_t : \quad (1 + \tau_{ct})\lambda_t C_t = \theta (1 - \sigma) U_t, \quad (17)
\]

\[
L_t : \quad (1 - \tau_{lt})\lambda_t w_t (1 - L_t) = (1 - \theta) (1 - \sigma) U_t, \quad (18)
\]

\[
I_t : \quad \lambda_t = \mu_t \left[ \delta \left( I_t / \delta \right)^{1-\phi} + (1 - \delta) K_t^{1-\phi} \right]^{\frac{\phi}{1-\phi}} \left( \frac{I_t}{\delta} \right)^{-\phi}, \quad (19)
\]

\[
K_t : \quad \mu_t = \beta E_t \left[ (1 - \delta) \mu_{t+1} \left[ \delta \left( I_{t+1} / \delta \right)^{1-\phi} + (1 - \delta) K_{t+1}^{1-\phi} \right]^{\frac{\phi}{1-\phi}} (K_{t+1})^{-\phi} + \lambda_{t+1} (r_{t+1} (1 - \tau_{k,t+1}) + \delta \tau_{k,t+1}) \right], \quad (20)
\]

\[
B_t : \quad \lambda_t (1 + \zeta B_t) = \beta R_tE_t (\lambda_{t+1}), \quad (21)
\]

where \(\lambda_t\) and \(\mu_t\) are Lagrangian multipliers for the budget constraint and capital accumulation equation, respectively. There are foreign country analogues to equations (12) to (21). Foreign variables are denoted by asterisks.

\(^{19}\)This equation is equivalent to the standard capital accumulation equation (e.g. Baxter and Crucini, 1995) when linearized.
3.2 Shocks and Governments

Productivity variable $A_t$ and $A_t^*$, representing stochastic components of the production functions of the two countries (home and foreign countries), follow a symmetric vector Markov process:

$$
\begin{bmatrix}
\log(A_t) \\
\log(A_t^*)
\end{bmatrix} =
\begin{bmatrix}
\rho_A & \nu_A \\
\nu_A & \rho_A
\end{bmatrix}
\begin{bmatrix}
\log(A_{t-1}) \\
\log(A_{t-1}^*)
\end{bmatrix} +
\begin{bmatrix}
\varepsilon_A \\
\varepsilon_A^*
\end{bmatrix}.
$$

where $E(\varepsilon_{A_t}) = E(\varepsilon_{A_t}^*) = 0$ and $E(\varepsilon_{A_t}^2) = \sigma_{\varepsilon_A}^2$, $E((\varepsilon_{A_t}^*)^2) = \sigma_{\varepsilon_A^*}^2$, and $\rho(\varepsilon_{A_t}, \varepsilon_{A_t}^*) = \psi_A$ for all $t$. $\rho_A$ is the persistence of productivity and $\nu_A$ represents the spillover effects—the degree of transmission of productivity with one period lag. A non-zero $\psi_A$ means that the innovations are contemporaneously correlated across countries.

We assume that two countries (governments) make arrangements to transfer a part of their fiscal revenue with each other in the same way as in the previous section. In this case, budget constraint of government in domestic country becomes

$$
\tau_{ct}C_t + \tau_{lw}L_t + \tau_{kt}(r_t - \delta)K_t = G_t + S_t
$$

where $S_t$ is the amount of fiscal revenue to be shared with the other country. $S_t$ is positive if there is fiscal surplus and negative if deficit exists. Note that the bond holding costs do not enter into government revenue and they simply disappear from the system. Lump sum transfers to households, $T_t$, is related to $S_t$ as follows;

$$
T_t = (1 - \kappa)S_t + \kappa \frac{S_t + S_t^*}{2},
$$

$$
T_t^* = (1 - \kappa)S_t^* + \kappa \frac{S_t + S_t^*}{2},
$$

where $\kappa$ denotes the degree of revenue sharing as in the previous section.

3.3 Equilibrium

Domestic equilibrium is given by optimizing the behavior of the household and the firm since we implicitly consider the behavior of the government by plugging its budget constraint into the household’s. Then, the country’s overall resource constraint becomes

$$
Y_t + R_{t-1}B_{t-1} + \frac{\kappa}{2}S_t^* - S_t = C_t + I_t + G_t + B_t + \frac{\zeta}{2} (B_t)^2
$$

For the world equilibrium, the model requires bond market-clearing condition that bonds should be in zero net supply since they denote the net bond holdings of each country:

$$
B_t + B_t^* = 0.
$$

3.4 Calibration

For the calibration, we adopt parameter values widely used in previous studies for business cycle models. We focus on the annual frequency data. Capital depreciation rate $\delta$ is set at 0.1. Labor share $\alpha$ is 0.64 which is a typically used value in the literature. The consumption share parameter $\theta$
is set to match the steady-state share of time devoted to market activities, 0.3. The representative agent’s discount factor $\beta$ is 0.96 so that the steady-state annual real interest rate is equal to 4%. We set the curvature parameter $\sigma$ which determines the household’s coefficient of relative risk aversion at 2 which is in line with many empirical estimates. The elasticity of bond holding adjustment costs, $\zeta$, determines the amount of international borrowing and we experiment with different values ranging from $10^{-4}$ to 0.5.\(^{20}\) Finally, we set the parameter value for $\phi$ in capital adjustment costs at 0.2 to match the volatility of investment in the data.

We assume that productivity shocks follow a persistent process with no spillovers, $\rho_A = 0.95$ and $v_A = 0$. For the standard deviation of productivity shocks, we adopt the parameter value used in Backus et al. (1992) and Baxter and Crucini (1995): $\sigma_{\xi_A} = 0.852\%$. In the benchmark case, we assume that productivity shocks ($\xi_A, \xi_A^*$) are independent across countries.

Measuring aggregate tax rates is a complex and difficult task and there is little consensus on effective tax rate measures. In this paper, we use the aggregate effective tax rates calculated by Trabandt and Uhlig (2009) which is based on Mendoza et al. (1994).\(^{21}\) They calculate annual effective tax rates for consumption, labor income and capital income for US and EU-14 countries from 1995 to 2007. This method divides actual tax payments by corresponding national accounts.\(^{22}\) These effective tax rates reflect governments’ policies on tax credits, deductions, and exemptions as well as information on statutory tax rates. Moreover, they are consistent with the concept of aggregate tax rates at the national level and with the representative assumption.

Average estimated tax rates for EU-14 countries in 1995-2007 are 17%, 41% and 33% for consumption, labor income and capital income taxes, respectively. We use these values as benchmark tax rates for simulation. Under the benchmark calibration, these tax rates generate the total tax revenue over GDP ratio at 37% which is not much different from the EU data (40% according to Carey and Tchilinguirian, 2000). Initial bond holding is set at zero ($B_Y = 0$). For the benchmark study, we set the government spending at 35.4% of GDP so that the steady-state net fiscal revenue ($S_t$) is at 1% of GDP. We follow Kim et al. (2008) and use the second-order perturbation method to solve the model for a correct welfare calculation.

4 Simulation Results

We analyze how welfare changes when countries adopt a certain revenue-sharing rule ($\kappa$ changes from zero to a certain positive number). We use conditional welfare measure which is calculated by taking discounted sum of periodic utility (measured by certainty-equivalent consumption) over time following a one-time change in $\kappa$.\(^{23}\) Time period is set at 500 periods in order to ensure convergence.

Numbers in Table 1 represent conditional welfare gains when $\kappa$ changes from 0 to a certain positive number, at different bond holding costs. When bond holding costs are low ($\zeta = 0.0001$ and

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\(^{20}\) Additional benefits of having bond holding costs is that the model becomes stationary. See Kim and Kose (2003) for the detailed explanations of the nonstationary property of the incomplete markets economy with bonds.

\(^{21}\) Their method is in the same line with Lucas (1990) and Razin and Sadka (1994). A number of papers have used this method to construct data on effective tax rates. See, for example, Mendoza and Tesar (1998), Carey and Tchilinguirian (2000) and Carey and Rabesona (2004).

\(^{22}\) Another widely-used alternative for data on tax rates is aggregate marginal tax rates. See Mendoza et al. (1994) for a detailed explanation and comparison of different computation methods.

\(^{23}\) We first calculate expected periodic utility given a certain degree of revenue sharing rule and then derive certainty-equivalent consumption level by fixing the labor at its steady state.
0.001), revenue sharing lowers welfare and the welfare loss increases as revenue sharing increases (higher $\kappa$). Welfare loss can be as large as 4.7% of permanent consumption (when $\zeta = 0.0001$ and $\kappa = 2$). Consumption smoothing channel through international borrowing is already in place and additional risk sharing by fiscal transfers can actually lower welfare.

However, when bond holding costs are high ($\zeta \geq 0.01$), revenue sharing increases welfare. Welfare gains increase as $\kappa$ increases and eventually attains maximum at $\kappa = 2.4$ with all three values of $\zeta \geq 0.01$ (it is above an economically meaningful range though). Welfare gains are $0.07\% \sim 0.08\%$ when $\kappa = 1$, and $0.14\% \sim 0.15\%$ when $\kappa = 2$. When international borrowing is restricted, government’s revenue sharing can work as a substitute to international borrowing and therefore increase welfare.

In order to understand the reasoning behind this result, we draw impulse responses to a positive productivity shock at home in the case of no revenue sharing ($\kappa = 0$) and full revenue sharing ($\kappa = 1$). Figure 3 shows impulse responses when there is little restrictions in international borrowing ($\zeta = 0.0001$) and Figure 4 presents the case with restricted borrowing ($\zeta = 0.1$).²⁴

Under unrestricted borrowing (Figure 3), facing a positive productivity shock, domestic agents increase output and consumption by accumulating bonds over time: the consumption smoothing channel. Under full revenue sharing ($\kappa = 1$, dotted line), output responds more and consumption responds less to a positive productivity shock, compared to the case with no revenue sharing ($\kappa = 0$, solid line). Risk sharing through fiscal transfers pushes agents in a more productive country to produce more and share increased output with the other country: an income effect similar to the one in the previous section. This can be seen in the impulse response of net fiscal transfers: there is positive net transfer from home to a foreign country when there is a positive productivity shock at home. In fact, this negative income effect generates a similar outcome as consumption smoothing channel. In terms of welfare, a full revenue sharing lowers welfare in the case of unrestricted borrowing. Optimal consumption smoothing is already in place with unrestricted international borrowing, and additional revenue sharing generates more consumption smoothing than necessary, which lowers overall welfare of the economy.

Impulse responses under restricted borrowing (Figure 4) exhibit a similar pattern to the case with unrestricted borrowing except that bond holdings now remain at near zero, and therefore investment and output respond less to a productivity shock—compared to the case with unrestricted borrowing. All foreign variables respond less to a productivity shock as well. A limited amount of consumption smoothing is in place due to high bond holding costs, and a full revenue sharing in this case improves welfare through income effects that produce a similar outcome to the consumption smoothing channel which is not present when borrowing is restricted.

### 4.1 Conditional vs. Unconditional Welfare

In Table 1, the numbers in parentheses are unconditional welfare gains that represent differences in welfare between initial state ($\kappa = 0$) and a new state with revenue sharing: measured by percentage changes in certainty-equivalent consumption. The table shows that unconditional welfare gains can be quite different from conditional welfare gains. For example, when $\zeta = 0.001$, unconditional welfare analysis shows sizable welfare gains from revenue sharing around $0.05\% \sim 0.27\%$. However, conditional welfare analysis indicates a welfare loss around $0.01\% \sim 0.25\%$. Unconditional welfare

²⁴Note that three tax rates are positive at the steady state, so the impulse responses are somewhat different from those in a distortion-free economy.
analysis neglects welfare changes in the transitional period from one state to another. Ignoring welfare changes during the transitional period may lead to misleading welfare results, especially if welfare costs during the transitional period are large enough to offset any long-term gains.\textsuperscript{25} Therefore, it is important to use conditional welfare measure in order to correctly capture the dynamic transitional effects of a fiscal policy.

In order to further analyze conditional vs. unconditional welfare, we draw time series plot of periodic utility when a full revenue sharing ($\kappa = 1$) is implemented (Figure 5) at different values of bond holding costs. When $\zeta = 0.001$, the world periodic utility initially drops to $-0.06\%$ and steadily improves. Utility gains become positive only after about 50 periods (years) and slowly approach towards the long term gains of $0.12\%$. However, the speed of improvement is too slow to generate overall positive welfare gains. Therefore, considering the discount factor, the conditional welfare calculation including the transitional periods shows a loss of $0.035\%$. When $\zeta > 0.01$, both transitional and long term utility show positive welfare gains from fiscal transfers.

### 4.2 Sensitivity Analysis

In the preceding section, we assume that distortions are generated by all three taxes. The amount and type of existing tax distortions affect the equilibrium of the economy and therefore welfare effects of fiscal transfers. In this section, we examine how welfare effects of fiscal transfers in the previous section change when we assume that only one type of tax exists in the economy. Figure 6 plots conditional welfare gains of revenue sharing at a different degree of revenue sharing ($\kappa$ changes from 0 to 2) with different bond holding costs.

The first row presents the case when the government levies only consumption tax at 40.6\%, 17.8\% and 1.4\%, while holding the other two taxes at zero. Government spending is fixed at 20\%, 10\% and 0\% of GDP, respectively, so that these tax rates always generate 1\% fiscal revenue (of GDP). In all cases, revenue sharing always generates welfare gains and welfare gains increase as $\kappa$ increases. The largest welfare gains attained when there is no restriction in international borrowing. Since consumption tax does not generate distortions in optimality conditions in production and only generates income effects, revenue sharing improves welfare.\textsuperscript{26}

The second and third rows in Figure 6 present the cases when the economy has only labor or capital income tax at different level: 35\%, 18.4\%, 1.7\% for labor income tax and 56.4\%, 37.5\%, 8\% for capital income tax. In all cases, we adjust government spending so that fiscal revenue remains at 1\% of GDP. The figure shows that the results in the previous section holds. With unrestricted international borrowing ($\zeta = 0.0001$), revenue sharing reduces welfare and a higher $\kappa$ generates more welfare loss. However, when there are restrictions in international borrowing, revenue sharing generates welfare gains. These results hold even when tax rates are very low. As long as there are distortions in production, there is a chance that revenue sharing may reduce welfare. These results provide an important policy implications. When a government implements fiscal transfers from a new revenue source, it is better to tap in the least distorting sources such as consumption tax (VAT), instead of income taxes.

In Table 2, we show how conditional welfare gains of revenue sharing change when we allow for positive or negative cross-country correlation of productivity shocks. The table presents the case

\textsuperscript{25}Since conditional welfare is measured by taking a discounted sum of periodic utility, utility loss in an immediate future counts more than utility gain in a remote future.

\textsuperscript{26}Note that we cannot directly compare the level of welfare in different cases, as the steady-state utility changes with different tax rates.
when bond holding costs are set at 0.1 (the case of positive welfare gains from revenue sharing) and 0.001 (the case of negative welfare gains of revenue sharing). In both cases, with a positive cross-country correlation of productivity shocks (at 0.5), revenue sharing generates lower welfare gains (higher welfare loss) compared to the benchmark model with zero correlation. With a negative shock correlation (at −0.5), revenue sharing generates a higher welfare gains (lower welfare loss). When two countries face similar shocks, their output follows a similar cycle and therefore provides less scope for risk sharing, and vice versa. This result is consistent with the conclusion from the risk sharing literature: negatively correlated shocks provide more welfare gains from risk sharing.

Finally, in order to verify that non-separability of utility function plays a key role in determining welfare gains of fiscal transfers (as shown in Section 2), we experiment with separable utility function by setting $\sigma = 1$ in (11). Welfare gains of revenue sharing under separable utility function are much larger than the case with non-separable utility function. For example, when $\zeta = 0.001$ in Table 1 under $\sigma = 2$, welfare gains are negative in all cases. However, with separable utility function ($\sigma = 1$), welfare gains become positive in all cases.

5 Conclusion

We summarize the welfare implications of revenue sharing as follows. First, revenue sharing improves welfare by moving the economy closer to the social planner’s solution: agents in a more productive country produce more and share increased output with other countries. Second, when agents have access to unrestricted international borrowing, revenue sharing can reduce welfare. However, when borrowing is restricted, revenue sharing improves welfare in most cases. Given the fact that most countries suffering from debt crisis have limited access to international capital markets (or have to pay a very high risk premium), there is room for welfare improvement from fiscal transfers. Finally, when governments need to raise tax revenue for the purpose of fiscal transfers, consumption tax (VAT) is the best option. If additional revenue is raised from distortionary taxes such as income tax, revenue sharing may reduce welfare.

Note that this paper examines a time zero problem where countries make transfer arrangements before shocks occur and assume that countries abide by the terms in the contract. Therefore, in this model, there is no incentive compatibility problem or moral hazard issue. Several extensions are possible. First, one can construct a model with more than two countries or asymmetric countries/shocks where initial conditions differ across countries. Second, welfare results depend on whether the fiscal-transfer rule is implemented before shocks occur or after. This may involve a time inconsistent solution for optimal tax-transfer rules. Finally, the results of this paper can also be applied to optimal revenue-sharing rule by local governments within a country, such as states or provinces.
A Welfare Calculation in the Labor Production Economy

In the labor-production economy in section 2, the equilibrium consists of the two countries’ resource constraints and the two private-sector optimality conditions. This system can be further reduced to only two equations that involve the two labor terms:

\[
(1 - \tau) A_1 L_1^{-\frac{1}{2}} = \left(1 - \frac{\kappa \tau}{2}\right) A_1 L_1 + \frac{\kappa \tau}{2} A_2 L_2
\]

\[
(1 - \tau) A_2 L_2^{-\frac{1}{2}} = \frac{\kappa \tau}{2} A_1 L_1 + \left(1 - \frac{\kappa \tau}{2}\right) A_2 L_2
\]

The steady state of this system is \( L_1 = L_2 = (1 - \tau)^{\frac{1}{1+\tau}} \), which is less than unity with positive taxes. Denoting the log deviation of each variable from its deterministic steady state (whose value depends on \( \tau \) and \( \varepsilon \)) with its lower case, the linearized version for this system is

\[
\left(1 - \frac{\kappa \tau}{2} + \frac{1}{\varepsilon}\right) l_1 + \frac{\kappa \tau}{2} l_2 \approx \frac{\kappa \tau}{2} (a_1 - a_2)
\]

\[
\left(1 - \frac{\kappa \tau}{2} + \frac{1}{\varepsilon}\right) l_2 + \frac{\kappa \tau}{2} l_1 \approx \frac{\kappa \tau}{2} (a_2 - a_1)
\]

From these equations, we can derive the linearized solutions for labor supply and consumption, (8) and (9).

In order to use the bias correction method, we assume that consumption and labor supply follow a normal distribution with a (possibly) non-zero mean:

\[
l_i \sim N\left(\mu_i, \sigma_i^2\right)
\]

\[
c_i \sim N\left(\mu_c, \sigma_c^2\right)
\]

Assuming that the two shocks are uncorrelated, the variance can be derived from the linearized solution

\[
\sigma_i^2 = \frac{1}{2} \left(\frac{\varepsilon \kappa \tau}{1 + \varepsilon - \varepsilon \kappa \tau}\right)^2 \sigma_a^2
\]

\[
\sigma_c^2 = \left[1 - \frac{\kappa \tau}{1 + \varepsilon - \varepsilon \kappa \tau} + \frac{1}{2} \left(\frac{\kappa \tau}{1 + \varepsilon - \varepsilon \kappa \tau}\right)^2\right] \sigma_a^2
\]

As the last preparation step, we take the expectation of the nonlinear equations for the two labor terms to compute the two mean terms:

\[
\mu_i = \left(\frac{\varepsilon}{1 + \varepsilon}\right) \left[\frac{- (1 + \varepsilon) \kappa \tau}{2 (1 + \varepsilon - \varepsilon \kappa \tau)} + \frac{(1 + \varepsilon)^2 (\kappa \tau)^2}{4 (1 + \varepsilon - \varepsilon \kappa \tau)^2}\right] \frac{\sigma_a^2}{2}
\]

\[
= \left(\frac{\varepsilon}{1 + \varepsilon}\right) \left[\frac{-2 (1 + \varepsilon)^2 \kappa \tau + (1 + \varepsilon)^2 (\kappa \tau)^2}{4 (1 + \varepsilon - \varepsilon \kappa \tau)^2}\right] \frac{\sigma_a^2}{2}
\]

and

\[
\mu_c = \frac{\mu_i}{\varepsilon}.
\]
When Utility is Non-Separable

In section 2, we only considered the case of a utility function that is separable between consumption and labor. In this appendix section, we relax this assumption and use a general form for utility that allows for non-separability between consumption and labor/leisure and examine whether the same conclusion for optimal revenue sharing rule applies or not. Specifically, we use the periodic utility function:

$$U_{it} = \frac{C_{it}^{\theta} (1 - L_{it})^{1-\theta}}{1 - \sigma} - 1,$$  \hspace{1cm} (36)

which is a constant relative risk aversion (CRRA) transformation of a Cobb-Douglas function of consumption and the amount of leisure. This function would be separable between consumption and leisure if and only if the degree of relative risk aversion is unity \((\sigma = 1)\).

In general, socially optimal allocation (i.e. complete markets solution) would require:

$$C_{1t}^{\theta(1-\sigma)-1} (1 - L_{1t})^{(1-\theta)(1-\sigma)} = C_{2t}^{\theta(1-\sigma)-1} (1 - L_{2t})^{(1-\theta)(1-\sigma)},$$  \hspace{1cm} (37)

which implies that marginal utilities of consumption of the two countries are equated. However, this condition does not necessarily imply that consumption levels in the two countries are same. With separable utility (or \(\sigma = 1\) in equation 36), the optimal output sharing rule \(\kappa_f = 1\) generates identical consumption between the two countries, which is a property of the socially optimal allocation. However, under non-separable utility function, identical consumption may not be the socially optimal allocation, which means that the previous optimal revenue sharing rule does not apply any more.

When bonds are introduced in the model with non-separable utility, it becomes more complicated to analyze optimal revenue sharing rule. The behavior of the bond economy can be expressed as a convex combination of financial autarky and the complete markets economy, and the weight depends on the parameter values (in particular, discount factor and persistence of shocks). In the case when the allocation of bond economy is not too far from the complete markets outcome, the additional risk sharing through fiscal transfers may not increase social welfare.
References


Table 1. Welfare gains of revenue sharing

Conditional welfare gains *(unconditional welfare gains)*

<table>
<thead>
<tr>
<th>Bond holding costs (ζ)</th>
<th>0.0001</th>
<th>0.001</th>
<th>0.01</th>
<th>0.1</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Degree of revenue sharing (κ)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.5</td>
<td>-0.386</td>
<td>-0.010</td>
<td>0.033</td>
<td>0.040</td>
<td>0.041</td>
</tr>
<tr>
<td></td>
<td>(-0.049)</td>
<td>(0.050)</td>
<td>(0.035)</td>
<td>(0.030)</td>
<td>(0.029)</td>
</tr>
<tr>
<td>1</td>
<td>-1.034</td>
<td>-0.035</td>
<td>0.069</td>
<td>0.081</td>
<td>0.082</td>
</tr>
<tr>
<td></td>
<td>(-0.219)</td>
<td>(0.114)</td>
<td>(0.075)</td>
<td>(0.062)</td>
<td>(0.060)</td>
</tr>
<tr>
<td>1.5</td>
<td>-2.226</td>
<td>-0.096</td>
<td>0.106</td>
<td>0.121</td>
<td>0.121</td>
</tr>
<tr>
<td></td>
<td>(-0.740)</td>
<td>(0.194)</td>
<td>(0.116)</td>
<td>(0.095)</td>
<td>(0.091)</td>
</tr>
<tr>
<td>2</td>
<td>-4.743</td>
<td>-0.250</td>
<td>0.138</td>
<td>0.152</td>
<td>0.151</td>
</tr>
<tr>
<td></td>
<td>(-2.378)</td>
<td>(0.274)</td>
<td>(0.154)</td>
<td>(0.123)</td>
<td>(0.119)</td>
</tr>
<tr>
<td>2.4</td>
<td>-9.376</td>
<td>-0.574</td>
<td>0.147*</td>
<td>0.162*</td>
<td>0.160*</td>
</tr>
<tr>
<td></td>
<td>(-6.410)</td>
<td>(0.224)</td>
<td>(0.163)</td>
<td>(0.137)</td>
<td>(0.134)</td>
</tr>
</tbody>
</table>

Numbers are percentage changes in welfare (measured in permanent changes in certainty-equivalent consumption) when revenue sharing is implemented with a degree κ. Initial state is no sharing (κ=0). Conditional welfare is calculated by taking discounted sum of utility over time (time period is set at 500 periods) following a one-time change in κ.

Numbers in parenthesis are unconditional welfare gains that represent percentage changes in unconditional welfare between initial state (κ=0) and a new state with revenue sharing with a degree κ (measured in permanent changes in certainty-equivalent consumption).

*Maximum conditional welfare gains are attained at κ=2.4 (optimal degree of revenue sharing).*
Table 2. Sensitivity analysis: cross-country correlations of shocks

Conditional welfare gains (*unconditional welfare gains*)

Bond holding cost ($\zeta$) is fixed at 0.1

<table>
<thead>
<tr>
<th>Degree of revenue sharing ($\kappa$)</th>
<th>Benchmark model</th>
<th>Positive cross-country shock correlation at 0.5</th>
<th>Negative cross-country shock correlation at -0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.5</td>
<td>0.033 (0.035)</td>
<td>0.020 (0.015)</td>
<td>0.060 (0.045)</td>
</tr>
<tr>
<td>1</td>
<td>0.069 (0.075)</td>
<td>0.041 (0.031)</td>
<td>0.121 (0.093)</td>
</tr>
<tr>
<td>1.5</td>
<td>0.106 (0.116)</td>
<td>0.060 (0.047)</td>
<td>0.181 (0.142)</td>
</tr>
<tr>
<td>2</td>
<td>0.138 (0.154)</td>
<td>0.076 (0.062)</td>
<td>0.229 (0.185)</td>
</tr>
<tr>
<td>2.4</td>
<td>0.147* (0.163)</td>
<td>0.081* (0.069)</td>
<td>0.242* (0.206)</td>
</tr>
</tbody>
</table>

Bond holding cost ($\zeta$) is fixed at 0.001

<table>
<thead>
<tr>
<th>Degree of revenue sharing ($\kappa$)</th>
<th>Benchmark model</th>
<th>Positive cross-country shock correlation at 0.5</th>
<th>Negative cross-country shock correlation at -0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.5</td>
<td>-0.010 (0.050)</td>
<td>-0.005 (0.025)</td>
<td>-0.015 (0.075)</td>
</tr>
<tr>
<td>1</td>
<td>-0.035 (0.114)</td>
<td>-0.018 (0.057)</td>
<td>-0.053 (0.171)</td>
</tr>
<tr>
<td>1.5</td>
<td>-0.096 (0.194)</td>
<td>-0.048 (0.097)</td>
<td>-0.144 (0.291)</td>
</tr>
<tr>
<td>2</td>
<td>-0.250 (0.274)</td>
<td>-0.125 (0.137)</td>
<td>-0.374 (0.412)</td>
</tr>
</tbody>
</table>

Contemporaneous correlation between two countries’ productivity shocks are set at 0.5 and -0.5. Benchmark case is zero correlation.

*Maximum conditional welfare gains are attained at $\kappa$=2.4 (optimal degree of revenue sharing).
Figure 1. Plot of welfare on degree of revenue sharing ($\kappa$) at different tax rates

Endowment economy ($\varepsilon = 0$)
Figure 2. Plot of welfare on degree of revenue sharing ($\kappa$) with and without steady-state correction ($\tau = 0.5, \varepsilon = 1$)
Figure 3. Impulse responses to 1% increase in productivity at home country ($\zeta=0.0001$)
Figure 4. Impulse responses to 1% increase in productivity at home country ($\zeta=0.1$)
Figure 5. Time series plot of welfare when degree of revenue sharing ($\kappa$) changes from 0 to 1

--- : $\zeta = 0.001$, ---- : $\zeta = 0.01$, ----- : $\zeta = 0.1$, : Bond holding costs
Figure 6. Conditional welfare gains of revenue sharing when only one tax is present

- Consumption tax only at 40.6%
- Consumption tax only at 17.8%
- Consumption tax only at 1.4%
- Labor income tax only at 35%
- Labor income tax only at 18.4%
- Labor income tax only at 1.7%
- Capital income tax only at 56.4%
- Capital income tax only at 37.5%
- Capital income tax only at 8%

---: $\zeta = 0.0001$, ---: $\zeta = 0.001$, ----: $\zeta = 0.01$, .....: $\zeta = 0.1$  $\zeta$: Bond holding costs