The Role of Labor Share in Relative Price Divergence

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May 2015

Abstract

Prices across different parts of the economy diverge as nations become wealthier. In this paper we consider what we can learn about the mechanisms underlying price divergence by analyzing disaggregate data. We consider both an expenditure decomposition of the economy, based on personal consumption expenditure items, as well as a value-added decomposition, based on individual industries. Our analysis reveals patterns in prices and resource allocation that are hidden under the three-sector decompositions that are commonly used in the extant literature. The expenditure decomposition reveals a negative relationship between price and real expenditure growth over the long run, suggesting that preferences play only a small role - if any - in explaining structural transformation in the economy. The value-added decomposition reveals that long run growth in output prices is closely associated with changes in both the labor share of income and real output, indicating that heterogeneities in labor substitutability together with technological change play key roles in explaining structural transformation in the economy.

Keywords: price divergence, inflation divergence, structural transformation, labor share, log t regression.

JEL Classification: E31, E37, C33

1 Introduction

The reallocation of resources across broad sectors of the economy is one of the salient features of economic growth (Kuznets, 1966). Production-based theories of this so-called structural transformation emphasize the role of relative prices across different sectors of the economy in the reallocation process (examples include Baumol, 1967; and Ngai and Pissarides, 2007, henceforth "NP"). Relative prices and employment levels fall in sectors of the economy that experience the largest gains in productivity. The associated analyses are typically based on decomposing the economy into a few broad aggregate sectors, such as agriculture, manufacturing and services (NP; Herrendorf, Rogerson and Valentinyi, 2013; Herrendorf, Herrington and Valentinyi, 2014) or goods and services when using a final expenditure decomposition of the economy (Kravis, Heston, and Summers, 1983).

There are good reasons to question whether these broad sectoral decompositions are appropriate when tackling questions of price divergence and the mechanisms driving broader structural transformation in the economy. The differences in productivity between sectors are relatively small (Baumol et al., 1985; Fixler and Siegel, 1999; Triplett and Bosworth, 2006), while there can be large differences in industry-level productivity within sectors (Wölfl, 2003; 2005). In addition, sectoral decompositions offer little insight when studying developed economies because the service sector typically dominates the economy (Jorgenson and Timmer, 2011; Duarte and Restuccia, 2012; also see Herrendorf and Valentinyi, 2011).

In this paper we consider what we can learn about relative price divergence by analyzing disaggregate data. Disaggregate data often reveal patterns that are concealed in broad sectoral aggregates, and these may be useful when evaluating different explanations of structural transformation. We take an empirically-driven approach to this task. We first characterize broad trends in the disaggregate data. We consider both the final expenditure side of the economy - by decomposing personal consumption expenditure into contistuent items - and the production (or value-added) decomposition of the economy - by utilizing the Jorgenson (2007) industry dataset (35 industries over 1960 to 2005).

We then build a model of structural transformation that yields predictions that are consistent with the observed trends in relative prices and other key variables. Heterogeneities in industry-level labor substitutability and technological growth (TFP) are both necessary to understand the observed patterns. While differences in technological progress across sectors are common in models of structural transformation (Baumol, 1967; NP), input substitutability has not received substantial attention in the extant literature (a recent exception is Alvarez-Cuadardo, Van Long and Poschke, 2014, henceforth "ACVLP"). Heterogeneity in input substitutabilities are required because of the clear role of labor share (i.e. the share of value-added going to labor) in the industry level data. There are large differences in long term trends in labor share across industries. Yet differences in labor substitutability alone are insufficient to explain what is going on. We also require the more conventional explanations such as heterogeneity in industry-level technology in order to explain the divergence in output between different industries. Our favored model thus has elements of both the production-based explanations of Baumol (1967) and NP as well as the input substitutability explanations of ACVLP.

Our empirics rely on comparatively large datasets. Our final expenditure decomposition is based on 69 PCE items over 1933 to 2012, while our value-added decomposition is based on 35 industries over 1960 to 2005. One drawback of this approach compared to the conventional two or three sector analyses is that it is difficult to succinctly characterize key trends in a large cross section of data over time. To overcome this problem when analyzing the disaggregate data we rely on the econometric tests of divergence and models of convergence developed by Phillips and Sul (2007, hereafter "PS"). In contrast to other common convergence tests and models, the PS methods are specifically built for datasets with a large cross section of data.

Our empirical analysis reveals key patterns in prices and other key variables (such as real expenditure, output, employment, and labor share) which are obfuscated when working with the conventional three-sector decomposition of the economy. First, relative prices within the conventional sectors of the economy are diverging. This result reflects the fact that some services have experienced very low rates of inflation, while some goods and manufacturing industries have experienced very high rates of inflation. Using econometric tests for convergence we look for similar trends in prices. Many of the low-inflation services appear to be capital intensive industries (e.g.: air transportation; telecommunication services), while many of the high-inflation goods are either labor-intensive (e.g.: educational books) or are natural resources subject to high marginal costs (gasoline). Similar patterns are found when we turn to analyzing prices on the value-added decomposition of the economy. This finding is particularly challenging for three-sector models of structural transformation since industries within each sector are assumed to have the same production function, and thereby exhibit similar patterns in prices.

Second, in our final expenditure decomposition of the economy, long term price growth and real expenditure growth are negatively correlated: Households have in general been consuming more of low inflation items. This finding contrasts with what we observe based on sectors: Consumers have been consuming more of the high inflation services sector. The finding favors that production-based theories of structural transformation over the preference-base theories (see, e.g., Kongsamut, Rebelo, Xie, 2001).

Third, in our value-added decomposition of the economy, we show that industry level shares of employment, output, capital and labor income are diverging within the conventional sectors of the economy. This finding further corroborates the need to use disaggregate data when modelling price divergence and structural transformation.

We then build a model of structural transformation to rationalize these findings. Our findings favor production-based explanations of structural transformation in both the final expenditure and value-added decomposition of the economy. Extant production-based models of structural transformation tend to focus on a single source of heterogeneity between sectors. NP use on heterogeneity in sector-specific technological progress; Acemoglu and Guirerra (2008, henceforth "AG") rely on sectoral heterogeneity in input shares; while more recently ACVLP rely on sectoral heterogeneity in labor substitutability. We argue that features of all these models are necessary to explain the broad trends in the data. Most importantly, we require heterogeneity in industry labor substitutability in order to explain the observed divergence in labor share between industries. Even if technology is neutral across industries, the labor inputs become relatively more expensive as technology increases. With a change in the relative price of inputs, industries change their input ratios. In industries where labor and capital inputs are substitutes, the capital inputs displace workers. Conversely, in industries where labor and capital are complements, relatively less capital is hired. This simple model generates the price, output and - most importantly - labor share divergence observed in the data: An industry with greater substitutability can reduce the cost of production more effectively through a reduction in labor-capital ratio and hence the labor share. This implies that the industries with greater input substitutability tend to have lower inflation rate, and the change in labor share is tightly related to the inflation rate in an industry.

The rest of the paper is organized as follows. In section 2 we present an overview of the empirical tools we use to summarize key trends in large datasets. Section 3 details our empirical results. In section 4 we present a structural transformation model to rationalize these findings. Section five concludes.

2 Econometrics for Long Run Growth Dynamics in Large Datasets

Conventional approaches to documenting structural transformation summarize key trends in prices, employment and output share in various sectors of the economy. Given the large amount of data we consider we must use other methods to summarize key trends in the data. In this section, we review the econometric techniques we use to examine the structural transformation.

We use the divergence tests, convergence and clustering algorithms of Phillips and Sul (2007) to identify common trends in a large cross section of a single time series variable (such as prices across disaggregate final expenditure categories). However, in order to identify commonalities across a large cross section of different variables (such as the correlation between prices and expenditures across final expenditure categories), we will rely on regression techniques. We discuss each tool below.

2.1 Tests for Relative Convergence

Let z_{it} denote a time series index i at time t. We adopt the approach of Phillips and Sul (2007) for testing for divergence in a large cross section of time series.

2.1.1 Relative Convergence

Following PS we work with the following definition of convergence of time series z_{it} and z_{jt}:

$$\lim_{t\to\infty}\frac{z_{it}}{z_{jt}}=\lim_{t\to\infty}\frac{b_{it}}{b_{jt}}=1 \text{ if } b_i=b_j. \tag{1}$$

Although (1) resembles an absolute convergence condition, when applied to time-series indexes it is in fact a form of relative convergence. Time series indexes reflect differences in an underlying time series at different points in time. They do not tell you about the overall level of that underlying time series, and thus we cannot use the indexes to make comparisons of the level across different underlying time series. When (1) does not hold, the times series are diverging.

The definition of convergence given in (1) is a more stringent condition than other common definitions of convergence. Under σ-convegrence, if the dispersion among $\{z_{it}\}_{i=1}^{n}$ increases (decreases), then $\{z_{it}\}_{i=1}^{n}$ satisfy $\sigma-$divergence ($\sigma-$convergence). In some cases time series can satisfy $\sigma-$convergence but not (1). To see this, consider the following simple example.

$$z_{it}=a_i t^{\alpha}+e_{it},\ e_{it}\sim iid\left(0,\sigma^2\right).$$

Evidently z_{it} diverges regardless of the value of α. However if $\alpha<0$, the cross sectional variance is decreasing over time, and thus $\sigma-$covergence holds.

In order to test for convergence we use the PS convergence test and clustering algorithm. For instructive purposes we briefly review the method. The PS model is based on the following simple nonlinear model

$$z_{it}=b_{it}\theta_t, \tag{2}$$

where θ_t is the common deterministic or stochastic trend component. The time varying factor loading b_{it} can be interpreted as transition path to a common trend θ_t. This time-varying behavior is modeled as

$$b_{it}=b_i+\xi_{it}\mathcal{L}\left(t\right)^{-1}t^{-\alpha_i},\ \text{for } \xi_{it}\sim iid\left(0,\sigma_i^2\right) \tag{3}$$

where $\mathcal{L}\left(t\right)$ is a function of t that is slowly varying at infinity (e.g. log$t$). When $\alpha_i\geq 0$, $z_{it}\to b_i\theta_t$ as $t\to\infty$. When $b_i=1$ and $\alpha_i\geq 0$ for all i, $z_{it}\to\theta_t$ as $t\to\infty$. The dynamics of individual series are dependent on the time varying factor loadings, b_{it}. Individual series, z_{it}, converge to θ_t if $b_i = b = 1$ and $\alpha_i \geq 0$ for all i. Meanwhile if either $b_i \neq b$ or $\alpha_i < 0$ for any i, y_{it} diverge over time.

The PS method provides a good method for analyzing structural transformation in a large dataset with many potential time series. Under conventional sectoral decompositions of the economy, individual firms within each sector are represented by a representative firm. The underlying idea being that firms within each sector are broadly subject to the same production functions and trends in technology so that the representative firm assumption is an accurate approximation. In a disaggregated dataset this would imply that industries within the same sector would experience common trends in prices, output, and employment. We would expect there to be convergence in these variables when grouped by sector. For example, under heterogeneity of technology but the homogeneous production function, real output in the ith sector can be written as

$$Y_{it} = A_{it} F\left(L_{it}, K_{it}\right) = A_{it} F_{it}. \tag{4}$$

where A_{it} is the heterogeneous technology, L_{it} is the labor input, K_{it} is the capital input, and F_i is a heterogeneous production function. Without loss of generality, define the overall technology across sectors as the geometric mean of A_{it}. That is,

$$A_t = \prod_{i=1}^{N} A_{it}^{\frac{1}{N}},$$

where N is the total number of sectors and a_{it} is the time varying weight parameter for the ith sector relative to A_t which satisfies $A_{it} = A_t^{\alpha_{it}}$.[1] Then we have

$$\ln Y_{it} = \left(a_{it} + \frac{\ln F_{it}}{\ln A_t} \right) \ln A_t.$$

Hence $b_{it} = a_{it} + \ln F_{it} / \ln A_t$ and $\theta_t = \ln A_t$. If $a_{it} \to a_i$, meaning that technology in each industry i follows a different growth path, $\ln Y_{it}$ diverges as $t \to \infty$.

Since the common factor is not observable, PS use the so-called "relative transition" parameter, defined as

$$h_{it} = \frac{z_{it}}{N^{-1} \sum_{i=1}^{N} z_{it}} = \frac{b_{it}}{N^{-1} \sum_{i=1}^{N} b_{it}}.$$

In contrast to the simple cross sectional variance of z_{it}, the cross sectional variance of h_{it} is free from the unobserved common factor θ_t. For example, the relative transition of the log real output under heterogeneous technology but homogeneous production function in (4) becomes

$$h_{it} = \frac{a_{it}}{N^{-1} \sum_{i=1}^{N} a_{it}} + o_p\left(1\right),$$

[1] By construction $\sum_{i=1}^{N} a_{it} = N$.

where the $o_p(1)$ term includes the $\ln F_t / \ln A_t$ term which goes to zero in probability as $t \to \infty$. Hence the relative transition approximates long run trends in heterogeneous technology (in this example (4)). In the empirical section we will rely on this finding to use long run growth in real output to approximate long run growth in unobserved technology.

PS use the following "log$t$" regression to test the null hypothesis of the overall convergence for all individuals.

$$\log\frac{H_1}{H_t} - 2\log(\log t) = a + \phi \log t + u_t, \quad \text{for} \quad t = rT, rT+1, \cdots, T, \tag{5}$$

where

$$H_t := N^{-1}\sum_{i=1}^{N}(h_{it}-1)^2, \tag{6}$$

and $0 < r < 1$. Several points regarding the properties of the $\log t$ regression model (5) are worth pointing out. First, under the null of convergence, the expected estimate of $\hat{\phi}$ must be equal to or greater than zero. Second, the second term in the left hand side in (5), which is $-2\log(\log t)$, acts as a penalty function. Third, an HAC estimator for the covariance should be used since the regression errors are serially correlated. Finally, only $(1-r)$ fraction of the sample is used for the regression. PS recommend setting $r = 0.3$. The regressor in the log$t$ regression is deterministic so that the power of the test is dependent both on N and T.

Throughout the paper, for indexed panel data we use the following long differenced series to avoid the base year problem.

$$z_{it}^* = \ln\left(Z_{it}/Z_{i1}\right).$$

Contrast to the relative level convergence, the relative long differenced convergence implies that the long run average of the growth rates of z_{it}^* becomes identical in the long run. To see this, we assume that z_{it}^* relatively converges to z_{jt}^*. Then

$$\lim_{t\to\infty}\frac{z_{it}^*}{z_{jt}^*} = \lim_{t\to\infty}\frac{z_{it}-z_{i1}}{z_{jt}-z_{j1}} = \lim_{t\to\infty}\frac{(z_{it}-z_{i1})/(t-1)}{(z_{jt}-z_{j1})/(t-1)} = 1.$$

Note that $(z_{it}-z_{i1})/(t-1)$ becomes the average growth rate of z_{it}. Since the relative convergence among the long differenced series implies that the long run growth trajectories are the same, we will use this measure to evaluate whether or not a panel of time series shares the same growth pattern.

One drawback of the PS test is that it requires all time series to be either positive or negative. For indexed panel data, some series are negative for a particular time t so that H_t suddenly increases since the cross sectional average of z_{it}^* under estimates their true mean. Adding a constant number leads to introduce an additional penalty function in the logt regression in (5) so that the null is more often rejected even when the null is true. In practice we will discard any time series if it has some negative numbers.

2.1.2 Sub-Convergence and Clustering Method

The alternative hypothesis is divergence within the set of individual time series. We can permit, however, sub-convergent clubs under the alternative hypothesis: Within a club, members are converging. That is, as $t \to \infty$, some of z_{it} can converge each other with a club, but diverge across clubs. For example, if there are two convergent clubs and one divergent group, then we can express them as

$$z_{it} \to \begin{cases} b_1 \theta_t \text{ if } i \in C_1 \\ b_2 \theta_t \text{ if } i \in C_2 \\ b_i \theta_t \text{ if } i \notin C_1 \text{ and } i \notin C_2 \end{cases}.$$

To identify potential club convergence within the group of individuals, PS propose a clustering method based on the last observation ordering. If some of z_{it} converges to a stochastic common factor, the last observations within a club must be similar each other as $t \to \infty$. By running the logt regression repeatedly with members of which the last observations are similar, PS suggest to select a few core members, which converge each other. Once the core members are selected, the rest of members are selected from the rest of non-core members by examining whether or not inclusion of each non-core member into the core members leads to the convergence. PS showed that their clustering method is consistently selecting the true members asymptotically as $T \to \infty$ regardless of the size of N.

By utilizing PS' clustering method, we can examine whether or not the traditional product category approach is well representing the structural changes. The growth theories predict that the prices of services items or sectors are growing faster than the prices of the other sectors. If so, overall prices must diverge and more importantly, the prices in the services sectors should converge. We will show that this is not the case, and the club clustering results show somewhat different result.

Later we will show but the prices diverge but form several sub-convergence clubs. Within a club, however, other key macro variables do not converge in general. If the prices are function of several variables, then it is not straightforward to find the source of the divergence and sub-convergence. We will discuss this issue further.

2.1.3 Co-Divergence

Structural transformation models typically predict similar patterns in a cross section of variables. For example, under production-based explanations we expect to see higher growth rates in prices and employment levels in sectors that have the lowest increases in productivity. Analyzing these common patterns across variables is thereby a method for validating a specific theory.

In this subsection we demonstrate that it is difficult to rely solely on convergence and clustering methods when looking for these common trends across variables, particularly when several mechanisms are driving structural transformation.

First we consider the possibility that the price divergence and sub-convergence is explained by a single variable such as the heterogeneity of technology progress across sectors.

Case 1: Single Variable with Homogeneous Slope Coefficients Let z_{it} be an index of an endogenous variable from a ST model. Suppose that the log price can be written as

$$p_{it} = \alpha z_{it}. \tag{7}$$

Without loss of generality, let $p_{it} = b_{p,it}\theta_{pt}$ and $z_{it} = b_{it}\theta_t$. then we have

$$\frac{p_{it}}{p_{jt}} = \frac{b_{p,it}}{b_{p,jt}} = \frac{z_{it}}{z_{jt}} = \frac{b_{it}}{b_{jt}} \text{ if } \alpha_i = \alpha.$$

If z_{it} is relatively converging ($b_{it} \to b$) as $t \to \infty$, then

$$\lim_{t\to\infty} \frac{p_{it}}{p_{jt}} = \lim_{t\to\infty} \frac{z_{it}}{z_{jt}} = \frac{b}{b} = 1 \text{ if } \alpha_i = \alpha.$$

In this case, the price divergence and sub-convergence in p_{it} is mirrored by the same patterns of divergence of sub-convergence with z_{it}.

Case 2: Single Variable with Heterogenous Slope Coefficients Suppose that the slope coefficients are heterogeneous across sectors. Then we have

$$p_{it} = \alpha_i z_{it} \text{ for some } i. \tag{8}$$

In this case, the convergence of z_{it} is reflected in divergence and sub-convergence of p_{it}. That is,

$$\lim_{t\to\infty} \frac{p_{it}}{p_{jt}} = \lim_{t\to\infty} \frac{\alpha_i z_{it}}{\alpha_j z_{jt}} = \begin{cases} \alpha_i/\alpha_j \neq 1 & \text{if } \alpha_i \neq \alpha \text{ for some } i \\ 1 & \text{if } \alpha_i = \alpha_j \text{ for } i, j \in C_1 \end{cases}.$$

If z_{it} diverges, then the divergence and sub-convergence of z_{it} may not be reflected in the divergence and sub-convergence of p_{it} due to the heterogeneous α_i. For example, assume that

$$\lim_{t\to\infty} \frac{z_{it}}{z_{jt}} = \frac{b_i}{b_j}.$$

Then it is possible that p_{it} converges relatively if $\alpha_i = 1/b_i$.

$$\lim_{t\to\infty} \frac{\alpha_i z_{it}}{\alpha_j z_{jt}} = 1 \text{ if } \alpha_i = 1/b_i \text{ for all } i.$$

Hence if z_{it} diverges and p_{it} is expressed like (8), then the price sub-convergence may not mirrored by sub-convergence of z_{it}.

We will show shortly that the prices in the model can be expressed as a linear function of several divergent macro variables also. In this case, we cannot use the simple logt concept to analyze the source of the price divergence. Hence we will use the following alternative method.

2.2 Long Run Growth Rates and Between Group Regressions

Conventional models of structural transformation imply certain commonalities in the long run growth paths of both endogenous and exogenous variables. For example, the Baumol (1967) model suggests that long run growth rates in prices and labor productivities across sectors will be correlated. Regression methods therefore provide a tractable method for modelling these predictions.

Because the concepts of divergence and sub-convergence are long run concepts, we will use the long run growth rate to analyze commonalities amongst variables. The long run growth rate for the generic variable z_{it} $(= \log Z_{it})$ is defined as

$$\Delta_T z_{iT} := \frac{1}{T-1} \sum_{t=2}^{T} (z_{it} - z_{it-1}) = (z_{iT} - z_{i1}) / (T-1) .$$

Under conventional structural transformation models a given endogenous variable can be expressed as a function of other variables in the model. Assume that the log prices p_{it} are a linear function of other macro variables x_{it},

$$p_{it} = \beta_i' x_{it} + \varepsilon_{it}, \tag{9}$$

where ε_{it} is an approximation error which may contains non-zero stochastic trends.[2]. Taking the first difference yields

$$\Delta p_{it} = \alpha_i + \beta_i' \Delta x_{it} + u_{it}. \tag{10}$$

To analyze the short run dynamics, one may run the panel fixed effects regression. By taking the first difference, the stochastic trend terms are eliminated so that the regression in (10) can be used for the short run analysis under the assumption that ε_{it} in (9) is nonstationary.[3]

However to analyze the long run dynamics, we need to approximate the long run growth rates by taking the time series means in (10).

$$\Delta_T p_{iT} = \alpha_i + \beta_i \Delta_T x_{iT} + u_{iT}, \tag{11}$$

where 'T' subscript stands for the long run differenced series. For example, $\Delta_T p_{iT} = (p_{iT} - p_{i1}) / (T-1)$. Next, we transform (11) as

$$\Delta_T p_{iT} = \alpha + \beta \Delta_T x_{iT} + v_{iT},$$

[2] In the model section, we will show that many structural transformation models, including our benchmark model, NP and AG models, have the price equation in (9).

[3] If the regression errors are stationary, then the cointegration analysis may be helpful to analyze the long run relationship. However, the regression errors may not be stationary and also are dependent across cross sectional units. More importantly no-cointegration does not imply divergence. For an example, $\varepsilon_{it} = \theta_t + \varepsilon_{it}^o$ where θ_t is $I(1)$. Then the regression error becomes nonstationary but converges to θ_t in the long run.

where

$$v_{iT} = (\alpha_i - \alpha) + (\beta_i - \beta)\,\Delta_T x_{iT} + u_{iT}. \tag{12}$$

The regression in (12) is the between group (BG) regression – cross-sectional regression with the long run growth rates. Also note that the slope coefficients capture the relationships between the relative long run inflation rate and the relative long run growth rates of the regressors since the unknown constant captures the averages of all variables across sectors.

Now we are ready to present the empirical evidences of the price divergence and sub-convergence.

3 Price Divergence in Disaggregate Data

In this section we document divergence in disaggregate prices. Prices are the key signalling mechanism through which production-based theories of structural transformation operate, making them a natural starting point for characterizing the data. We then move on to documenting divergence in other key variables.

We consider two distinct decompositions of the economy. There are various sectoral decompositions of the macroeconomy. As illustrated by Herrendorf, Rogerson and Valentinyi (2012), one can come to very different conclusions regarding the driving forces behind structural transformation depending on which decomposition of the economy is adopted. They argue that preference-based explanations such as KRX do better when using the final expenditure decomposition, whereas production-based explanations do better when using a value added decomposition. We will consider both the final expenditure and the production decompositions of the economy.

We conclude the section with a list of stylized facts obtained from the exercise. These stylized facts inform the structure of a model

3.1 Final Expenditure Decomposition

In this section we examine prices and output by individual consumption items.

3.1.1 Data

Our PCE price and real expenditure data set consists of a panel of 69 annual indices spanning 1933 through to 2013. Finer levels of disaggregation are possible, but these only begin in 1959, and a long time series is required to overcome the "base year problem" discussed below.[4] This also

[4] While the majority of PCE items contained in our 69 item dataset are the most detailed type of product available in the NIPA tables, some higher-level components are included due to this data availability constraint. For example,

permits us to avoid computational problems that arise when a new price index is introduced into PCE, since all 69 indices span the entire 1933 to 2013 time period. The selected PCE components used in our analysis are comprised of 29 goods (16 durables and 13 non-durables) and 40 services.[5] Among 29 the goods items, three items are related to food consumption. See the appendix A for the full description of the 69 PCE items. Data are obtained from NIPA Table 2.4.4 and 2.4.6 for PCE price and real expenditure indexes at the BEA website (www.bea.gov).

In the data, both PCE price and real expenditure indexes are normalized to 100 in the base year. By construction, the indexes will converge before the base year and diverge after the base year. The problem can be overcome by re-normalizing the base year to be the first year in the sample, as PS suggest. Let $y_{it} = \ln(Y_{it}/Y_{i1})$. Then $y_{i1} = 0$ for all i but $y_{iT} = \ln(Y_{iT}/Y_{i1})$. As we discussed before, the modified series become backward moving averages after dividing it by $t-1$. Hence the last observations for PCE price and real expenditure become the entire averages of the inflation and the growth rates of real expenditure, respectively.

PCE data includes the consumption on imported goods and services while excludes the exported goods and services. Naturally PCE data does not provide any information of the production-side at all such as the labor share, employment and labor productivity. We use Jorgenson data to analyze the production side economy.

3.1.2 Divergence within Sectors

We discard the first m observations to avoid the impact of the initial condition.[6] We also exclude two items in PCE price and two items in real expenditure since some of their values are negative.[7] As a robustness check, we further discard the initial r fraction of the sample and test for overall convergence. Table 1 presents the $\log t$ test results with some selected $r \in (0.15, 0.2, 0.3)$ which corresponds to the actual starting years of 1973, 1975 and 1980, respectively. Evidently, the convergence is strongly rejected since estimated ϕ is significantly less than zero for any of the r values we considered. This is strong evidence against the single component model for inflation.

our data includes "garments" because further detail on this category, such as "women's and girls' clothing," "men's and boy's clothing," and "children's and infants' clothing," are not available before 1959.

[5] Note that PCE component classifications change periodically with BEA benchmark revisions to the national income and product accounts, and hence the number of PCE components at the highest level of disaggregation differ across studies.

[6] Let $\ln y_{it}^o = \delta_{it}^o \ln y_t^o + e_{it}$ where y_{it}^o is the latent value free from the base year or normalization. We observe $\ln Y_{it} = \ln(Y_{it}^o/Y_{i1}^o) = [\delta_{it}^o - \delta_{i1}^o(\ln Y_1^o / \ln Y_t^o) + (e_{it} - e_{i1})/\ln Y_t^o] \ln Y_t^o = \delta_{it} \ln Y_t^o$. The common trend Y_t^o usually has a trend component, so that we have $\ln Y_t^o = O_p(t^\alpha)$ for some $\alpha > 0$. For large t, so the impact of the initial condition on δ_{it} disappears as $t \to \infty$, and more rapidly the stronger the trend (or larger α).

[7] In prices, we exclude *Video, audio, photographic,information processing equipment and media,* and *Water supply and sanitation*; in deflated expenditures we exclude *Food produced and consumed on farms, Fuel oil and other fuels.*

We also examine if there is a potential reduction in in cross-sectional variance among the items in a specific major product category, non-service items and services. Table 1 evidently shows that such a convergence within a major product category does not appear as $\hat{\phi}$ is significantly less than zero for each category. Hence the log t convergence test results convincingly suggest that there is no evidence for PCE price convergence and it is unlikely that use of conventional major product category is helpful in understanding overall price divergence.[8]

Table 1 also reports the convergence test results with real expenditures. Similar to the divergence of relative prices, real expenditures are diverging based on the typical categories.

Table 1: Convergence Test Results: PCE Prices and Real Expenditure

		Statistics	1973	1975	1980
Price	All items	$\hat{\phi}$	-0.621	-0.605	-0.583
		$t_{\hat{\phi}}$	-42.76	-66.03	-39.55
	Goods items	$\hat{\phi}$	-0.634	-0.625	-0.668
		$t_{\hat{\phi}}$	-31.40	-36.52	-37.85
	Services items	$\hat{\phi}$	-0.611	-0.593	-0.546
		$t_{\hat{\phi}}$	-27.27	-29.79	-31.52
Real Exp.	All items	$\hat{\phi}$	-0.535	-0.621	-0.859
		$t_{\hat{\phi}}$	-7.959	-8.634	-13.90
	Good items	$\hat{\phi}$	-0.484	-0.557	-0.763
		$t_{\hat{\phi}}$	-7.151	-8.824	-18.66
	Services items	$\hat{\phi}$	-0.411	-0.495	-0.736
		$t_{\hat{\phi}}$	-6.280	-5.991	-13.79

3.1.3 Convergence and Clustering in Prices

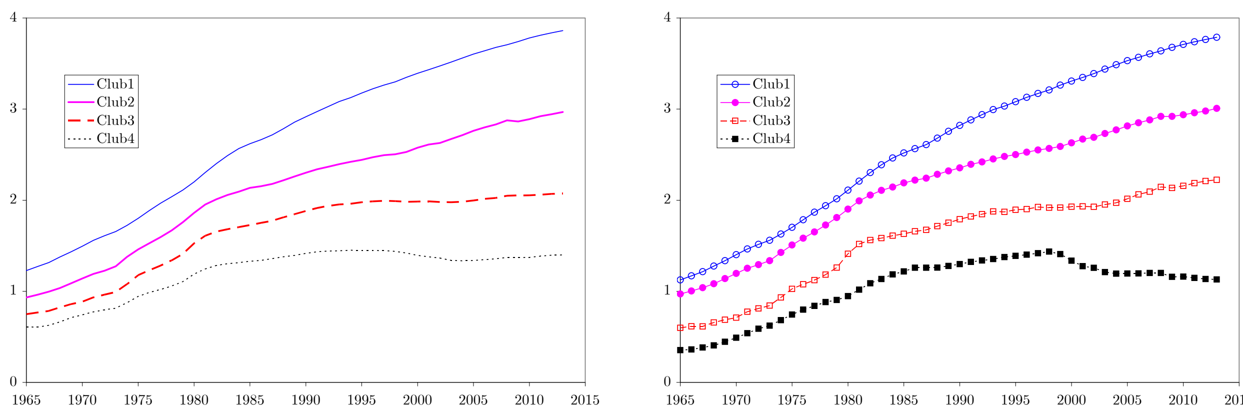
Since the $\log t$ convergence test for PCE prices rejects the null of convergence in the presence of only one divergent series, if we find in favor of the alternative there could exist subgroups that may converge. The idea that there are common drivers to inflation rates has been explored by Boivin, Giannoni, and Mihov (2009), Bils and Klenow (2004), and Carvalho and Lee (2011). Whereas the focus in these papers is on inflation rates, in our framework we model price levels.

We consider identifying convergence clubs based on disaggregate PCE items price levels. Items within each club follow the same common trend. PS propose a clustering procedure that

[8] Note that the use of a finer product category did not change this conclusion.

involves the stepwise application of $\log t$ regression tests based on the ordering of last observations. If some series are converging each other, their last observations must not be far different from each other. The clustering procedure differs from previous studies on clustering methods, such as Durlauf and Johnson (1995) and Hobijn and Franses (2000), in the sense that their algorithm focuses on how idiosyncratic transitions behave over time in relation to the common factor component.

Using the clustering algorithm we find that there are four convergent clubs and one divergent group as shown in Table 2. We set $r = 0.3$ so that the first observation used in the regression becomes 1980. The convergent results are not dependent on the choice of $r \in (0.15, 0.2, 0.3)$. For all convergence clubs the point estimates of ϕ are significantly greater than zero. As we confirmed in Table 1, the nature of convergence clubs is not highly correlated with the major product categories. That is, any of convergence clubs does not include price items that belongs to a particular product category. For instance, although many service items are included in high-inflation groups, clubs 1 and 2, a substantial number of PCE items in clubs 3 and 4, low-inflation groups, are associated with services category.



Panel A: Average prices for each club   Panel B: Average services' prices for each club

Figure 1: Estimated Common Stochastic Trends for Each Convergent Club

We estimate the common stochastic trend for each convergent club by taking the sample cross sectional mean of the prices for each club. See Figure 1: Panel A shows estimated common stochastic trends for all prices, while panel B shows estimated common stochastic trends for services prices only. The estimated common price for club 1 is consistently highest, meanwhile that of club 4 is consistently lowest. The common prices for club 1 and 2 have increased monotonically, but the growth rate of club 1 is relatively higher than that of club 2. Meanwhile the common price for club 3 seems to have stablized after 1980. The common price for club 4 also seems to be stablized after

1980 and then decreased after 2000. Since such patterns are observed both in Panel A and B, we can conclude that the convergence clubs identified by clustering analysis have no tight nexus with the major product category.

It is worth mentioning potential reasons why there exist distinct common trends across convergence clubs. First, the presence of different groups of services in terms of capital-skill complementarity may play a sizeable role in explaining why there exist the different long-run price dynamics. A number of studies, e.g., Krusell, Ohanian, Rios-Rull, and Violante (2000) and Jeong, Kim, and Manovskii (2015), have stressed the importance of different groups of services, for example high-skilled and low-skilled group, and documented relative supply and price of skilled labor has been increasing steadily since postwar US economy. The convergence club classification (presented in Table A1) supports the view that member services items in clubs 1 and 2, such as health, financial, and recreational services, tend to require high-skilled workers while the items included in clubs 3 and 4 may be associated with more capital-intensive service sectors and hence relatively low-skilled workers. As a result, the rising skill premium over time that are apparent in the data may be an important source of the different long-run price dynamics; services items in clubs 1 and 2 display strong tendency of rising prices whereas services in clubs 3 and 4 have initially rising and subsequently stagnant prices.

Second, as Elsby, Hobijn, and Sahin (2013) pointed out, offshoring of labor-intensive component can be a possible explanation of different services groups since it is realtively harder for the services sectors listed in clubs 1 and 2 to outsourcing of technical and administrative services supporting domestic and global operations from outside the US. Third, the log-run trend of the prices in clubs 3 and 4 (group 5) initially rises and subsequently stagnant prices may be the fact that the price of equipment has declined substantially. Interestingly, except two durable goods of the PCE data, 'net purchase of used motor vehicles' and 'educational books,' all other durables fall into the category that displays stagnant (clubs 3 and 4) or even falling prices (group 5). This is consistent with a vast literature dealing with the declining price of equipment. For example, using post-war US data, Greenwood, Hercowitz, and Krusell (1997) document that the relative price of equipment has fallen sustainably while there has been substantial increase in the accumulation of new equipment due to technological advances. It is not clear at this point which one is a leading explanation about relative price divergence, but those potential reasons above have a common conclusion that the labor substitutability may be a key ingredient of the explanation as the elasticity of substitution is higher between capital equipment and unskilled labor than between capital equipment and skilled labor.

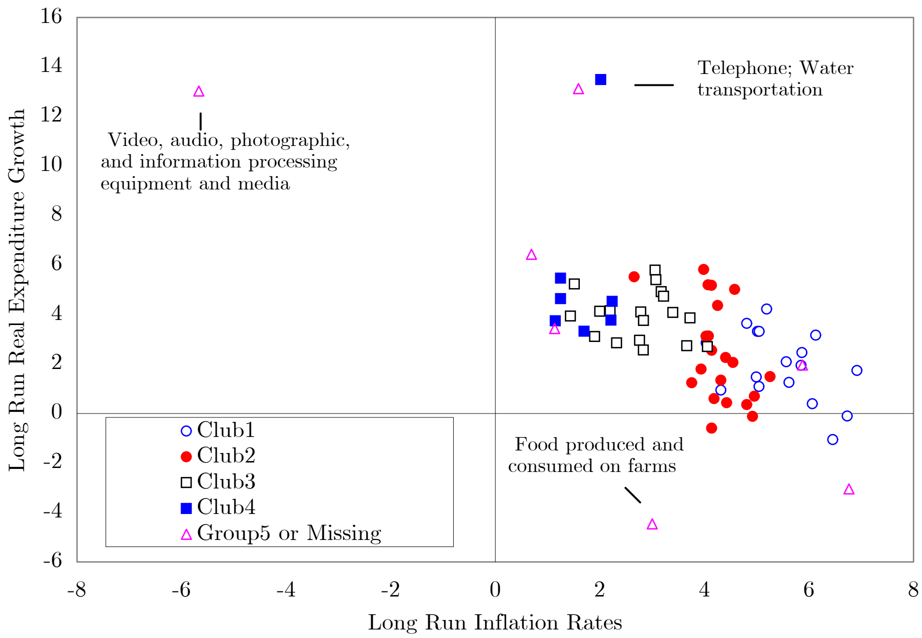
Table 2: Convergence Club Membership by Major Product Category

Statistics		Club 1 [19]	Club 2 [21]	Club 3 [20]	Club 4 [7]	Group 5 [3]
$\hat{\phi}(t_{\hat{\phi}})$		0.74 (8.42)	0.97 (22.7)	0.63 (11.3)	0.29 (7.78)	n.a.
Item Categories						
Durable	16	2	0	8	5	3
Nondurable	13	2	4	6	1	0
Services	40	15	17	6	2	0

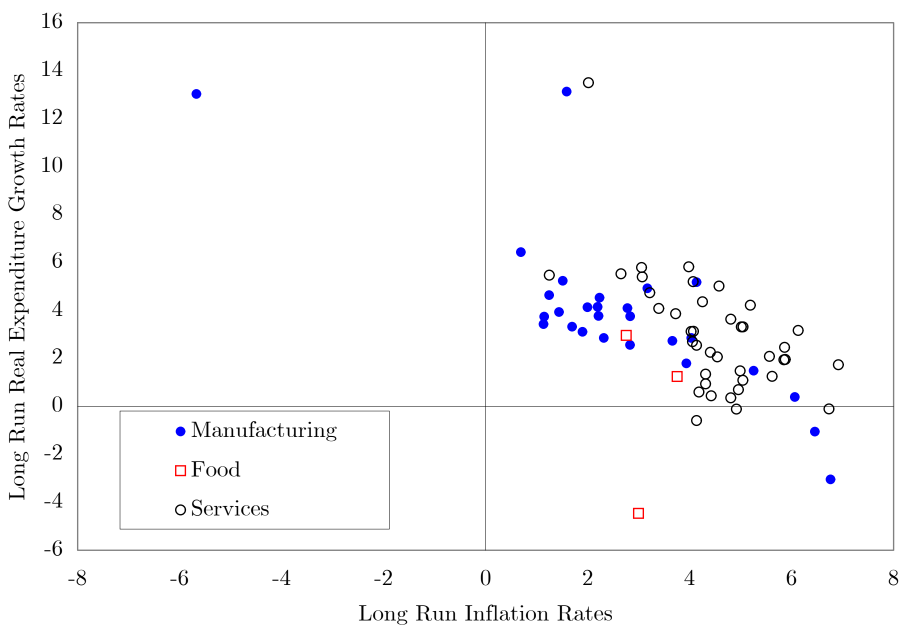
It is also important to note that the within a price convergence club, real expenditures are diverging in general. It is not surprising since the evolution of the prices are a function of heterogeneous consumption elasticity and real expenditures. Hence the divergence and sub-convergence of the price may not be explained by the real expenditure if the consumption elasticity is heterogeneous across goods. Figure 1, which consists of two figures, shows the long run average growth rates of inflation and real expenditure for each item, which are equivalent to the last observations of the long differenced series. Panel A shows the long run relationship between the two by the price clubs, meanwhile Panel B exhibits the relationship by the product categories. The club membership does not have one for one relationship with the ordering of the last observations but overall there is a positive relationship. In other words, the long run inflation rates in club 1 are overall higher than the rest of clubs. Overall the long run inflation rates are negatively correlated with the long run real expenditure once we discard three outliers: Two outliers on the north-east section (telephone, water transportation) and one outlier on the south-east section (food produced and consumed on farm). However except for club 3, the real expenditure are diverging for each price convergence club. Nonetheless, the general negative relationship between the two implies that consumers have increased their expenditures on cheaper goods. For example, the prices of video, audio and photographic items have been decreased significantly. The long run inflation rates between 1933 and 2013 is around -6%. Meanwhile their real expenditure growth rate is more than 13%. Typical demand-side structural changes such as lower consumption on the foods or agricultural products, and higher consumption on the service items are not shown in Panel A. For example, as it shown in Table 2, the club 1 encompasses 19 PCE components and the vast majority are services; including 4 out of 5 "health care" sub-categories, 4 out of 6 "financial services and insurance" sub-categories, and all 3 sub-categories of "educational services". However Panel A shows that overall real expenditure growth rates in club 1 are relatively lower than those in other clubs.

Figure 1: The relationship between long run inflation and real expenditure growth rates

Panel A: Clustered by Prices



Panel B: Clustered by Sector



Panel B displays the same relationship in the view of the product categories. Services items are overall centered on the north-east side, meanwhile the manufacturing items (durable and non-durable goods excluding the food items) are widely spread out. In fact, the majority of manufacturing items show higher long run real expenditure growth rates than that of the average services item. Also except for the item of food produced and consumed on farms, the long run inflation rates and real expenditure growth rates of the two food items are not relatively lower than those of services items.

3.2 Value-Added Decomposition

In this subsection we document price divergence on in a production or value-added side decomposition of the economy. We decompose production into different industries (rather than sectors).

3.2.1 Data

We use the input-output dataset compiled by Jorgensen (2007) which contains many variables that are useful for characterizing other aspects of structural transformation. The dataset consists of 35 industries: 8 services, and 27 non-services. Starting and ending years are 1960 and 2005 (total 46 annual observations), respectively. Unfortunately there is only one agricultural industry in the dataset, so we group agriculture and manufacturing into "non-services".

The key variables from the dataset we use are nominal output, capital and labor compensation (all measured in current dollars), together with the associated price deflators (which are normalized to 1 in 1996). The dataset also includes employment and capital. To obtain real output and capital we deflate the nominal values by their respective price deflators. For labor share we use nominal labor cost divided by value added, the sum of the nominal labor and capital compensations.

3.2.2 Price Divergence within Sectors

As above we apply the PS test of divergence. We discard at least the first 16 observations to avoid the base year effect.

Table 3 illustrates price divergence for all 35 industries and within both the non-service and services sectors. In the non-service sector, there is weak convergence when the starting year is 1976. However, this convergence result is rather weak, and disappears as the starting year increases

Table 3: Divergence in Prices within Sectors

	Statistics	Starting Year 1976	1978	1980
All industries (33/35)	$\hat{\phi}$	-0.142	-0.248	-0.364
	$t_{\hat{\phi}}$	-2.028	-3.242	-4.546
Non-service (25/27)	$\hat{\phi}$	-0.084	-0.221	-0.361
	$t_{\hat{\phi}}$	-0.700	-1.697	-2.437
Service (8)	$\hat{\phi}$	-0.155	-0.177	-0.249
	$t_{\hat{\phi}}$	-1.711	-2.403	-4.616

Table 4 exhibits divergence within sectors between other key variables, including deflated output (Y), deflated capital (K), employment (L), and labor share (LS). In testing for divergence we use the log level series (as opposed to the long-differenced series). As such we do not need to vary the starting year to account for the base year problem: All starting years are 1960 in Table 4.[9]

Table 4: Divergence of Other Variables based on Product Category

	Statistics	$\ln Y$	$\ln K$	$\ln L$	ln(LS)
All (35)	$\hat{\phi}$	-0.869	-0.622	-1.058	-0.413
	$t_{\hat{\phi}}$	-68.00	-43.88	-22.81	-2.949
Non-serv. (27)	$\hat{\phi}$	-0.998	-0.571	-1.116	-0.539
	$t_{\hat{\phi}}$	-20.29	-100.8	-12.62	-6.189
Services (8)	$\hat{\phi}$	-0.789	-0.477	-1.015	-0.421
	$t_{\hat{\phi}}$	-58.41	-29.07	-20.50	-2.657

Note: $\ln Y$ = log real output, $\ln K$ = log capital input, $\ln L$ = log employment, ln(LS) = log labor share

Within each sector all of the variables are diverging. The results exhibited in Tables 3 and 4 cast doubt on the use of sectors in value-added decompositions of economy when tackling the causes of price divergence and structural transformation. Within sectors, output prices, output,

[9] Regardless, these results are robust when we long-difference the time series and omit time series that are negative.

employment, capital and labor shares are diverging. It therefore appears that the individual industries within each conventional sector are subject to very different trends in technological growth and production technologies.

3.2.3 Convergence and Clustering in Output Prices

We next consider whether there are sufficient commonalities in long run trends in industrial output prices to warrant categorization into convergence clubs. We consider 33 of the 35 industries: As discussed above, two industries are not included because the long-differenced series contain negative values.

Table 5: Clustering Results

Club	Member Items
Club 1 [12] $\hat{\phi}(t_{\hat{\phi}})$: 0.24 (1.21)	Metal mining; Oil and gas extraction; Construction; Tobacco; Printing, publishing and allied; Chemicals; Petroleum and coal products; *Electric utilities; Gas utilities; Finance Insurance and Real Estate; Services; Government enterprises*
Club 2 [21] $\hat{\phi}$ $(t_{\hat{\phi}})$: 0.65 (7.69) Missing output	Agriculture; Coal mining; Non-metallic mining; Food and kindred products; Textile mill products; Apparel; Lumber and wood; Furniture and fixtures; Paper and allied; Rubber and misc plastics; Leather Stone, clay, glass; Primary metal; Fabricated metal; Motor vehicles; Transportation equipment & ordnance; Instruments; Misc. manufacturing; *Transportation; Communications; Trade*
Missing Prices [2]	Machinery, non-electrical; Electrical machinery

Note: Service sectors are in italic.

We find evidence of two convergence clubs. Table 5 exhibits club membership. The first club ("club 1") includes 12 industries that have experienced high inflation. Five of the industries are services. The second club ("club 2") consists of 21 industries, including agriculture, three services and 17 manufacturing industries. Overall, the long run inflation rates – which are equivalent to the last observations of the long differenced series – in club 1 are higher than those in club 2. The threshold point is around 3.8% per annum. A couple of the long run inflation rates in the club 1 (the club 2) are lower (higher) than the threshold point, but overall the long run inflation rates in club 1 are higher than 3.8%.

The number of output price convergence clubs is smaller than the number of final expenditure (PCE) price clubs. This may ultimately reflect the fact that the industry-level PPIs do not contain imported goods and services, or the fact that the number of industries is much smaller than the number of PCE items.

In Table 6 we consider whether there is convergence in the other key variables (employment, capital, output, and labor share) when grouped according to the price convergence clubs. The data suggest this is not the case: All variables are diverging within the price based convergence clubs.

Table 6: Divergence of Other Variables based on Price Clubs

	Statistics	Real Output	Capital Stock	Employment	Labor Share
Club 1 (12)	$\hat{\phi}$	-1.002	-0.655	-1.105	-0.482
	$t_{\hat{\phi}}$	-39.74	-86.93	-17.09	-4.097
Club 2 (21)	$\hat{\phi}$	-0.836	-0.574	-1.008	-0.566
	$t_{\hat{\phi}}$	-58.50	-30.05	-45.70	-3.938

3.3 Price Divergence and Structural Transformation in Disaggregate Data: Stylized Facts and Key Conclusions

We summarize our findings and their implications as follows.

1. Conventional sectoral decompositions of the economy are inappropriate when analyzing the causes of price divergence and structural transformation in the economy. Within the conventional sectors (services, manufacturing and agriculture in value-added decompositions, or goods and services in final expenditure decompositions), prices are diverging. Other key variables (output, expenditure, employment, and capital) are also diverging within those sectors. This implies that conventional sectors may obfuscate much of the underlying mechanisms at work.

2. Long run growth in prices and quantities are negatively correlated in both the final expenditure and the value added decompositions. This favors production-based explanations of

structural transformation, and not the preference-based explanations of KRX. In our subsequent discussions of the empirical findings we will focus on the production based explanations, and assume a constant returns to scale preference over varity.

3. Long term trends in industry level data indicate several requisite features of a production-based model of structural transformation:

   (a) Heterogenous technology (TFP) at the industry level. As discussed earlier in relation to (4), the relative transition of the log real output is a good approximation of technological growth at the industry level. Industry level deflated output is diverging across all industries, indicating heterogenous technological growth. However, the fact that output diverges within convergence clubs indicates that heterogenous technology is not an *exhaustive* explanation of structural transformation. If this were the case, we would observe similar long term trends in prices and output. Other mechanisms must simultaneously be at work. We conclude that the technology-based models of Baumol (1967) and NP are part of - but not the entire - story.

   (b) Heterogeneities in input substitutability across different industries. Labor shares diverge across all industries and within both the conventional sectors ("services" and "non-services") and each price convergence club. This is suggestive of large heterogeneities in input substitutability across different industries. As technology increases, industries in which capital and labor are compliments will use relatively more labor, leading to increases in labor share. Industries in which capital and labor are substitutes will use relatively more capital, leading to a fall in labor share. These dynamics hold even if technological progress is the same across industries. But the fact that labor share diverges within each convergence club indicates that labor substitutability is not an *exhaustive* explanation of structural transformation. If this were the case, we would observe similar long term trends in prices and labor share. We conclude that the input substitutability models of ACVLP are part of, but not the entire story.

4 The Model

In this section we present a model of structural transformation to investigate the relationship between prices, output, labor share and productivity we observe in the data. Sectoral heterogeneity in both technology and labor substitutability play critical roles in explaining the empirical findings. Heterogeneity in technological progress (TFP) generates the observed divergence in output, while heterogeneity in labor substitutability generates the observed divergence in labor shares. We also argue that several mechanisms must be at play in order to explain the relative convergence in labor productivity. Due to the observed negative correlation between long-term inflation and expenditure share growth, preferences are not a key feature of the model.

Our benchmark model nests alternative models of structural transformation, including those of NP, AG and ACVLP. However, as we discuss in more detail below, each model in itself cannot account for all of the observed empirical findings. The NP and AG models cannot account for the observed divergence in labor share. The required model in fact contains elements each model.

We conclude the section by returning to the empirical dataset, showing how incorporating all potential sources of heterogeneity helps us explain the long term trends in disaggregate prices.

4.1 The Benchmark Economy

The representative household has preferences over aggregate consumption with

$$U = \int_0^\infty e^{-(\rho-n)t} \frac{c(t)^{1-\vartheta} - 1}{1-\vartheta} dt, \tag{13}$$

where $\rho > 0$ is the subjective time discount rate, $\vartheta \geq 0$ is inverse of the intertemporal elasticity of substitution, $n \in [0, \rho)$ is the growth rate of population, and $c(t)$ is consumption per capita at time t.[10] The representative household supplies one unit labor inelastically. The total labor supply in the economy is $L(t) = L(0) e^{nt}$. The final good $Y(t)$ is composed of goods produced in N sectors with a constant returns to scale function

$$Y(t) = \mathcal{Y}(Y_1(t), \cdots, Y_N(t)), \tag{14}$$

where $Y_i(t)$ is the good produced in sector i, and the function $\mathcal{Y}(\cdot)$ is increasing in each element.[11] Total capital used for production $K(t)$ depreciates with the rate of $\delta \geq 0$. The resource constraint of the final good is given by

$$\dot{K}(t) + \delta K(t) + C(t) = Y(t), \tag{15}$$

where $C(t) = L(t) c(t)$, total consumption.

Each sector, $i \in \{1, ..., N\}$, hires capital $K_i(t)$ and labor $L_i(t)$ with the rental rate of capital $R(t)$ and the wage rate $W(t)$ given its total factor productivity $A_i(t)$. The elasticity of substitution between two inputs is $\gamma_i > 0$ which may be different across sectors. The production function in sector i is given by,

$$Y_i(t) = A_i(t) \left[\alpha_{Li} L_i(t)^{\frac{\gamma_i - 1}{\gamma_i}} + \alpha_{Ki} K_i(t)^{\frac{\gamma_i - 1}{\gamma_i}} \right]^{\frac{\gamma_i}{\gamma_i - 1}}, \tag{16}$$

[10] We assume that $\rho - n$ is sufficiently high so that the transversality condition holds.

[11] We focus on the production side of the economy for the explanation of divergence/convergence of prices. So, we use a general aggregate function.

where $\alpha_{Li} \in (0,1]$ and $\alpha_{Ki} = 1 - \alpha_{Li}$ are the input share parameters. All markets are perfectly competitive. Producers' profit maximization problem gives the capital and labor input demands as

$$\begin{aligned} L_i(t) &= \left[\frac{W(t)}{\alpha_{Li} p_i(t)}\right]^{-\gamma_i} A_i(t)^{\gamma_i - 1} Y_i(t), & (17) \\ K_i(t) &= \left[\frac{R(t)}{\alpha_{Ki} p_i(t)}\right]^{-\gamma_i} A_i(t)^{\gamma_i - 1} Y_i(t). & (18) \end{aligned}$$

Let $P_i(t)$ be the price of good produced in sector i. With the zero profit condition of $P_i(t) Y_i(t) = W(t) L_i(t) + R(t) K_i(t)$ under the perfectly competitive market, the price of good in sector i is given by

$$P_i(t) = A_i(t)^{-1} \left[\alpha_{Li}^{\gamma_i} W(t)^{1-\gamma_i} + \alpha_{Ki}^{\gamma_i} R(t)^{1-\gamma_i}\right]^{\frac{1}{1-\gamma_i}}. \tag{19}$$

4.2 Evolution of Prices, Technologies and Labor Shares

For notational convenience, we denote the growth rate of a variable $X(t)$ as $\widehat{X}(t) = \dot{X}(t)/X(t)$. We will consider the economy where sectoral productivity grows over time at a constant rate, $\mu_i := \widehat{A}_i(t) > 0$. With positive productivity growths we assume that the wage rate grows over time $\widehat{W}(t) > 0$, while the rental rate of capital is relatively stationary so that $\xi(t) \equiv \widehat{W}(t) - \widehat{R}(t) > 0$.

From the labor and capital demand functions in sector i, (17) and (18), we have

$$\frac{R(t) K_i(t)}{W(t) L_i(t)} = \left(\frac{\alpha_{Ki}}{\alpha_{Li}}\right)^{\gamma_i} \left[\frac{W(t)}{R(t)}\right]^{\gamma_i - 1}. \tag{20}$$

In growth rates, we have

$$\left[\widehat{\frac{R(t) K_i(t)}{W(t) L_i(t)}}\right] = (\gamma_i - 1)\,\xi(t). \tag{21}$$

Thus, the higher is γ_i, the lower the growth rate of labor income share. Let S_i be the labor share in sector i,

$$S_i(t) = \frac{W(t) L_i(t)}{P_i(t) Y_i(t)} = \left[1 + \frac{R(t) K_i(t)}{W(t) L_i(t)}\right]^{-1}. \tag{22}$$

In growth rates, we have

$$\begin{aligned} \widehat{S}_i &= -\left[1 - S_i(t)\right] \left[\widehat{\frac{R(t) K_i(t)}{W(t) L_i(t)}}\right] \\ &= -\left[1 - S_i(t)\right] (\gamma_i - 1)\,\xi(t). \end{aligned} \tag{23}$$

We then have the following relationship between the labor share $S_i(t)$ and the elasticity of substitution γ_i.

Proposition 1. *Under positive productivity growth,* $\mu_i > 0$ for all i:

(i) *if* $\gamma_i < 1$ *(*> 1), $S_i(t)$ *converges to 1 (0); and*

(ii) *if* $\gamma_i < \gamma_j$, *eventually* $S_i(t) > S_j(t)$ regardless of the initial values of $S_i(0)$ and $S_j(0)$.

Proof: *See the appendix.* ∎

The labor share converges to either 1 or 0 depending on the sign of $\gamma_i - 1$. However, this does not mean that the convergence of the relative labor share $S_i(t)/S_j(t)$ depends only on the signs of $\gamma_i - 1$ and $\gamma_j - 1$.

Proposition 2. *Under positive productivity growth,* $\mu_i > 0$ for all i, *the relative labor share* $S_i(t)/S_j(t)$

(i) *monotonically decreases over time and converges to 1 if* $\gamma_i = \gamma_j < 1$,

(ii) *monotonically increases over time and converges to* $\left(\frac{1-S_j(0)}{S_j(0)}\right) / \left(\frac{1-S_i(0)}{S_i(0)}\right)$ *if* $\gamma_i = \gamma_j > 1$, *and*

(iii) *diverges if* $\gamma_i \neq \gamma_j$ and $\max\{\gamma_i, \gamma_j\} > 1$.

Proof: *See the appendix.* ∎

Note that when $\gamma_i < \gamma_j < 1$, the relative labor share $S_i(t)/S_j(t)$ converges to 1, since $\exp\left[(\gamma_i - 1)\int_0^t \xi(\tau)\,d\tau\right]$ converges to 0. However, convergence can be non-monotonic because the growth rate of $S_i(t)/S_j(t)$ depends on both γ_i and $S_i(0)$ in both sectors.

The evolution of prices is closely related to the evolution of labor shares and technologies. Dividing the price of good in sector i, $P_i(t)$ in (19), with $W(t)$ and applying (20) and (22), we have

$$P_i(t) = \alpha_{L_i}^{\frac{\gamma_i}{1-\gamma_i}} S_i(t)^{\frac{1}{\gamma_{i-1}}} A_i(t)^{-1} W(t). \tag{24}$$

In growth rates, we have

$$\hat{P}_i(t) = \frac{1}{\gamma_i - 1}\hat{S}_i(t) - \mu_i + \hat{W}(t). \tag{25}$$

Since $\hat{W}(t)$ is common across sectors, the relative inflation rate $\hat{P}_i(t)$ is closely related to the relative labor share growth rate $\hat{S}_i(t)$. For example, consider two sectors with identical $\gamma > 1$ and $\mu > 0$, but different relative input share parameters, $\alpha_{Ki}/\alpha_{Li} < \alpha_{Kj}/\alpha_{Lj}$. In this case, $S_i(t) > S_j(t)$ for all t from (20) and $\hat{S}_j(t) < \hat{S}_i(t) < 0$ from (23). This gives

$$\hat{P}_i(t) - \hat{P}_j(t) = \frac{1}{\gamma - 1}\left[\hat{S}_i(t) - \hat{S}_j(t)\right] > 0. \tag{26}$$

Thus, the relative labor share growth rate is positively related to relative inflation rate when γ_i are identical. Also note that when $\gamma_i = \gamma_j$, labor share converges. This means that price also converges when $\gamma_i = \gamma_j$ and technological growths are identical. Now consider that $\gamma_i < 1$ whereas $\gamma_j > 1$. In this case, the labor share $S_i(t)$ diverges and the elasticity adjusted labor share $S_i(t)^{\gamma_i - 1}$ diverges. This results in price divergence even if the technological growths are the same.

The benchmark model predicts that there are strong relationships between the elasticity of substitution and the convergence club of labor share, and the convergence club of price. Empirically, we do find a strong relationship between the long term growth rates of labor share and prices after conditioning on techonological growth. The benchmark model nests two models, the NP and AG models. In the following subsection, we briefly discuss these models' predictions on the evolutions of labor share and price.

4.3 Price Divergence in the NP and AG models

In this subsection, we briefly discuss the NP and AG models' predictions on price divergence, labor productivity divergence, and the evolution of labor shares. These models' predictions are qualitatively equivalent to those in the benchmark model with restrictions on parameter values.

Ngai and Pissarides (2007) Model: Ngai and Pissarides (2007) model assumes the same production function across sectors with different productivity growth. Specifically, the production function is given by

$$Y_i(t) = A_i(t) F\left[K_i(t), L_i(t)\right], \tag{27}$$

where function F is common across sectors and has a constant returns to scale and other standard production function conditions, but $A_i(t)$ growth rates are different across sectors. This model has the same qualitative results in the benchmark model with the restrictions of the same elasticity $\gamma_i = \gamma$ and the same labor share parameter $\alpha_{Li} = \alpha_L$. We can rewrite the production function as

$$Y_i(t) = A_i(t) L_i(t) f\left(\widetilde{K}_i(t)\right), \tag{28}$$

where $f\left(\widetilde{K}_i(t)\right) = F\left(1, \widetilde{K}_i(t)\right)$ with $\widetilde{K}_i(t) = K_i(t)/L_i(t)$, the capital per worker in sector i. The cost minimization problem gives

$$\frac{W(t)}{R(t)} = \frac{f\left(\widetilde{K}_i(t)\right)}{f'\left(\widetilde{K}_i(t)\right)} - \widetilde{K}_i(t). \tag{29}$$

Since $W(t)/R(t)$ and the function $f(\cdot)$ are the same across sectors, $\widetilde{K}_i(t)$ should be the same across sectors, $\widetilde{K}_i(t) = \widetilde{K}(t)$. Rearranging (29), we have

$$S_i(t) = 1 - \frac{\widetilde{K}(t) f'\left(\widetilde{K}(t)\right)}{f\left(\widetilde{K}(t)\right)} = S(t). \tag{30}$$

So, the labor share $S_i(t)$ and its growth rates, $\widehat{S}_i(t)$ are the same across sectors regardless of $A_i(t)$ and its growth rate μ_i. From the first order conditions of the profit maximization problem, we have

$$P_i(t) = \left\{A_i(t)\left[f\left(\widetilde{K}(t)\right) - \widetilde{K}(t) f'\left(\widetilde{K}(t)\right)\right]\right\}^{-1} W(t) = \frac{1}{A_i(t)}\left(\frac{W(t)}{f\left(\widetilde{K}(t)\right) S(t)}\right). \tag{31}$$

With the same $\widetilde{K}(t)$ and $f(\cdot)$, we have

$$\hat{P}_i(t) - \hat{P}_j(t) = \mu_i - \mu_j. \tag{32}$$

So, the relative inflation rate only depends on the relative productivity growth. The heterogeneous productivity growth cannot explain heterogeneous evolution of labor share, and the relationship between relative inflation rate and the evolution of labor share.

Acemoglu and Guerrieri (2008) Model: In their model, each sector has a Cobb-Douglas production function with heterogeneous labor share. Specifically, the production function is given by

$$Y_i(t) = A_i(t) K_i(t)^{\alpha_{Ki}} L_i(t)^{\alpha_{Li}} \tag{33}$$

with $\alpha_{Ki} + \alpha_{Li} = 1$. This model is equivalent to the benchmark model with the restriction of $\gamma_i = \gamma = 1$. Without loss of generality, let sector 1 be more labor intensive, $\alpha_{L1} > \alpha_{L2}$. From the property of a Cobb-Douglas production function, the labor share is given by

$$S_i(t) = \alpha_{Li}. \tag{34}$$

Thus, the labor shares in two sectors are time invariant, $\widehat{S}_i(t) = 0$. The price of good in sector i is given by

$$P_i(t) = A_i(t)^{-1}\left[\frac{R(t)}{\alpha_{Ki}}\right]^{\alpha_{Ki}}\left[\frac{W(t)}{\alpha_{Li}}\right]^{\alpha_{Li}}. \tag{35}$$

Acemoglu and Guerrieri (2008) show that if the elasticity of substitution between two sector goods is less than 1, $\varepsilon < 1$, and the labor share adjusted productivity in sector 2 is higher than that in the other sector, $\frac{\mu_2}{\alpha_{L2}} > \frac{\mu_1}{\alpha_{L1}}$, there exists a constant growth path with $\widehat{R}(t) = 0$.[12] Along the path, the price growth rate is given by

$$\widehat{P}_i(t) = -\mu_i + \alpha_{Li}\widehat{W}(t),$$

where $\widehat{W}(t) = \mu_1/\alpha_{L1}$.[13] Thus, the price divergence occurs in their model,

$$\widehat{P}_1(t) - \widehat{P}_2(t) = \alpha_{L2}\left(\frac{\mu_2}{\alpha_{L2}} - \frac{\mu_1}{\alpha_{L1}}\right) > 0. \tag{36}$$

Although their models can explain the price divergence with diverging productivity, they cannot explain the evolution of labor share and its relation with the price divergence. To explain the data, the results from these models suggest that a model requires heterogeneous systematic time varying labor share α_{Li} possibly with heterogeneous $f(\cdot)$ as in our benchmark model.

4.4 Return to Empirical Evidence on Labor Share

The evolution of the log price in three models, the benchmark, NP and AG models, can be written, under on (25), in a general form of

$$\Delta_T p_{iT} = \frac{1}{\gamma_i - 1}\Delta_T s_{iT} - \Delta_T \ln A_{iT} + \beta_i \Delta_T w_{iT}, \tag{37}$$

where $\Delta_T x_{iT} = \log(X_{iT}/X_{i1})/(T-1)$. This reduced form nests both NP and AG models. In the benchmark model, $\beta_i = 1$. In a version of NP model with the CES production function, $\gamma_i = \gamma$ and $\beta_i = 1$. In the AG model, $\Delta s_{iT} = 0$, $\beta_i = \alpha_{Li}$.[14]

Technology growth $\Delta_T \ln A_{iT}$ is of course unobservable. We can however use long term output growth as a proxy technology growth. Our reasoning is as follows. We have

$$\ln Y_{it} = \ln A_{it} + \ln F_{it} = \left(1 + \frac{\ln F_{it}}{\ln A_{it}}\right)\ln A_{it}.$$

[12] In the AG model, the CES aggregate function is given by

$$Y(t) = \left[\kappa_1 y_1(t)^{\frac{\varepsilon-1}{\varepsilon}} + \kappa_2 y_2(t)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

See Acemoglu and Guerrieri (2008) for detailed discussion on the constant growth path in the model.

[13] Note that in the constant growth path the wage growth rate depends only on the relatively low productivity growth, $\frac{\mu_1}{\alpha_{L2}} < \frac{\mu_2}{\alpha_{L2}}$.

[14] Strictly speaking, in the benchmark model, $\lim_{\gamma_i \to 1} \Delta s_{iT}/(\gamma_i - 1) = -(1-\alpha_{Li})\Delta w_{iT}$. Thus, the log price equation in the benchmark, (25), becomes that in the AG model as

$$\Delta p_{iT} = -\Delta \ln A_{iT} + \alpha_{Li}\Delta w_{iT}.$$

In the limit, we have

$$\lim_{t\to\infty}\left(1+\frac{\ln F_{it}}{\ln A_{it}}\right)=g_i,$$

because $\ln F_{it}/\ln A_{it}$ converges to a constant, g_i-1, that can be different across sectors. Taking the long run growth rate of $\ln Y_{it}$ with g_i, we have

$$\Delta_T \ln Y_{iT}=g_i\Delta_T \ln A_{iT}+o_p(1)$$

Incorporating this into (37) we have

$$\Delta_T p_{iT}=\beta_{1i}\Delta_T y_{iT}+\Delta_T w_{iT}+\beta_{3i}\Delta_T s_{iT}+u_{iT}, \tag{38}$$

where $\beta_{1i}=g_i^{-1}$ and $\beta_{3i}=(1-\gamma_i)^{-1}$, and u_{iT} incorporates the $o_p(1)$ approximation error. Note that the parameters on labor share and output growth are unidentified since these vary with each cross section. We run the following empirical counterpart to (38).

$$\Delta_T p_{iT}=\alpha+\beta_1\Delta_T \ln Y_{iT}+\beta_2\Delta_T \ln W_{iT}+\beta_3\Delta_T \ln S_{iT}+v_{iT}, \tag{39}$$

where

$$v_{iT}=(\alpha_i-\alpha)+(\beta_{1i}-\beta_1)\Delta_T \ln Y_{iT}+(\beta_{2i}-\beta_2)\Delta_T \ln W_{iT}+(\beta_{3i}-\beta_3)\Delta_T \ln S_{iT}+u_{iT}.$$

where we impose homogeneity on $\beta_3 = \mathrm{E}(\gamma_i-1)^{-1}$. However the regression in (39) becomes problematic since the sign of $(\gamma_i-1)^{-1}$ is dependent of the value of γ_i. Suppose that some of γ_i are less than unity but the others are greater than unity. And then there is possibility that β_3 becomes zero. To prevent this, we consider the following price convergence club dummy variable regression.

$$\Delta_T p_{iT}=a+\alpha\Delta_T y_{iT}+\beta\Delta_T w_{iT}+\lambda_1\Delta_T s_{iT}+\lambda_2(\Delta_T s_{iT}\times D_i)+\epsilon_{iT}, \tag{40}$$

where $D_i=1$ if the ith sector is in the second price convergence club, otherwise $D_i=0$. Note that the long run inflation rates in the second price convergence club are relative low. This implies that the long run growth rates of the log labor share in the second club are larger than unity. Hence the expected signs become $\lambda_1<0$ but $\lambda_2>0$.

Table 7 shows the results with various regression specifications. When we impose restriction on $\beta=\lambda_1=\lambda_2=0$, the $\bar{R}^2$ statistic is around 0.37. The sign of α becomes negative, and $\hat{\alpha}$ is significantly different from zero. Next, when we impose the restriction (Reg 2) on $\alpha=\beta=0$ but $\lambda_1=\lambda_2=\lambda_0$, we find that $\hat{\lambda}_0$ becomes around -0.4 but is not significantly different from zero. Also $\bar{R}^2$ is near to zero. When we relax the restriction of $\alpha=0$ but maintain other restrictions (Reg 3), the estimation results do not change much. The point estimate of $\hat{\lambda}_0$ is not statistically different from zero. However when we run (40), which is the correct specification of the long run inflation rate, we find that both $\hat{\lambda}_1$ and $\hat{\lambda}_2$ are significantly different from zero (Reg 4), and as our model predicts, the signs of $\hat{\lambda}_1$ and $\hat{\lambda}_2$ are negative and positive, respectively. Lastly we relax the restriction of $\beta = 0$ (Reg 5). The inclusion of the long run wage growth rates does not change the regression results.

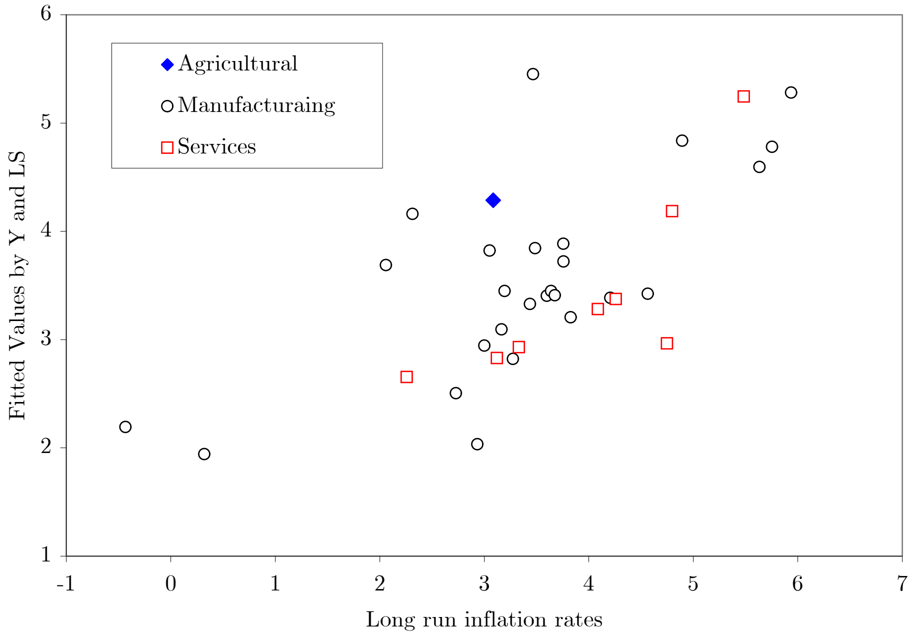
Table 7: Between Group Regressions for Long Run Inflation Rates

Independent	Reg 1	Reg 2	Reg 3	Reg 4	Reg 5
constant	0.045	0.033	0.044	0.045	0.021
	(15.61)	(11.50)	(12.67)	(14.01)	(1.219)
real output	-0.419		-0.408	-0.404	-0.399
	(-4.259)		(-4.124)	(-4.458)	(-4.532)
labor share		-0.440	-0.234	-0.711	-0.699
		(-1.146)	(-0.738)	(-2.054)	(-2.080)
LS dummy				1.125	0.899
(for Price Club 2)				(2.543)	(1.966)
wage					0.456
					(1.439)
$\bar{R}^2$	0.368	0.066	0.378	0.478	0.509

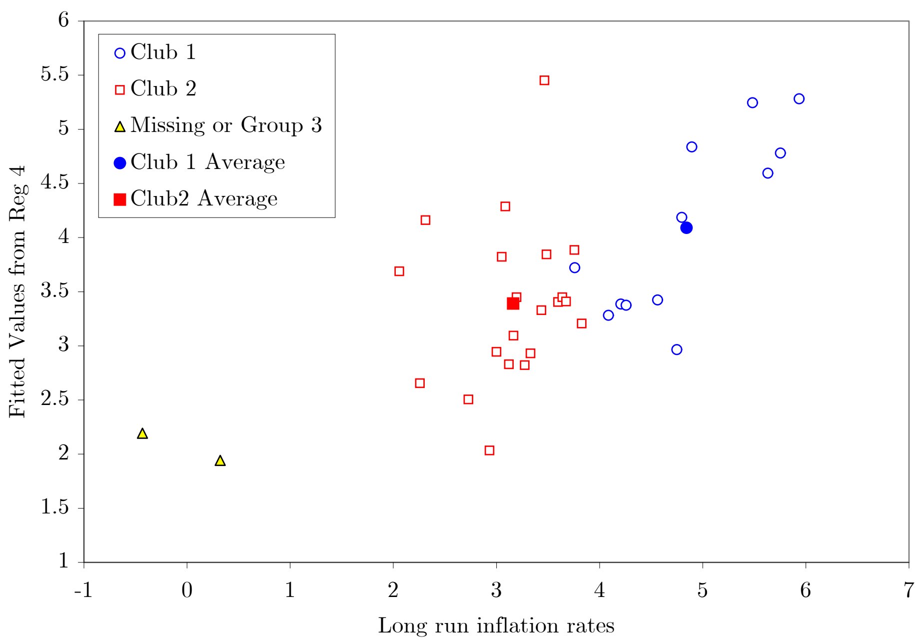
Figure 4 displays the fitted values from Reg 4 in Table 7 against the long run inflation rates. Panel A and B show the same figure but with different decompositions. In Panel A, we group industries according to sectors. Obviously, grouping by sectors does not explain the general pattern of the fitted values and the long run inflation rates. As we showed earlier in Table 1, 3 and 4, structural transformation cannot be explained well by sectoral decompositions. In Panel B, we group industries based on the price convergence club. Evidently, the approach based on the price convergence clubs explain the long run relationship with disaggregate and aggregate data.

Figure 4: Fitted Values of Long Run Inflation Rates

Panel A: Sectors



Panel B: Price Clubs



5 Conclusion

Prices in different parts of the economy diverge as economies grow wealthier. Typically this stylized fact is documented based on broad aggregate sectors, such as agriculture, manufacturing and services, and as part of a broader study of structural transformation in the economy. In this paper we consider what we can learn about price divergence by looking at disaggregate data. We argue that disaggregate data are more appropriate for analyzing price divergence and broader questions related to structural transformation because there is substantial heterogeneity in the long-term inflation rates of items within the conventional goods and services sectors of the economy. Broad sectors are therefore likely to obfuscate key trends in the composition of resource allocation over long time horizons. For example, long term growth in prices and quantities are negatively correlated in both expenditure and production decompositions of the economy, lending support to production-based (as opposed to preference-based) theories of structural transformation. We also show that there is substantial heterogeneity in long term changes in labor share at the industry level. We build a basic model of structural transformation to account for these findings. The key ingredients of our static model are heterogeneity in labor substitutability across industries coupled with differences in industry-specific technological growth.

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Appendix

Proof of Proposition 1 From (23), we have

$$S_i(t) = \left\{1 + \left[\frac{1 - S_i(0)}{S_i(0)}\right] e^{(\gamma_i - 1)\int_0^t \xi(\tau)d\tau}\right\}^{-1}, \tag{41}$$

with $S_i(t) = [0,1]$. Here $\int_0^t \xi(\tau)\,d\tau = \ln[W(t)/R(t)] - \ln[W(0)/R(0)]$. Clearly, if $\gamma_i > 1$, then $\lim_{t\to\infty} S_i(t) = 0$, and if $\gamma_i < 1$, then $\lim_{t\to\infty} S_i(t) = 1$.

From (41) we have

$$\left[\frac{1}{S_1(t)} - 1\right]\left[\frac{1}{S_2(t)} - 1\right]^{-1} = \left[\frac{1}{S_1(0)} - 1\right]\left[\frac{1}{S_2(0)} - 1\right]^{-1} e^{-(\gamma_2 - \gamma_1)\int_0^t \xi(\tau)d\tau}. \tag{42}$$

If $\gamma_1 < \gamma_2$, eventually, $[1/S_1(t) - 1] < [1/S_2(t) - 1]$ regardless of the value of $S_1(0)/S_2(0)$. This means that eventually the economy has $S_1(t) > S_2(t)$. ∎

Proof of Proposition 2 First, consider the case with identical $\gamma = \gamma_1 = \gamma_2$. From (41), we have

$$\frac{S_1(t)}{S_2(t)} = \frac{1 + \left(\frac{1 - S_2(0)}{S_2(0)}\right) e^{(\gamma-1)\int_0^t \xi(\tau)d\tau}}{1 + \left(\frac{1 - S_1(0)}{S_1(0)}\right) e^{(\gamma-1)\int_0^t \xi(\tau)d\tau}}. \tag{43}$$

Clearly, $S_1(t) > S_2(t)$ iff $S_1(0) > S_2(0)$. From (23) we have

$$\widehat{S}_1(t) - \widehat{S}_2(t) = [S_1(t) - S_2(t)](\gamma - 1)\,\xi(t). \tag{44}$$

So, when $\gamma > 1$ (< 1), the relative labor share $S_1(t)/S_2(t)$ is monotonically increasing (decreasing) over time. If $\gamma < 1$, $\lim_{t\to\infty} e^{(\gamma-1)\int_0^t \xi(\tau)d\tau} = 0$. This gives $\lim_{t\to\infty}[S_1(t)/S_2(t)] = 1$. If $\gamma > 1$, we can rewrite (43) as

$$\frac{S_1(t)}{S_2(t)} = \frac{e^{-(\gamma-1)\int_0^t \xi(\tau)d\tau} + \left(\frac{1 - S_2(0)}{S_2(0)}\right)}{e^{-(\gamma-1)\int_0^t \xi(\tau)d\tau} + \left(\frac{1 - S_1(0)}{S_1(0)}\right)}. \tag{45}$$

Since $\lim_{t\to\infty} e^{-(\gamma-1)\int_0^t \xi(\tau)d\tau} = 0$ if $\gamma < 1$, $\lim_{t\to\infty}[S_1(t)/S_2(t)] = \left(\frac{1 - S_2(0)}{S_2(0)}\right) / \left(\frac{1 - S_1(0)}{S_1(0)}\right)$.

If $\gamma_1 < \gamma_2$ and $\gamma_2 > 1$, we have

$$\frac{S_1(t)}{S_2(t)} = \frac{e^{-(\gamma_2-1)\int_0^t \xi(\tau)d\tau} + \left(\frac{1 - S_2(0)}{S_2(0)}\right)}{e^{-(\gamma_2-1)\int_0^t \xi(\tau)d\tau} + \left(\frac{1 - S_1(0)}{S_1(0)}\right) e^{-(\gamma_2-\gamma_1)\int_0^t \xi(\tau)d\tau}}. \tag{46}$$

Clearly, the numerator converges to $[1 - S_2(0)]/S_2(0)$ whereas the denominator converges to 0. Thus, $\lim_{t\to\infty}[S_1(t)/S_2(t)] = \infty$. ∎

Table A1: Convergence Club Classification

Club	$\hat{\phi}$ ($t_{\hat{\phi}}$)	$\hat{\beta}$ ($t_{\hat{\beta}}$)	Member Items
Club 1 [19]	0.74 (8.72)	-0.01 (-2.52)	Net purchases of used motor vehicles; Educational books; Fuel oil and other fuels; Tobacco; Water supply and sanitation; Physician services; Dental services; Hospitals; Nursing homes; Accommodations; Life insurance; Net household insurance; Net health insurance; Net motor vehicle and other transportation insurance; Higher education; Nursery, elementary, and secondary schools; Commercial and vocational schools; Professional and other services; Household maintenance
Club 2 [21]	0.97 (22.71)	-0.01 (-3.23)	Food and nonalcoholic beverages purchased for off-premises consumption; Motor vehicle fuels, lubricants, and fluids; Pharmaceutical and other medical products; Magazines, newspapers, and stationery; Rental of tenant-occupied nonfarm housing; Imputed rental of owner-occupied nonfarm housing; Rental value of farm dwellings; Group housing; Natural gas; Paramedical services; Motor vehicle maintenance and repair; Ground transportation; Membership clubs, sports centers, parks, theaters, and museums; Gambling; Other recreational services; Purchased meals and beverages; Food furnished to employees; Financial services furnished without payment; Postal and delivery services; Personal care and clothing services; Social services and religious activities
Club 3 [20]	0.63 (11.30)	-0.02 (-4.21)	New motor vehicles; Furniture and furnishings; Glassware, tableware, and household utensils; Sports and recreational vehicles; Recreational books; Therapeutic appliances and equipment; Luggage and similar personal items; Telephone and facsimile equipment; Alcoholic beverages purchased for off-premises consumption; Food produced and consumed on farms; Other clothing materials and footwear; Recreational items; Household supplies; Personal care products; Electricity; Other motor vehicle services; Air transportation; Audio-video, photographic, and information processing equipment services; Financial service charges, fees, and commissions;Foreign travel by US residents
Club 4 [8]	0.29 (7.78)	-0.01 (-1.74)	Motor vehicle parts and accessories; Tools and equipment for house and garden; Sporting equipment, supplies, guns, and ammunition; Musical instruments; Jewelry and watches; Garments; Water transportation; Telecommunication services
Group 5 [1]			Household appliances;