Optimal Group Contest*

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Abstract

Types of group contest include *weakest-link group contests* (comparing the weakest performances of each group), *average-performance group contests* (comparing the average performances), and *best-shot group contests* (comparing the best performances). We show that, if the designer of a contest puts more weight on the performance of high-ability members – star players, for example – than on low-ability members, the designer encourages the low-ability member to free-ride on the high-ability members, so that the high-ability members exert even greater efforts. To this end, the contest designer's optimal choice approximates the format of the best-shot group contest. With more weight on the low-ability member's performance, the optimal choice approximates the weakest-link group contest to undermine the free-riding. We also show that the approximations of the aforementioned results work better with more convex effort-cost function *and/or* smaller heterogeneity of members in a group.

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1 Introduction

Why do different group sports have different criteria for selecting the winner? For example, the rule of *four-ball stroke play* in golf stipulates that each group's score is measured by the best score of the members in the group (*best-shot group contest*), while many other group contests such as *shooting* and *archery* sum the scores of all group members to determine the winning team (*the average performance contest*). Another type of group contest considers the performance of the weakest player in each group as the group's performance, e.g., *team pursuit race* in speed skating and cycling event (*the weakest-link contest*).[1] We address the reason for using different criteria from the perspective of a contest designer who chooses a rule for a group contest.

We consider a group contest in which n symmetric groups consisting of two members compete against each other. The two members in each group have different valuations on winning but have identical effort-cost structures. The different valuations of the members in each group can be interpreted as different motivations of the members or different levels of ability.[2] We construct a continuum set of feasible group contests using *constant elasticity of substitution* (henceforth, CES) impact functions, which include the three aforementioned group contests: the best-shot, the average-performance, and the weakest-link contest. First, for a given contest rule, we characterize the behaviors of the players. Then, based on these behaviors, we consider how the contest designer chooses the optimal group contest rule, i.e., the type of contest that maximizes the objective function. The objective function of the contest organizer is described as the weighted sum of the players' efforts. For instance, the designer may put more (or less) weight on the better performing players to attract more attention from the fans and consequently increase entrance fee revenue. Based on the different criteria, the weight will be

[1] In the case of team pursuit in cycling, each team consists of four riders and its final record is measured as the finishing time of the third rider. Considering that it is common for one rider in each team to be a "death puller" – a player at the front of his team's formation to increase the pace at the cost of exhausting himself – a cycling team pursuit race has the feature of *the weakest-link contest*.

[2] We show this in Section 2, i.e., the different valuation with identical cost functions within a group are mathematically equivalent to equal valuations with different cost functions.

different across sports.

Our main results are as follows. The low-ability member's free-riding on the high-ability member within each group becomes more severe as the contest becomes closer to the best-shot type of contest and becomes weaker as the contest becomes closer to the weakest-link type of contest. Given this observation, let us suppose that the contest designer puts more weight on the performance/effort of high-ability members than on that of the low-ability members in the groups. By shifting the contest toward the best-shot type, the designer can worsen the low-ability members' free riding problem, which in turn encourages the high-ability members to exert more effort. At the margin, the benefit must exceeds the cost because the high-ability players' efforts are highly appreciated; thus, the optimal contest must become more like the best-shot type contest as the weight on the high ability member's effort increases. On the other hand, the optimal contest moves closer to the weakest-link type as the weight on the low ability member's effort increases. We show that as the weight on the high ability member approaches infinity (compared to the weight on the low-ability member), the optimal contest converges to the best-shot type. On the other hand, as the weight on the high ability member nears zero, the optimal contest converges to the weakest-link type.

We also find that as the convexity of the effort-cost function increases or the heterogeneity of members within the groups decreases, the free riding problem becomes less severe and, accordingly, the above mentioned approximation works better. Suppose that the designer puts more weight on the high ability member's effort. A more convex cost function results in less free-riding of low ability members, which means that the designer should shift the contest more towards the best-shot type in order to ensure the sufficient free-riding of the low-ability member (so that the high ability member exerts enough effort). On the other hand, suppose that the designer puts more weight on the low ability member's effort. A more convex cost function results in less free-riding or, equivalently, provides more incentive for a stronger effort from the low ability member; therefore, the designer shifts the contest more toward the weakest-link type in order to ensure that the low-ability member free-rides even less on the high-ability member (so that the low ability member exerts more effort). Thus, the approximation works better with a more convex effort-cost function. With the same logic, we see that less heterogeneity between the two different-ability members within each group makes the approximation work better.

Beginning with the seminal work of Katz et al. (1990) who studied a contest between groups where the players in a group are symmetric and their efforts are perfectly substitute, the literature on the theory of group contests has been growing rapidly.[3] Baik (1993, 2008) considers a group contest in which the individual members' efforts within each group are perfect substitutes and the members in the group have asymmetric valuations on winning the contest. He shows that there exists an extreme free riding problem at equilibrium, where only the highest-valuation players in each group supply positive effort and the rest of the players do not supply anything. Departing from the perfect substitute technology in measuring a group effort from its individual members' efforts, Lee (2012) and Chowdhury et al. (2013) studied the weakest-link group contest and the best-shot group contest. Lee (2012) assumes that the members' efforts in each group are perfectly complementary, i.e. each member has a kind of veto power in its group, and shows that there is no free riding at equilibrium, such that all the members in each group exert equal effort. Chowdhury et al. (2013) adopted the best-shot technology in a group contest setting where each group's effort is defined as the highest effort level within the group. They showed that there always exists an equilibrium in which the highest-valuation players in each group are exploited by other low-valuation players, as in Baik (1993, 2008), and they also found that there exist perverse equilibria where the high-valuation players in each group free-rides on other low-valuation players in the group. Sheremeta (2011) studied the three types of group contests – the perfect-substitutes, the weakest-link, and the best-shot group contest – experimentally and ascertained the theoretical predictions on them. Further, Kolmar and Rommeswinkel (2013) used a generalized CES-form group impact function that allows for different degrees of complementarity among individual members' efforts in each group, ranging from the weakest-link to the perfect-substitute technology (not including the

[3]For details, see Corchón (2007), Garfinkel and Skaperdas (2007), and Konrad (2009).

best-shot technology), and investigate the effect of the complementarity among group members on the equilibrium behaviors of the players in the contest. All of these papers assume that the probability of winning of a group in group contests is determined stochastically, i.e., they follow the Tullock contest success function, while one strand of the literature on the theory of group contests employs the all-pay auction contest success function (Baye et al., 1996) that determines the winning group in the perfectly deterministic way: Baik et al. (2001), Topolyan (2013), Chowdhury et al. (2013), and Barbieri et al. (2013).

Among the aforementioned papers, our paper is closest to that of Kolmar and Rommeswinkel (2013) in which they generalize the group contest models by using the class of CES-form group impact functions and characterize the equilibrium. Standing on the basis of their analytical/technical contributions, we aim to contribute to the literature on group contests by studying the endogenous choice of the group impact function in a contest. We provide a rationale for choosing a CES impact function that will be applied as a rule in a group contest. While Kolmar and Rommeswinkel (2013) considered an n-group contest in which each group is composed of an arbitrary number of heterogenous players and it has different strength relatively to the other groups, we consider a symmetric n-group contest in which each group consists of two heterogenous players and the players face a convex effort-cost function. In our simplified n-group symmetric contest, we study the endogenous determination of a group impact function from the viewpoint of a contest designer and the effect of the heterogeneity of the players in the groups and the convexity of the effort-cost structure on the contest designer's decision on the group impact function.

Since our paper considers the optimal choice of a contest designer, it is also related to the literature on optimal contests: Michaels (1998), Dasgupta and Nti (1998), Glazer and Hassin (1998), Gradstein and Konrad (1999), Moldovanu and Sela (2001, 2006), Szymanski (2003), Nti (2004), Szymanski and Valletti (2005), Runkel (2006), Cohen et al. (2008), Sisak (2009), Fu and Lu (2010), Riis (2010), Fu et al. (2011), Ryvkin (2011), Moldovanu et al. (2012), Epstein et al. (2013), and Franke et al. (2013). These studies consider the problem of designing a contest in order to elicit maximum aggregate efforts from the contestants and suggest the various solutions in terms of the optimal number of the contestants, the optimal timings for the contestants to move, the optimal structure of the contest success function, optimal reward systems (allocation of the prizes of the contest), the optimal information structure, the optimal sorting of contestants, and the optimal bias of contest rule. To the best of our knowledge, however, none of them explore the optimal *group* contest, i.e., the optimal type of group impact function which will be applied to determine each group's production and select the winning group.

Lastly, our paper is relevant to the literature on the non-cooperative provision of public goods and the free-riding, because winning the contest in our paper is assumed to benefit all players within the winning group regardless of their contributions to the winning. To name just a few, the relevant studies are as follows: Bergstrom et al. (1986), Hirshleifer (1983), Vicary (1997), Vicary and Sandler (2002), Cornes and Hartley (2007), and Barbieri and Malueg (2008).

The paper proceeds as follows. In Section 2, we set up our model; it is analyzed in Section 3. Section 4 presents our conclusions. Proofs are in the Appendix.

2 The Model

Let us consider a group contest in which n groups compete against each other to win the contest, where $n \geq 2$. Each group consists of two players who exert effort to win the contest and who have different valuations for winning the contest, b_H and b_L, respectively, where $b_H > b_L$. We do not argue that this reflects the real world, but we assume these conditions to simplify our analysis and comparative statics.

Parameters b_H and b_L measure the degree of heterogeneity between the players within the group; more specifically, they show the efficiency of players in a contest or their motivation level to win the contest. Within the group, the player whose valuation is b_H is the high-ability or highly motivated player, and the player whose valuation is b_L is the low-ability or poorly motivated player.

Let x_{iH} and x_{iL} represent, respectively, the effort levels expended by the high-ability player (player H) and the low-ability player (player L) in group i. The effort of player j in group i, x_{ij}, incurs the cost $c \cdot {x_{ij}}^k$ for $j \in \{H, L\}$, where $k \geq 1$. Namely, the players face the same non-decreasing marginal cost. Although we assume that the coefficient c is identical for all the players, this is not a restriction.[4]

Using the Tullock-form contest success function that is extensively used in the literature on the theory of contests, group i's probability of winning the contest, p_i, is given by

$$p_i = p_i(X_1, X_2, \cdots, X_n) = \begin{cases} \frac{X_i}{\sum_{\ell=1}^n X_\ell} & \text{if } \sum_{\ell=1}^n X_\ell > 0 \\ 1/n & \text{if } \sum_{\ell=1}^n X_\ell = 0 \end{cases} \tag{1}$$

where X_i is group i's aggregate effort (or performance) and is determined by group i's impact function which maps from the efforts of the individual players in group i, x_{iH} and x_{iL}, to the aggregate level. We define X_i as $({x_{iH}}^\rho + {x_{iL}}^\rho)^{\frac{1}{\rho}}$ which is a CES (constant elasticity of substitution) group impact function, where the elasticity of substitution between the players' efforts, σ, is $\frac{1}{1-\rho}$. For different values of ρ, the iso-impact curves, i.e. $X_i = a\ constant$, look like those in Figure 1.

[Figure 1 about here.]

The limit case where $\rho \to +\infty$ is represented by impact function $X_i = \max\{x_{iH}, x_{iL}\}$, and this impact function implies *the best-shot* group contest (Chowdhury et al. (2013)) in which only the best performance within each group is measured as the performance of the group. The case where $\rho = 1$ ($\sigma = \infty$; perfect-substitute case) is represented by impact function $X_i = x_{iH} + x_{iL}$, and this implies *the average performance* contest (Baik (1993, 2008)), in which the summation

[4]Consider an alternative model in which player j's valuation for winning the contest is B_j and his effort-cost function is $c_j \cdot {x_{ij}}^k$. If we reformulate this model so that player j has valuation $b_j := \frac{B_j}{c_j}$ and effort-cost function ${x_{ij}}^k$, then the reformulated model is isomorphic to the original (for detail, refer to equation (2)). Thus, in our model, b_H and b_L summarizes the heterogeneity between the players within the group in terms of the valuation on the winning and/or the efficiency in exerting effort, and in this sense, the player having high valuation b_H can be thought of as being high-ability and/or highly motivated.

of the players' performances within each group is used as the group's performance. When ρ goes to 0 ($\sigma = 1$; unit elasticity), X_i has the Cobb-Douglas form, or $X_i = x_{iH} \cdot x_{iL}$. Finally the limit case where $\rho \to -\infty$ ($\sigma = 0$; perfect-complementary case) is represented by impact function $X_i = \min\{x_{iH}, x_{iL}\}$, which implies *the weakest-link* group contest (Lee (2012)), in which only the lowest performance within each group is measured as the group's overall performance.

Note that the contest is parametrized by the value of ρ that determines the curvature of an iso-impact curve and thus the type of group contest. So, the contest designer (with an appropriately defined objective function – which will be given in section 3.3) will choose the value of ρ in order to obtain the goal.

Letting π_{ij} represent the expected payoff of player $j \in \{H, L\}$ in group $i \in \{1, 2, \cdots, n\}$, the payoff of player j in group i is

$$\pi_{ij} = b_j \cdot p_i(X_1, \cdots, X_n) - c \cdot {x_{ij}}^k, \tag{2}$$

where $X_i = ({x_{iH}}^\rho + {x_{iL}}^\rho)^{\frac{1}{\rho}}$.

We formally consider the following game. First, the contest designer determines the value of ρ. Second, after observing the value of ρ chosen by the designer, all the players in all the groups choose their effort levels independently and simultaneously. All of the above is common knowledge among the players and the contest designer, and we employ Nash equilibrium as our solution concept.

3 Analysis of Model

3.1 Transformation of contest

As in Epstein and Mealem (2009), we first transform the contest into one where the players have the same linear effort-cost function. This transformation is not necessary to calculate the equilibrium in principle; however, it makes computation simpler.

We define y_{ij} as follows:

$$y_{ij} := {x_{ij}}^k. \tag{3}$$

Then the impact function of group i with arguments (y_{iH}, y_{iL}) becomes

$$X_i = ({x_{iH}}^\rho + {x_{iL}}^\rho)^{\frac{1}{\rho}} = ({y_{iH}}^{\frac{\rho}{k}} + {y_{iL}}^{\frac{\rho}{k}})^{\frac{1}{\rho}}. \tag{4}$$

This new contest has the following primitives: player j in group i has cost function $c \cdot y_{ij}$ and valuation of winning the contest b_j. Thus, the expected payoff for player j in group i is

$$\pi_{ij} = b_j \cdot p_i(X_1, \cdots, X_n) - c \cdot y_{ij}, \tag{5}$$

where $X_i = ({y_{iH}}^{\frac{\rho}{k}} + {y_{iL}}^{\frac{\rho}{k}})^{\frac{1}{\rho}}$. In this transformed contest, the players' marginal cost of exerting effort is "normalized" to a constant, c. Note also that ${y_{iH}}^{\frac{\rho}{k}} + {y_{iL}}^{\frac{\rho}{k}}$ within the parenthesis of X_i becomes linear, i.e., $y_{iH} + y_{iL}$, when $\rho = k$, which means that the value of k that determines the curvature of the cost function in the original contest is equivalent to the value of ρ that determines the curvature of the iso-impact curve in the original CES impact function.

3.2 Characterization of player decisions on exerting efforts

We analyze the players' decisions (primarily, how much effort they will exert) in a certain type of group contest that is chosen by the contest designer. After observing the value of ρ chosen by the contest designer, each player chooses his effort level for winning, and thus the players' decisions depend on the value of ρ. We derive the equilibrium effort levels of the players for given contest rule ρ.

3.2.1 Case 1: $\rho < k$

First, we calculate the equilibrium for the given ρ that is less than k. Given the value of ρ, player j in group i chooses y_{ij} which maximizes his expected payoff π_{ij} in equation (5). Thus, the best responses of player H and player L in each group i are characterized by the following first-order conditions, respectively:

$$b_H \frac{d}{dy_{iH}}\left(\frac{X_i}{\sum_\ell X_\ell}\right) = c \quad \text{and} \quad b_L \frac{d}{dy_{iL}}\left(\frac{X_i}{\sum_\ell X_\ell}\right) = c. \tag{6}$$

By using the first-order conditions for maximizing the payoffs of the players in each group i and concentrating on the symmetry between the groups, we derive the players' effort levels to be exerted for a given value of ρ.[5] Lemma 1 shows this.

Lemma 1 *When $\rho < k$, player H and player L in each group exert efforts x_H and x_L, respectively, as follows:*

$$x_H = \left(\frac{n-1}{ckn^2} \cdot \frac{b_L}{(b_H/b_L)^{\frac{\rho}{k-\rho}} + 1}\right)^{\frac{1}{k}} \left(\frac{b_H}{b_L}\right)^{\frac{1}{k-\rho}}, \quad x_L = \left(\frac{n-1}{ckn^2} \cdot \frac{b_L}{(b_H/b_L)^{\frac{\rho}{k-\rho}} + 1}\right)^{\frac{1}{k}}.$$

Note that $x_H > x_L > 0$ because $\rho < k$, i.e., all the players exert positive efforts and the high-ability players exert more than the low-ability players.

3.2.2 Case 2: $\rho = k$

If the value of ρ is equal to k, group i's impact function in (4) becomes

$$X_i = (y_{iH} + y_{iL})^{\frac{1}{\rho}}. \tag{7}$$

Note that the term within the parenthesis in (7) is linear. This implies that group i's performance depends on the linear combination of the individual players' effort levels in the group. Therefore, in this case of $\rho = k$, the group contest has exactly the same feature as the one in Baik (1993, 2008), where each group's total effort is determined as the sum of its members' efforts and the players have a constant maginal cost of exerting effort. We thus have the same result as in Baik (1993, 2008).

Lemma 2 *When $\rho = k$, player H in each group exerts positive effort and player L does not exert any effort:*

$$x_H = \left(\frac{(n-1)b_H}{ckn^2}\right)^{\frac{1}{k}}, \quad x_L = 0.$$

[5]The second-order conditions for maximizing players' payoffs are satisfied. For details, see the Appendix A.1.

From Lemma 2 we see that there appears a serious free riding problem within the groups if the value of ρ equal to k is chosen.

3.2.3 Case 3: $\rho > k$

As in Section 3.2.1, we compute the players' effort levels for given value of ρ, which is greater than k, by solving the first-order conditions for maximizing the players' expected payoffs in equation (5). Consequentially, we have the same first-order conditions as in (6) and obtain the same solutions x_H and x_L that satisfy the first-order conditions as in Lemma 1. The only difference when $\rho > k$ is the relative size of x_H and x_L, comparing to those in Lemma 1. Since $\rho > k$, we have $x_H < x_L$. (Note that $x_H > x_L$ in Lemma 1.) Interestingly, it implies that the low-ability players in each group exert more effort than the high-ability players. (This is opposite to in Lemma 1.) However, these interesting looking effort levels x_H and x_L are not the equilibrium effort levels of the players, because they are the local maximizers, not the global ones. In other words, in case of $\rho > k$, although each player's effort x_H and x_L satisfy the first-order and the second-order condition for maximizing the expected payoff of each player for given the other players' efforts, they maximize the expected payoffs locally, not globally. Hence, we cannot claim that they are the equilibrium effort levels of the players.

More specifically, when ρ is greater than k, the solutions to the first-order conditions in (6) are meaningful as the maximizers only when the following two conditions hold: $\rho < 2k - 1 + \frac{2}{n}$ and $\frac{b_H}{b_L} < \left(\frac{n(k-1)+2}{n(\rho-k)}\right)^{\frac{\rho-k}{\rho}}$. Unless these conditions are met, the solutions to the first-order conditions fail to satisfy the second-order conditions for the maximization.[6] However, even though these conditions are met, i.e., the second-order conditions are satisfied, it does not assure that the solutions satisfying the first-order conditions are the global maximizers; indeed, they are the local maximizers in our model.[7] Therefore, the players' effort levels x_H and x_L

[6]See the Appendix A.2 for detail.

[7]See the following numerical example: $\rho = 1.5$, $k = 1$, $n = 2$, $b_H = 1.25$, $b_L = 1$, and $c = 1$. The conditions required for satisfying the second-order conditions are satisfied with these parameter values. We then have the numerical solutions to the first-order conditions, $x_H = 0.1058$ and $x_L = 0.1653$. At these solutions, the high-ability player has its expected payoff $\pi_H = 0.5191$, while she gets higher expected payoff $\pi_H^d = 0.5394$

obtained from the first-order conditions are not the equilibrium outcomes in case of $\rho > k$.

Pérez-Castrillo and Verdier (1992), Baye et al. (1994), and Cornes and Hartley (2005) investigated a contest among n individual players in which the contest success function for player i is defined as $p_i = \frac{x_i^r}{\sum_{j=1}^n x_j^r}$ where $r(>0)$ represents the technology of players' production function in the contest, and showed that the structure of the equilibrium in the contest changes according to the value of r: when $r \leq \frac{n}{n-1}$, there is a unique symmetric pure-strategy Nash equilibrium. Otherwise, there exist asymmetric pure-strategy equilibria or no pure-strategy equilibrium. The case of $\rho > k$ in our model seems closely related to the work of Baye et al. (1994). They show that, in two-player individual contest, the symmetric solution computed from the players' first and second-order conditions does not necessarily yield a global maximum when $r > 2$, i.e., the production functions of the players have the nature of increasing return in the contest success function. Although, in this paper, we consider the contest among groups not individuals, our issue on the equilibrium structure in case of $\rho > k$ must be relevant to theirs. Since we deal with the complementarites (ρ) between the individual players' efforts within each group as well as the technologies (k) of the players in the contest, the issue on the equilibrium structure would be more complicated than in the individual contest setting and beyond the scope of this paper. So, we exclude the case of $\rho > k$ from our consideration and leave it for our future research.[8]

3.2.4 Case 4: $\rho \to -\infty$

As the value of ρ goes to $-\infty$, the group impact function for group i becomes $X_i = \min\{x_{iH}, x_{iL}\}$. This means that each group's performance is measured by the lowest effort level among the group members. Hence, in the case where ρ goes to $-\infty$, the group contest becomes the

when she deviates from $x_H = 0.1058$ to $x_H^d = 0$. This means $x_H = 0.1058$ is not the global maximizer for given the other players' effort levels. Actually, the numerical plot of her expected payoff reveals that $x_H = 0.1058$ is a local maximizer and the global maximizer is 0. We appreciate the anonymous referee's giving this numerical example and relevant insightful comments on that.

[8]Fortunately, the exclusion of the case of $\rho > k$ does not result in any critical hole that makes us difficult in having the main results and findings of this paper.

weakest-link contest as in Lee (2012) where efforts of the players in each group are perfect complements. We thus have the same results as in Lee (2012): there exist multiple pure-strategy Nash equilibria in which all the players in each group choose an equal effort level and a unique coalition-proof equilibrium. As in Lee (2012), in this paper, we focus on the coalition-proof equilibrium effort levels of the players. Lemma 3 presents them.

Lemma 3 *When ρ goes to $-\infty$, player H and player L in the groups exert the same level of effort:*

$$x_H = x_L = \left(\frac{(n-1)b_L}{ckn^2}\right)^{\frac{1}{k}}.$$

3.2.5 Case 5: $\rho \to +\infty$

Although our analysis is confined to the case where $\rho \leq k$ due to its analytical intricacy in Section 3.2.3, the limit case where $\rho \to +\infty$ is still solvable. As the value of ρ goes to $+\infty$, the group impact function for group i becomes $X_i = \max\{x_{iH}, x_{iL}\}$. This means that each group's performance is measured by the highest effort level among the group members. Hence, in this case, the group contest becomes the best-shot contest, as in Chowdhury et al. (2013) where each group has the best-shot type of group impact function, and we thus have the same results as in Chowdhury et al. (2013): there always exists an equilibrium where the high-ability players in each group are active in the contest and there may exist another type of equilibria where the active players in some (or all) groups are not the high-ability ones. Here we focus on the players' effort levels in the ever-present equilibrium, and they are given in the following Lemma.

Lemma 4 *When ρ goes to $+\infty$, player H and player L in each group exert efforts x_H and x_L, respectively, as follows:*

$$x_H = \left(\frac{(n-1)b_H}{ckn^2}\right)^{\frac{1}{k}}, \quad x_L = 0.$$

Lemma 4 says that the high-ability players in each group exert positive effort and the low-ability players do not exert any effort.

3.3 The optimal type of group contest

So far, we have examined the player decisions on their effort levels for a given type of group contest (i.e. given ρ). The effort levels of the players are summarized in Figure 2, and the ratio of the high-ability player's effort to that of the low-ability player is given in Figure 3.

[Figure 2 about here.]

[Figure 3 about here.]

The solid line and the dotted line in Figure 2 represent, respectively, the effort levels of the high-ability player (x_H) and the low-ability player (x_L) in each group for different values of ρ. We can see that the effort levels of the players have different characteristics according to the value of ρ. When $\rho < k$, the high-ability player supplies more efforts than the low-ability player, i.e. $x_H > x_L$, and x_H decreases; however, x_L increases as the value of ρ decreases. When $\rho = k$, extreme free riding occurs: $x_H > x_L = 0$. In the limit case of $\rho \to -\infty$, both players expend equal efforts, $x_H = x_L$. When $\rho \to +\infty$, extreme free riding occurs as in the case of $\rho = k$. Figure 3 shows the efforts ratio between x_H and x_L, $\frac{x_H}{x_L}$, for each value of ρ.

Now we consider the contest designer's problem of choosing the value of ρ. We assume that the designer tries to generate the maximum total effort from the players participating in the contest so that the players' efforts or performances can excite and attract the largest possible audience. Specifically, the designer's objective function is defined as the following:

$$\max_{\rho} \sum_{i=1}^{n} (x_{iH} + \alpha x_{iL}), \tag{8}$$

where $\alpha \geq 0$ measures how much the designer values the effort (performance) of the low-ability player in each group relative to the effort of the high-ability player. If $\alpha < 1$, it implies that the designer puts more weight on the effort of the high-ability players in the contest than the low-ability players. For example, sports fans often pay more attention to star players; thus, increasing the effort of the star players will be more important to the contest designer than inducing effort from the other players. If the audiences are more interested in watching a high degree of cooperation/coordination among the members within each group rather than the performance of the most outstanding players in the groups, then the designer may focus more on the low-ability player's performance, because it is crucial to shape the overall performance of a group and will thus try to induce the low-ability player's effort as much as the high-ability player's effort. In this case, $\alpha \geq 1$.

As we saw in the previous section, the efforts of the players, x_H and x_L, vary according to the value of ρ. We first confine our attention to the case $\rho \in (-\infty, k)$ where the players' effort levels are the interior solutions which satisfy the first and second-order conditions for their maximum payoffs. We then find the optimal ρ for the given α in the designer's objective function while considering the findings in the other cases, $\rho \to -\infty$, $\rho = k$, and $\rho \to +\infty$.

From the results of the previous section, when $\rho \in (-\infty, k)$, all the players exert positive effort, and the effort levels are symmetric across the groups. Based on this information, the contest designer seeks to maximize the objective function:

$$\max_{\rho} \; n \cdot (x_H + \alpha x_L), \tag{9}$$

where x_H and x_L are the effort levels in Lemma 1. Solving the maximization problem for the contest designer, we determine the optimal value of ρ.

Proposition 5 *The contest designer chooses the optimal ρ as follows:*

$$\rho = \frac{\log(\frac{b_H}{b_L}) + k\log(\frac{1}{\alpha})}{\log(\frac{b_H}{b_L}) + \log(\frac{1}{\alpha})}. \tag{10}$$

Proposition 5 shows that the optimal value of ρ depends on the values of α, $\frac{b_H}{b_L}$, and k. The optimal ρ is presented in Figure 4 as a function of α for given values of $\frac{b_H}{b_L}$ and k. Figure 4 shows that (i) the optimal value of ρ is greater than 1 for $\alpha < 1$, it stays relatively high for α significantly smaller than 1 and approaches k as α goes to 0, (ii) the optimal value of ρ is equal to 1 for $\alpha = 1$, and (iii) the optimal value of ρ is smaller than 1 for $1 < \alpha < \frac{b_H}{b_L}$, and it becomes smaller and approaches negative infinity as α goes to $\frac{b_H}{b_L}$. Note that, as the value of ρ

increases from $-\infty$ to 1 and from 1 to $+\infty$, the group contest changes from the weakest-link contest to the average-performance contest and from the average-performance contest to the best-shot contest, respectively.

[Figure 4 about here.]

The intuition behind the first observation is the following. When the weight for the low-ability player's performance is low, i.e. $\alpha < 1$, the designer wants to increase the high-ability player's effort. The designer increases the free-riding of the low-ability player by increasing ρ (i.e., the contest becomes closer to best-shot type), so that the high-ability player has to further increase his effort. Since the weight α is smaller than 1, the weighted sum of efforts must increase at the margin. More specifically, observe Figures 2 and 3: as the value of ρ increases from $-\infty$ to k, the high-ability player's effort increases, while the low-ability player's effort decreases to zero. This implies that the low-ability player's free-riding on the high-ability player becomes more severe as the value of ρ increases, i.e., the contest becomes more like the best-shot type.

The intuition for the second observation is the following. If the designer puts the same weight on the efforts of high-ability players and low-ability players, i.e., only cares about the sum of players' effort levels regardless of who the active players are, she chooses $\rho = 1$ so that the group impact function for each group, equation (4), coincides with the objective function. Note that, as shown in Figure 2, both the high-ability player and the low-ability player in a group exert strictly positive efforts, which is different from the results in Baik (1993, 2008), because we employ a convex cost structure, i.e., $k > 1$, as in Epstein and Mealem (2009).

The intuition for the third observation is similar to that of the first observation: as the designer gives greater weight to the low-ability player's effort, i.e. $\alpha > 1$, the free-riding of the low-ability player should be minimized by selecting the weakest-link type of contest. As seen in Figures 2 and 3, as the value of ρ decreases and goes to $-\infty$, the effort gap between the high-ability and the low-ability player decreases and converges to the same level of effort. Note that the players exert an equal level of effort in the limit case $\rho \to -\infty$.

The following proposition summarizes the characteristics of the optimal ρ explained above and comparative statics on the optimal ρ with respect to α, k and $\frac{b_H}{b_L}$.

Proposition 6 *A characterization of optimal ρ is as follows.*

1. *For $\alpha = 0$, $\rho = k$.*
2. *For $0 < \alpha < 1$, (i) $1 < \rho < k$, (ii) $\frac{d\rho}{d\alpha} < 0$, (iii) $\frac{d\rho}{dk} > 0$, (iv) $\frac{d\rho}{d(b_H/b_L)} < 0$.*
3. *For $\alpha = 1$, $\rho = 1$.*
4. *For $1 < \alpha < \frac{b_H}{b_L}$, (i) $\rho < 1 < k$, (ii) $\frac{d\rho}{d\alpha} < 0$, (iii) $\frac{d\rho}{dk} < 0$, (iv) $\frac{d\rho}{d(b_H/b_L)} > 0$.*

The comparative static analysis in terms of k and $\frac{b_H}{b_L}$ can be understood as follows. First, consider the case $\alpha < 1$. In this case, the contest designer more highly appreciates the efforts exerted by the high-ability players than the low-ability players and thus selects ρ greater than 1, i.e., the best-shot type of contest. Now suppose that k increases to k', namely, the effort-cost structure for the players becomes more convex, which means the marginal effort-cost for the players increases for sufficiently high effort levels. The players then will exert less effort, and the low-ability player's free-riding decreases compared to that before the change of the cost structure (x_H, x_L, and x_H/x_L decrease; see Lemma 1), and the contest designer will choose a higher value of ρ in order to induce more efforts from the high-ability players (or supplement the reduced high-ability players' efforts due to the change of the cost structure). Although the increased ρ brings less efforts from the low-ability players, its net effect on the contest designer's objective function is positive because the designer more highly values the high-ability players' efforts compared to the low-ability players' efforts. In other words, the designer will increase the value of ρ to offset the negative effect of the increased effort cost, especially the high-ability players' cost.

Now suppose that $\frac{b_H}{b_L}$ increases to $\frac{\bar{b}_H}{\bar{b}_L}$. This implies that the heterogeneity of the players in a group increases. Then the ratio of the effort levels between the high-ability and the low-ability players ($\frac{x_H}{x_L}$) increases (see Lemma 1), which means that the high-ability player will expend more and the low-ability player will expend less, relatively, compared to the original efforts before the change of $\frac{b_H}{b_L}$; the low-ability player's free-riding will also become more severe. Hence, the contest designer will select a lower value of ρ in order to induce more efforts exerted by the low-ability player (or supplement the low-ability players' reduced efforts due to the increased heterogeneity). In a sense, the increased heterogeneity between the players gives too strong an incentive to the high-ability players but too weak an incentive to the low-ability players, and hence the contest designer will try to balance them by decreasing the value of ρ.

We can also understand the results for the case of $\alpha > 1$ by applying similar logic to that mentioned above. The comparative-static analysis with respect to k and $\frac{b_H}{b_L}$ is summarized in Figure 5 and 6.

[Figure 5 about here.]

[Figure 6 about here.]

4 Conclusion

We consider the optimal type of group contest from the perspective of a contest designer who seeks to maximize the weighted sum of the efforts by the players. By considering CES-type group impact functions, we calculate the effort levels exerted by the players for any given group contest (that is characterized by a parameter for a CES-type group impact function). Then, using the calculated effort levels of the players, we calculate the solution of the contest designer's problem of choosing the optimal group contest.

The main results are as follows. First, if the contest designer pays more attention on the performance/effort of the high-ability members – for example, star players – than that of the low-ability members in each group, the optimal choice of ρ approximates the best-shot type of group contest. Second, if the contest designer cares more about the low-ability members' performance/effort, the optimal choice approximates the weakest-link group contest. Lastly, as the convexity of the effort-cost increases *and/or* as the heterogeneity of the members within the group decreases, the approximations of the two aforementioned results work better.

A Appendix

A.1 Proof of Lemma 1.

The first-order condition for maximizing player H's payoff in (6) is:

$$b_H\Big[\frac{1}{k}y_{iH}{}^{\frac{\rho}{k}-1}\Big(y_{iH}{}^{\frac{\rho}{k}}+y_{iL}{}^{\frac{\rho}{k}}\Big)^{\frac{1}{\rho}-1}\frac{1}{\sum_\ell X_\ell}-\frac{(y_{iH}{}^{\frac{\rho}{k}}+y_{iL}{}^{\frac{\rho}{k}})^{\frac{1}{\rho}}}{\Big(\sum_\ell X_\ell\Big)^2}\frac{1}{\rho}(y_{iH}{}^{\frac{\rho}{k}}+y_{iL}{}^{\frac{\rho}{k}})^{\frac{1}{\rho}-1}\frac{\rho}{k}y_{iH}{}^{\frac{\rho}{k}-1}\Big]=c$$

$$\Leftrightarrow\quad b_H\frac{1}{k}y_{iH}{}^{\frac{\rho}{k}-1}\Big(y_{iH}{}^{\frac{\rho}{k}}+y_{iL}{}^{\frac{\rho}{k}}\Big)^{\frac{1}{\rho}-1}\Big[\frac{1}{\sum_\ell X_\ell}-\frac{(y_{iH}{}^{\frac{\rho}{k}}+y_{iL}{}^{\frac{\rho}{k}})^{\frac{1}{\rho}}}{\Big(\sum_\ell X_\ell\Big)^2}\Big]=c$$

The second-order condition is satisfied because $\rho < k$:

$$b_H\frac{1}{k^2}y_{iH}{}^{\frac{\rho}{k}-2}\left(y_{iH}{}^{\frac{\rho}{k}}+y_{iL}{}^{\frac{\rho}{k}}\right)^{\frac{1}{\rho}-2}\frac{\sum_{\ell\neq i}X_\ell}{(\sum_\ell X_\ell)^3}\times$$
$$\left[X_i\left(-(k+1)y_{iH}{}^{\frac{\rho}{k}}-(k-\rho)y_{iL}{}^{\frac{\rho}{k}}\right)+\sum_{\ell\neq i}X_\ell\left(-(k-1)y_{iH}{}^{\frac{\rho}{k}}-(k-\rho)y_{iL}{}^{\frac{\rho}{k}}\right)\right]<0.$$

Concentrating on the symmetric behaviors of the players across the groups, i.e. $y_{iH}=y_H$ and $y_{iL}=y_L$ for $i=1,\dots,n$, the term in the bracket of the first-order condition becomes $\frac{(n-1)}{n^2(y_H{}^{\frac{\rho}{k}}+y_L{}^{\frac{\rho}{k}})^{\frac{1}{\rho}}}$. The first-order condition for player H is now as follows:

$$b_H\frac{1}{k}y_H{}^{\frac{\rho}{k}-1}\Big(y_H{}^{\frac{\rho}{k}}+y_L{}^{\frac{\rho}{k}}\Big)^{\frac{1}{\rho}-1}\Big[\frac{(n-1)}{n^2(y_H{}^{\frac{\rho}{k}}+y_L{}^{\frac{\rho}{k}})^{\frac{1}{\rho}}}\Big]=c. \tag{11}$$

Similarly, we also have the first-order condition for player L:

$$b_L\frac{1}{k}y_L{}^{\frac{\rho}{k}-1}\Big(y_H{}^{\frac{\rho}{k}}+y_L{}^{\frac{\rho}{k}}\Big)^{\frac{1}{\rho}-1}\Big[\frac{(n-1)}{n^2(y_H{}^{\frac{\rho}{k}}+y_L{}^{\frac{\rho}{k}})^{\frac{1}{\rho}}}\Big]=c. \tag{12}$$

The second-order condition for player L is satisfied as well. By solving equation (11) and (12) in terms of y_H and y_L, we have:

$$y_H=\frac{\left(\frac{1}{b_H}\right)^{\frac{k}{\rho-k}}}{\frac{ckn^2}{n-1}\Big[\left(\frac{1}{b_H}\right)^{\frac{\rho}{\rho-k}}+\left(\frac{1}{b_L}\right)^{\frac{\rho}{\rho-k}}\Big]},\quad y_L=\frac{\left(\frac{1}{b_L}\right)^{\frac{k}{\rho-k}}}{\frac{ckn^2}{n-1}\Big[\left(\frac{1}{b_H}\right)^{\frac{\rho}{\rho-k}}+\left(\frac{1}{b_L}\right)^{\frac{\rho}{\rho-k}}\Big]}. \tag{13}$$

Finally, since $x_H = y_H^{\frac{1}{k}}$ and $x_L = y_L^{\frac{1}{k}}$ from (3), we have the following symmetric effort levels of the players when a certain value of ρ less than k is chosen:

$$x_H = \left(\frac{n-1}{ckn^2} \cdot \frac{b_L}{(b_H/b_L)^{\frac{\rho}{k-\rho}} + 1} \right)^{\frac{1}{k}} (b_H/b_L)^{\frac{1}{k-\rho}}, \quad x_L = \left(\frac{n-1}{ckn^2} \cdot \frac{b_L}{(b_H/b_L)^{\frac{\rho}{k-\rho}} + 1} \right)^{\frac{1}{k}}. \tag{14}$$

A.2 Conditions for satisfying the second-order conditions when $\rho > k$

The first-order conditions which maximize the players' payoffs are the same as those in the case where $\rho < k$, and we consequently obtain the same effort levels of the players in Lemma 1 by solving the first-order conditions. However, when ρ is greater than k, the second-order conditions are not always satisfied. Hence, we evaluate the second-order conditions with the solutions that satisfy the first-order conditions and determine whether the second-order conditions are satisfied at those solutions. That is, we determine whether the solutions to the first-order conditions are the maximizers or not, although it is not assured that they are global maximizers.

The second-order condition for player H in group i is as follows:

$$\begin{aligned} &b_H \frac{1}{k^2} {y_{iH}}^{\frac{\rho}{k}-2} \left({y_{iH}}^{\frac{\rho}{k}} + {y_{iL}}^{\frac{\rho}{k}} \right)^{\frac{1}{\rho}-2} \frac{\sum_{\ell \neq i} X_\ell}{(\sum_\ell X_\ell)^3} \times \\ &\left[X_i \left(-(k+1){y_{iH}}^{\frac{\rho}{k}} - (k-\rho){y_{iL}}^{\frac{\rho}{k}} \right) + \sum_{\ell \neq i} X_\ell \left(-(k-1){y_{iH}}^{\frac{\rho}{k}} - (k-\rho){y_{iL}}^{\frac{\rho}{k}} \right) \right]. \end{aligned}$$

The sign of the second-order condition depends on the sign of the terms within the bracket. Focusing on the symmetric effort levels of the players across the groups, we have $y_{iH} = y_H$ and $y_{iL} = y_L$ for any group i. From the first-order conditions (11) and (12), we also have $y_H = (b_L/b_H)^{\frac{k}{\rho-k}} y_L$. Plugging these into the terms within the bracket, we have

$$\begin{aligned} &X_i \left(-(k+1){y_H}^{\frac{\rho}{k}} - (k-\rho){y_L}^{\frac{\rho}{k}} \right) + (n-1)X_i \left(-(k-1){y_H}^{\frac{\rho}{k}} - (k-\rho){y_L}^{\frac{\rho}{k}} \right) \\ &= X_i \left(-(n(k-1)+2)(b_L/b_H)^{\frac{\rho}{\rho-k}} + n(\rho-k) \right) {y_L}^{\frac{\rho}{k}}. \end{aligned}$$

Thus, we see that the second-order condition is satisfied under the following condition:

$$n(\rho-k) < (n(k-1)+2)(b_L/b_H)^{\frac{\rho}{\rho-k}}$$
$$\Leftrightarrow \frac{b_H}{b_L} < \left(\frac{n(k-1)+2}{n(\rho-k)}\right)^{\frac{\rho-k}{\rho}}.$$

Finally, because $\frac{b_H}{b_L} > 1$, the numerator $n(k-1)+2$ should be greater than the denominator $n(\rho-k)$ in the parenthesis of the right side, which requires that $\rho < 2k-1+\frac{2}{n}$.

A.3 Proof of Proposition 5.

We simplify $x_H + \alpha x_L$ as:

$$x_H + \alpha x_L = \left(\left[\frac{1}{b_H}\right]^{\frac{1}{\rho-k}} + \alpha\left[\frac{1}{b_L}\right]^{\frac{1}{\rho-k}}\right)\left(\frac{ckn^2}{n-1}\right)^{\frac{1}{-k}}\left(\left[\frac{1}{b_H}\right]^{\frac{\rho}{\rho-k}} + \left[\frac{1}{b_L}\right]^{\frac{\rho}{\rho-k}}\right)^{\frac{1}{-k}}.$$

Differentiating the above with respect to ρ, we derive:

$$\left(-\left(\frac{1}{b_H}\right)^{\frac{1}{-k+\rho}}\left(\frac{1}{b_L}\right)^{\frac{\rho}{-k+\rho}} + \left(\frac{1}{b_H}\right)^{\frac{\rho}{-k+\rho}}\left(\frac{1}{b_L}\right)^{\frac{1}{-k+\rho}}\alpha\right)\times$$
$$\frac{\left(\left(\frac{1}{b_H}\right)^{\frac{\rho}{-k+\rho}} + \left(\frac{1}{b_L}\right)^{\frac{\rho}{-k+\rho}}\right)^{-\frac{1+k}{k}}\left(\frac{ckn^2}{-1+n}\right)^{\frac{-1}{k}}\left(\log\left[\frac{1}{b_H}\right] - \log\left[\frac{1}{b_L}\right]\right)}{(k-\rho)^2}.$$

Thus, the optimal ρ is determined by:

$$-\left(\frac{1}{b_H}\right)^{\frac{1}{-k+\rho}}\left(\frac{1}{b_L}\right)^{\frac{\rho}{-k+\rho}} + \left(\frac{1}{b_H}\right)^{\frac{\rho}{-k+\rho}}\left(\frac{1}{b_L}\right)^{\frac{1}{-k+\rho}}\alpha = 0.$$

Solving the equation for ρ, we derive:

$$\rho = \frac{\log(\frac{b_H}{b_L}) + k\log(\frac{1}{\alpha})}{\log(\frac{b_H}{b_L}) + \log(\frac{1}{\alpha})}. \tag{15}$$

The second-order condition holds for a large enough k. Differentiating the objective function twice with respect to ρ and applying the first-order condition, we can derive the following necessary and sufficient condition for the second-order condition:

$$\frac{\alpha\left(-1+k\alpha^{\frac{2}{\rho-1}} + (k-1)\alpha^{\frac{\rho}{\rho-1}}\right)(k-\rho)\log(\alpha)}{\rho-1} < 0.$$

On one hand, assuming $\alpha < 1$, we know that $\log(\alpha) < 0$ and that $k > \rho$ and $\rho > 1$ from equation (15). Thus, all we have to show is:

$$-1 + k\alpha^{\frac{2}{\rho-1}} + (k-1)\alpha^{\frac{\rho}{\rho-1}} > 0.$$

Note that $\alpha^{\frac{2}{\rho-1}} > \alpha^{\frac{2\rho}{\rho-1}}$ since $\alpha < 1$. Therefore, we have

$$-1 + k\alpha^{\frac{2}{\rho-1}} + (k-1)\alpha^{\frac{\rho}{\rho-1}} > -1 + k\alpha^{\frac{2\rho}{\rho-1}} + (k-1)\alpha^{\frac{\rho}{\rho-1}} = (k\alpha^{\frac{\rho}{\rho-1}} - 1)(\alpha^{\frac{\rho}{\rho-1}} + 1).$$

Thus, a sufficient condition to validate the second-order condition is:

$$k\alpha^{\frac{\rho}{\rho-1}} > 1.$$

Plugging equation (15) into the above, we derive

$$k\alpha^{\frac{\rho}{\rho-1}} = k\alpha^{\frac{\log\left(\frac{b_H}{b_L}\right) - k\log(\alpha)}{-(k-1)\log(\alpha)}} = k\left(\frac{b_L}{b_H}\right)^{\frac{1}{k-1}} \alpha^{\frac{k}{k-1}} - 1.$$

Note that for large k, $\left(\frac{b_L}{b_H}\right)^{\frac{1}{k-1}} \approx 1$ and $\alpha^{\frac{k}{k-1}} \approx \alpha$. Thus, for large k, the condition is reduced to $k\alpha - 1 > 0$.

On the other hand, assuming $\alpha > 1$, we know that for large k, $\log(\alpha) > 0$ and that $\rho > k > 1$ from equation (15). Thus, what we have to show is identical to that of the previous case:

$$-1 + k\alpha^{\frac{2}{\rho-1}} + (k-1)\alpha^{\frac{\rho}{\rho-1}} > 0.$$

Again, the condition is satisfied for large k.

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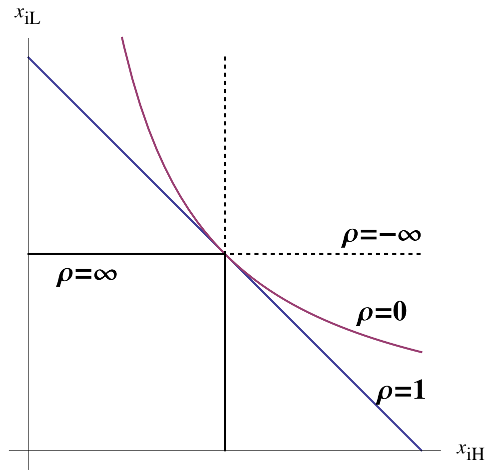

Figure 1: The iso-impact curves of CES impact functions

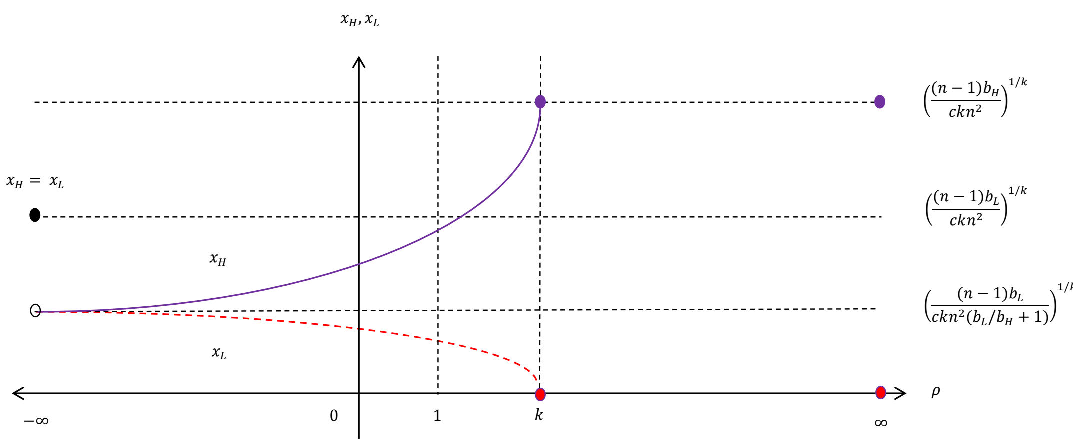

Figure 2: The effort levels of player H and player L

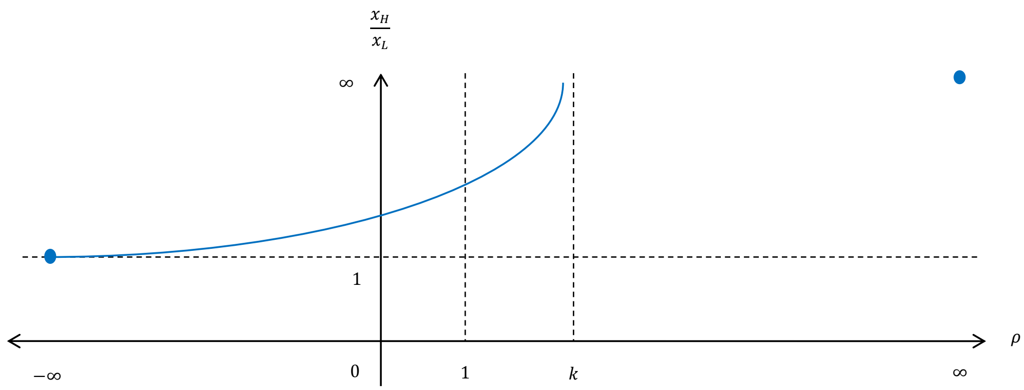

Figure 3: The ratio of effort levels of player H and player L

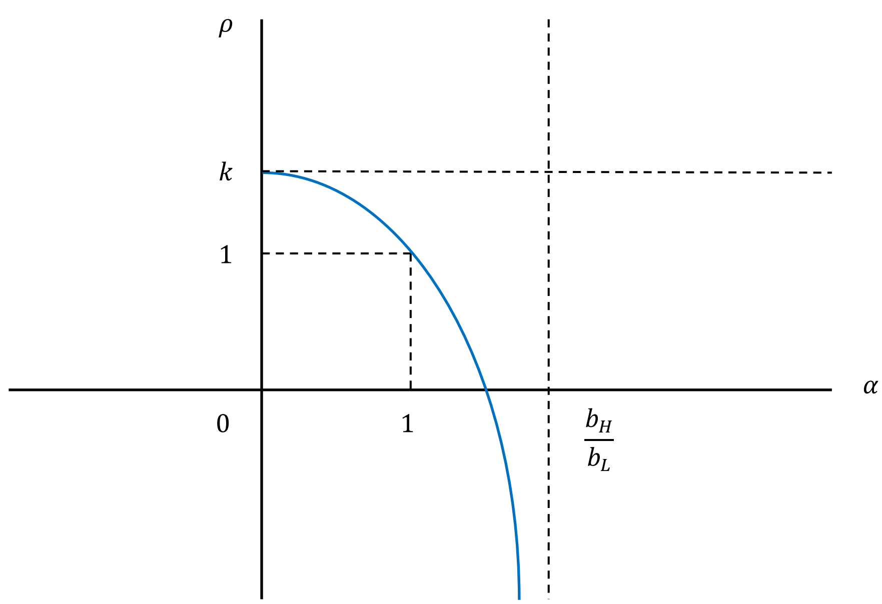

Figure 4: Optimal value of ρ

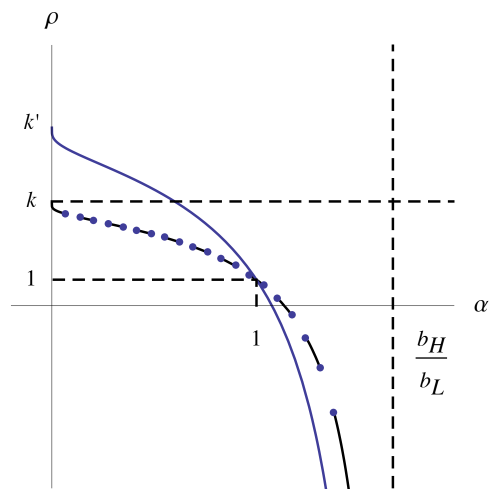

Figure 5: Optimal ρ for $k' > k$

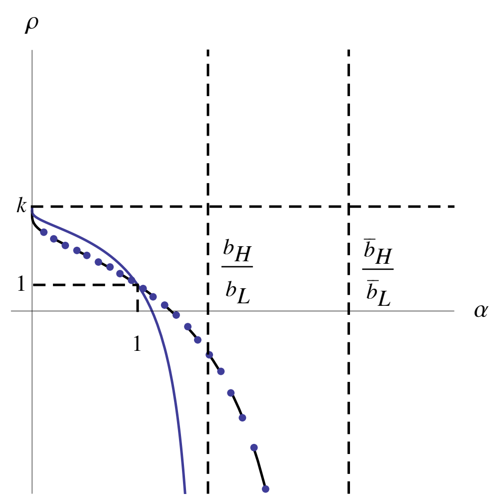

Figure 6: Optimal ρ for $\frac{\bar{b}_H}{\bar{b}_L} > \frac{b_H}{b_L}$