# Trend Inflation, Firm's Backward-looking Behavior, and Inflation Persistence

Insu Kim* and Myung-Soo Yie†

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**Abstract**

It is often believed that inflation persistence can be attributed to variation in the Federal Reserve's long-run inflation target rather than firm's backward-looking pricing behavior in the literature. In this regard, this paper investigates whether there is a need for the role of firm's backward-looking behavior in accounting for inflation persistence even after trend inflation is taken into account. To this end, we study the role of past output gaps in determining the inflation gap, defined as the deviation of inflation from its long-run path. Our findings are threefold. First, there is a long-lasting effect of the output gap on inflation gap and the maximum effect of the output gap on inflation gap occurs after four to seven quarters. Second, most fluctuations in the inflation gap are attributable to past output gaps than current and expected future output gaps. Finally, firm's backward-looking behavior is an essential ingredient in accounting for observed dynamic correlation between the output gap and inflation gap while the Federal Reserve's long run inflation target as a source of inflation persistence is not. Moreover, moments of key macroeconomic variables are best explained when both forward- and backward-looking behavior are equally important in determining the inflation gap. These results disagree with the forward-looking New Keynesian Phillips Curve (NKPC).

Keywords: Inflation Persistence, Backward-looking Behavior, Trend Inflation, Inflation Target.

JEL Classification: E31

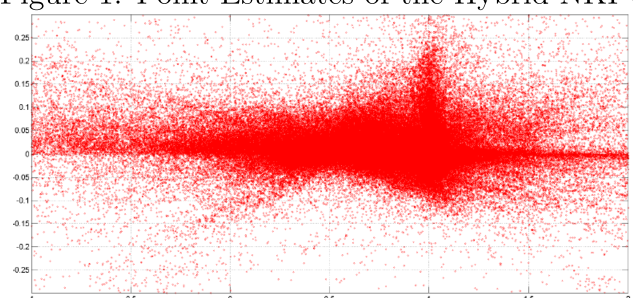
*Department of Economics, Sungkyunkwan University, Seoul, Korea (email: insu.kim@skku.edu). This work was supported by the National Research Foundation of Korea Grant funded by the Korean Government (NRF-2014S1A5B8060964).

†Corresponding author: Economic Research Institute of the Bank of Korea (email: yie@bok.or.kr). The views expressed in this paper are those of the authors and do not necessarily reflect the official views of the Bank of Korea.

# 1 Introduction

A large body of literature has employed the hybrid NKPC to study monetary policy and business cycles. However, there is still disagreement on the issue of intrinsic inflation persistence introduced by Fuhrer and Moore (1995). This issue matters because the presence of inflation persistence alters the welfare loss function that the central bank aims to minimize (Woodford, 2003). The survey papers written by Schorfheide (2008) and Mavroeidis, Plagborg-Møller, and Stock (2014) discover that the estimate of the degree of inflation persistence is substantially different across papers. In particular, Mavroeidis et al. (2014) re-estimate various specifications of the hybrid NKPC based on a wide range of estimation techniques considered in the literature. They presented Figure 1 which shows point estimates of the slope of the hybrid NKPC (y-axis) and the coefficient on inflation expectations (x-axis). The figure does not conclude whether inflation persistence matters in explaining inflation dynamics owing to the fact that the estimates of the coefficient on inflation expectations are distributed very widely between minus one and plus two rather than being centered around a particular point.

Figure 1: Point Estimates of the Hybrid NKPC




Source: Mavroeidis et al. (2014). Note: The y-axis represents the slope of the Phillips curve and the x-axis is the coefficient on inflation expectations. The output gap is considered as the driving force of inflation in the figure.

In recent literature, the hypothesis that inflation persistence is a consequence of variation in the Fed's long run inflation target (equivalently, trend inflation) has gained attention. However, there is still no consensus on the estimation of the hybrid NKPC even after the trend inflation component is eliminated from inflation. Ireland (2007) estimates a dynamic stochastic general equilibrium (DSGE) model incorporating the Federal Reserve's long-run inflation target and finds that the lagged inflation term of the hybrid NKPC does not play any role in explaining inflation dynamics. Cogley and Sbordone (2008) also document evidence that there is no need for the lagged inflation term once the low frequency component is eliminated from the inflation rate. Their results imply that the level of inflation is persistent, but the inflation gap is not. In contrast to these works, Barnes, Gumbau-Brisa, Lie, and Olivei (2011) report that the empirical results of Cogley and Sbordone (2008) is sensitive to model specifications. In addition, their Monte Carlo exercises reveal that estimates of the hybrid NKPC can be biased toward the forward-looking component.

This article aims to evaluate if firm's backward-looking behavior is an essential feature in accounting for inflation gap dynamics. Since estimation results of the hybrid NKPC is highly controversial, this article does not aim to estimate the hybrid NKPC. Instead, we assess the importance of backward-looking behavior with respect to how long the output gap, defined as the deviation of output from its trend, impacts the inflation gap. The NKPC suggests that the inflation gap depends on current and expected future output gaps while the hybrid NKPC implies that past output gaps also play an important role in determining inflation gap dynamics due to firm's backward-looking behavior. This distinction permits us to gauge the empirical relevance of firm's backward-looking behavior without relying on direct estimation of the hybrid NKPC. In our model framework, the degree of inflation gap persistence is proportional to the length of time that the output gap impacts the inflation gap. We demonstrate that the duration is closely associated with the degree of firm's backward-looking behavior.

This article undertakes reduced form analysis to determine whether the output gap has a long-lasting effect on the inflation gap. We also provide structural analysis that helps us to assess the contribution of firm's backward-looking behavior to inflation gap dynamics. A model's ability to match the second moments of key macroeconomic variables is often considered as a crucial criterion of empirical success in the literature. Thus, we investigate whether the (hybrid) NKPC is able to match standard deviations of key macroeconomic variables using a DSGE model framework. The (hybrid) NKPC is designed to provide a description of the dynamic relationship between the output gap and the inflation gap. Therefore, we also explore whether it does a reasonable job of describing the joint behavior of the two variables. A desirable Phillips curve is expected to perform well in matching basic macroeconomic moments and the dynamic correlation between the output gap and inflation gap.

Our findings are threefold. First, the output gap has a long-lasting effect on the inflation gap. It takes around three years for the inflationary impact of the output gap to die out completely. The maximum effect of the output gap on the inflation gap occurs after four to seven quarters. These empirical results cannot be reconciled by the NKPC which implies an immediate maximum impact of economic shocks on the inflation gap.

Second, a weighted average of current and past output gaps delivers a very good approximation of inflation gap dynamics. The evidence is in contrast to the prediction of the NKPC that current and future output gaps play a dominant role in enhancing likelihood. Our findings show that most fluctuations in the inflation gap are attributable to past output gaps rather than expected future output gaps.

Finally, firm's backward-looking behavior is an essential feature in accounting for observed dynamic correlation between the output gap and the inflation gap. The data show that current output gap is positively related to future inflation gap and negatively to lagged inflation gap. The reverse dynamic correlation between the two variables is observed even after the low frequency component is eliminated from the inflation rate. The correlation structure cannot be replicated by the forward-looking NKPC and the poor performance of the NKPC can be ascribed to the inability of the model to generate the slow and delayed impact of the output gap on the inflation gap. Our counterfactual analyses reveal that the (hybrid) NKPC in which forward-looking behavior plays a dominant role in determining the inflation gap leads to an unrealistically high contemporaneous correlation between the output gap and the inflation gap even though the estimated contemporaneous correlation is very close to zero. We also document evidence that that the moments of key macroeconomic variables are best explained by the hybrid NKPC in which both backward- and forward-looking behaviors are equally important in determining inflation gap dynamics.

The remainder of the paper is organized as follows. Section II explores how the inflation gap evolves within a DSGE model economy with agents forming expectations in a model-consistent fashion. In particular, we study the role of past output gap as a determinant of the inflation gap using a DSGE framework. Section III illustrates trend inflation estimates used to measure the inflation gap in this article. Section IV implements new tests for the importance of intrinsic inflation persistence using a variety of output and inflation gap measures. Section V investigates how the backward-looking component helps explain observed moments of key macroeconomic variables and dynamic correlation between the output gap and the inflation gap. Section VI conducts subsample analysis to investigate whether there is a structural change in inflation gap persistence. The last section concludes.

# 2 DSGE Model and Inflation Persistence

This section illustrates that the fundamental difference between the forward-looking NKPC and the hybrid NKPC is associated with whether past output gaps play a crucial role in determining current inflation gap. In order to deliver the intuition behind the distinction that helps assess the importance of intrinsic inflation persistence, as in Woodford (2008), we consider a simple DSGE model economy with trend inflation, which can be described as follows

$$\pi_t - \pi_t^* = \theta E_t(\pi_{t+1} - \pi_{t+1}^*) + (1-\theta)(\pi_{t-1} - \pi_{t-1}^*) + \gamma y_t \tag{1}$$

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1}) \tag{2}$$

$$i_t = \pi_t^* + a_\pi(\pi_t - \pi_t^*) + a_y y_t \tag{3}$$

where $y_t$ denotes the output gap, $i_t$ represents the short-term interest rate, and $\pi_t$ represent the inflation rate. We assume that the long-run path of inflation $\pi_t^*$ is set by the Federal Reserve and the inflation gap $\pi_t - \pi_t^*$ is defined as the deviation of inflation from its trend.

We consider the forward-looking IS curve, (2), that links the current output gap $y_t$ to the expected future output gap and real interest rates. It is also assumed that policymakers set interest rates only in response to changes in the inflation gap and output gap as shown in (3). Interest rate smoothing is not considered in the monetary policy rule for simplicity. We will use the DSGE model outlined above as a benchmark. Fuhrer (2009, page 42) demonstrates that adding a lagged output gap term to the forward-looking IS curve has a negligible contribution to inflation persistence and that interest rate smoothing in the policy rule even lowers inflation persistence. Later in the paper, we will attach a lag of the output gap to the IS curve and interest rate smoothing to the monetary policy rule for structural analysis. Autocorrelated cost-push shocks in the hybrid NKPC will be also taken into account later. Since adding additional model features mentioned above to the model does not change our results, we will first focus on the benchmark DSGE model to deliver intuition behind our proposed test for intrinsic inflation persistence based on the difference between the NKPC and the hybrid NKPC.

The forward-looking NKPC is a special case of the hybrid NKPC (1). When $\theta = 1$, (1) can be written as

$$\pi_t - \pi_t^* = E(\pi_{t+1} - \pi_{t+1}^*) + \gamma y_t. \tag{4}$$

Equation (4) implies that one period-ahead inflation expectations are formulated as $E_t(\pi_{t+1} - \pi_{t+1}^*) = \gamma \sum_{k=1}^{\infty} E_t y_{t+k}$ under the assumption of rational and model-consistent inflation expectations. Using (2), (3), and (4), we can demonstrate that the sum of the expected future output gaps, $\sum_{k=1}^{\infty} E_t y_{t+k}$, is determined by the current output and inflation gaps. Equation (2) suggests that the one-period ahead output gap expectations can be written as $E_t y_{t+1} = y_t + \frac{1}{\sigma}(i_t - E_t \pi_{t+1})$. Plugging (3) and (4) into the IS curve and rearranging the equation leads to $E_t y_{t+1} = \left(1 + \frac{\gamma + a_y}{\sigma}\right) y_t + \left(\frac{a_\pi - 1}{\sigma}\right)(\pi_t - \pi_t^*)$. Similarly, we can demonstrate that the two-period ahead output gap expectations can be written as $E_t y_{t+2} = \left[\left(1 + \frac{\gamma + a_y}{\sigma}\right)^2 - \left(\frac{a_\pi - 1}{\sigma}\right)\gamma\right] y_t + \left(2 + \frac{\gamma + a_y}{\sigma}\right)\left(\frac{a_\pi - 1}{\sigma}\right)(\pi_t - \pi_t^*)$. Note that the expected future output gaps are linked to the output and inflation gaps at time $t$. Once we repeat this algebra, we can conjecture that the $N$-periods ahead output gap expectations are also a function of the output and inflation gaps at time $t$ for all $N$. Therefore, model-consistent inflation expectations can be written as

$$E_t(\pi_{t+1} - \pi_{t+1}^*) = \gamma \sum_{k=1}^{\infty} E_t y_{t+k} = a y_t + b(\pi_t - \pi_t^*) \tag{5}$$

where $a \equiv a(\gamma,\ \sigma,\ a_\pi,\ a_y)$ and $b \equiv b(\gamma,\ \sigma,\ a_\pi,\ a_y)$. Plugging (5) into (4) results in

$$\pi_t - \pi_t^* = \gamma \sum_{k=0}^{\infty} E_t y_{t+k} = \frac{a+\gamma}{1-b} y_t. \tag{6}$$

The forward-looking NKPC implies that the inflation gap is solely determined by the current output gap $y_t$ in the DSGE model economy.[1]

Now we turn to the hybrid NKPC to see the relationship between the output and inflation gaps. Under the assumption of model-consistent inflation expectations, the hybrid NKPC can be expressed as follows

$$(1 - \tilde{\rho}L)(\pi_t - \pi_t^*) = \frac{\gamma}{\theta} \sum_{k=0}^{\infty} E_t y_{t+k} \tag{7}$$

where $\tilde{\rho} \equiv \frac{1-\theta}{\theta}$ and $L$ is the lag operator. Rearranging the three equations (1) - (3) that constitute the DSGE model, the expected future output gaps can be written as a linear function of the current and past inflation gaps as well as the current output gap,

$$\frac{\gamma}{\theta} \sum_{k=1}^{\infty} E_t y_{t+k} = c(\pi_t - \pi_t^*) + d(\pi_{t-1} - \pi_{t-1}^*) + e y_t \tag{8}$$

where $c \equiv c(\theta,\ \gamma,\ \sigma,\ a_\pi,\ a_y)$, $d \equiv d(\theta,\ \gamma,\ \sigma,\ a_\pi,\ a_y)$, and $e \equiv e(\theta,\ \gamma,\ \sigma,\ a_\pi,\ a_y)$. Plugging (8) into (7) gives rise to

$$\pi_t - \pi_t^* = \rho(\pi_{t-1} - \pi_{t-1}^*) + \tilde{\gamma} y_t \tag{9}$$

where $\rho \equiv \frac{\tilde{\rho}+d}{1-c}$ and $\tilde{\gamma} \equiv \frac{e+\gamma/\theta}{1-c}$. If the forward-looking NKPC is the true model for inflation gap dynamics, in other words, when $\theta = 1$, $\tilde{\rho} = 0$ and $d = 0$. In this case, the parameter $\rho$ turns out to be zero. The highly significant estimates of the parameter $\rho$ that have been repeatedly reported in the literature can be interpreted as supporting the empirical relevance of the backward-looking component. Interestingly, the algebra shows that (9) based on the hybrid NKPC has the same functional form as the backward-looking Phillips curve. It shows that, even though the true model for inflation gap dynamics is assumed to be the hybrid NKPC, the observed inflation dynamics could be derived in the same way as the implication, under the assumption that expectations are formulated in a DSGE model-consistent way, of the backward-looking model.

[1] Adding a lag of the output gap to the IS curve does not change our results. For simplicity, let's assume that output gap dynamics are approximated by $y_t = \alpha E_t y_{t+1} + (1-\alpha) y_{t-1}$. A rational expectation solution (transition equation) of the model suggests that the current output gap $y_t$ is solely determined by $y_{t-1}$. That is, $y_t$ follows an AR(1) process. Then the summation term $\sum_{k=0}^{\infty} E_t y_{t+k}$ can be expressed as a function of $y_t$. Later in section 5, we will take into account the conventional IS curve $y_t = \alpha E_t y_{t+1} + (1-\alpha) y_{t-1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1}) + \epsilon_t^y$ to discuss the importance of the backward-looking component.

Iterating the Phillips curve (9) backward leads to the functional form as follows

$$\pi_t - \pi_t^* = \tilde{\gamma} \sum_{k=0}^{\infty} \rho^k y_{t-k}. \tag{10}$$

When we impose a restriction of $\rho = 0$ on (10), the model collapses into (6). The past output gaps play no significant role in predicting the inflation gap when all firms set their prices in a forward-looking manner. On the other hand, if a significant fraction of firms act in a backward-looking manner in setting their prices, the inflation gap will be largely accounted for by the past output gaps.

The difference between the forward-looking NKPC and the hybrid NKPC, as shown in (6) and (10), permits us to assess the importance of backward-looking pricing behavior with respect to predictability of the past output gaps for the inflation gap. We estimate (10) to evaluate the empirical relevance of the past output gaps in accounting for the inflation gap. Our interest is to see if the parameter $\rho$ is sufficiently different from zero so that the past output gaps have a substantial impact on the inflation gap.

# 3 Inference on Trend Inflation

This article attempts to assess the importance of the backward-looking component in accounting for inflation gap dynamics. A statistical model is used to estimate trend inflation instead of DSGE models since the estimates vary with Phillips Curve specifications and coefficients of Phillips curve are not robustly identified in DSGE models (See Ireland, 2007 and Schorfheide, 2008).

Two statistical models are often adopted to estimate trend inflation in the literature. Cogley and Sargent (2005) and Cogley and Sbodone (2008) employ a Bayesian Vector Autoregression (VAR) model with drifting coefficients. An alternative model is the Unobserved Components (UC) model adopted by Stock and Watson (2007) in which they decompose inflation into a trend component and a serially uncorrelated innovation. However, as Ascari and Sbordone (2015) point out, this approach is "unsuitable" for the study of inflation gap persistence because the assumption of the serially uncorrelated innovation imposes no persistence in the inflation gap. In this respect, the Bayesian VAR model with drifting coefficients has an advantage for the study of the persistence of the inflation gap over the UC model.

Recently Kim, Manopimoken, and Nelson (2014) embedded a cyclical component that shows some serial correlation to the UC model employed by Stock and Watson (2007). This model framework, along with the VAR framework, is compatible with existing macroeconomic models in which inflation is determined by a cyclical component such as output gap (or labor's share of income) in addition to the trend inflation component as shown in (6) and (10).

We estimate trend inflation using a UC model with a cyclical component. The unobserved component model is described by

$$\begin{bmatrix} \pi_{t,GDP} \\ \pi_{t,NFB} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \pi_t^* \\ c_t \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \epsilon_{t,GDP} \\ \epsilon_{t,NFB} \end{bmatrix} \tag{11}$$

$$\begin{bmatrix} \pi_t^* \\ c_t \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \kappa \end{bmatrix} \begin{bmatrix} \pi_{t-1}^* \\ c_{t-1} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \epsilon_t^{\pi^*} \\ \epsilon_t^c \end{bmatrix} \tag{12}$$

where $\pi_t^*$ denotes a driftless random walk process and $c_t$ represents a cyclical component of inflation that follows an AR(1) process with the autoregressive parameter $\kappa$. This paper uses inflation measures based on the GDP and non-farm business sector (NFB) deflators. The innovations $\epsilon_{t,GDP}$, $\epsilon_{t,NFB}$, $\epsilon_t^{\pi^*}$, and $\epsilon_t^c$ are independent white noise processes. Let $\epsilon_{t,GDP}$ ($\epsilon_{t,NFB}$) denote an innovation to the GDP (NFB) inflation rate with mean zero and standard deviation $\sigma_{GDP}$ ($\sigma_{NFB}$). Let $\epsilon_t^{\pi^*}$ ($\epsilon_t^c$) denote an innovation to the trend (cyclical) component with mean zero and standard deviation $\sigma_{\pi^*}$ ($\sigma_c$). The data are taken from the Federal Reserve Bank of St. Louis' Fred dataset. The innovation $\epsilon_{t,GDP}$ ($\epsilon_{t,NFB}$) in (11) is likely to capture both measurement errors and a component specific to GDP (NFB) inflation measure. We assume that the trend inflation component is common to the inflation measures since trend inflation can be viewed as being governed by the Fed's long run inflation target in the literature (e,g, Ireland, 2007 and Cogley and Sbordone, 2008). We also consider the cyclical component $c_t$ as being common to the inflation measures because individual prices for the NFB deflator constitutes a subset of all the prices for the GDP deflator and the transitory price movements of the farm sector are less likely to be associated with business cycles. Notice that the highly transitory effects of individual prices of the farm sector on the GDP deflator are excluded in the NFB deflator. The fluctuations of GDP inflation associated with the transitory price movements are likely to be captured by the innovation $\epsilon_{t,GDP}$.[2]

We estimate trend inflation using the method of maximum likelihood. The sample starts in 1960 and ends in 2003. We focus on the same sample period considered in Cogley and Sbordone (2008) to see whether empirical results are sensitive to trend inflation estimates from the UC and time-varying Bayesian VAR models. We utilize Cogley and Sbordone (2008)' trend inflation estimate that is obtained from a time-varying Bayesian VAR model as a benchmark. The variables considered in the time-varying Bayesian VAR model are GDP inflation, real GDP growth, real unit labor cost, and the Federal Funds rate. As in Cogley and Sbordone (2008), we use the data from 1954 to 1959 as a training sample.

Table 1 presents the estimates of the UC model parameters. The results show that the standard deviations of the innovations to the trend and cyclical components are estimated

[2] Allowing the cyclical component to vary with each inflation measure does not change our results.

Table 1: Estimation Results of UC Model

| parameter | $\kappa$ | $\sigma_{\pi^*}$ | $\sigma_c$ | $\sigma_{GDP}$ | $\sigma_{NFB}$ | likelihood |
|---|---|---|---|---|---|---|
| | 0.795 (0.031)*** | 1.273 (0.148)*** | 2.054 (0.559)*** | 0.0001 (0.0049) | 0.0001 (0.7168) | 405.3 |

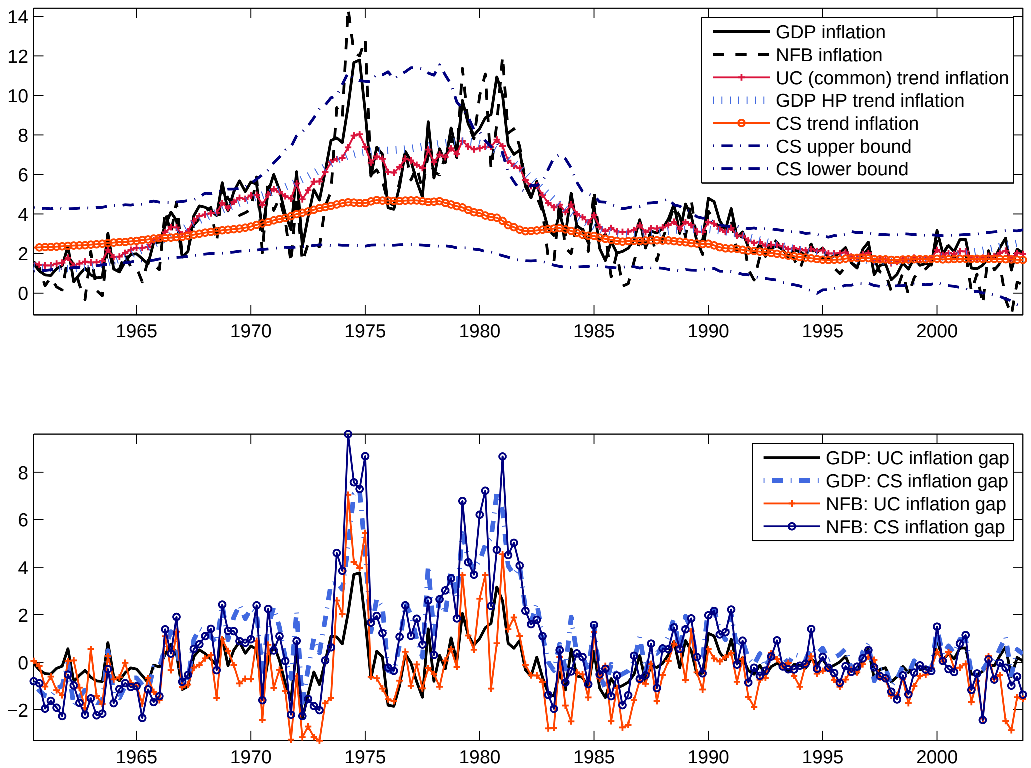
Notes: This table reports the estimates of the parameters for the UC model. The parameter $\kappa$ denotes the AR(1) coefficient of the cyclical component in the UC model. $\sigma_{\pi^*}$ ($\sigma_c$ ) denotes the standard deviation of an innovation to the trend inflation component (the cyclical component). $\sigma_{GDP}$ is the standard deviation of $\epsilon_{t,GDP}$ and $\sigma_{NFB}$ is the standard deviation of $\epsilon_{t,NFB}$. The superscript $***$ denotes significance at the one percent level. The case without the superscript denotes no signicance at the five percent level.

to be statistically different from zero, indicating the two components matter in accounting for inflation gap dynamics. The autoregressive parameter $\kappa$ is estimated to be 0.795 and is highly significant at the one percent level. The estimate is closely related to that of the autocorrelation of output gap. When the NKPC is adopted for inflation gap dynamics, as shown in section II, the current inflation gap can be written as a linear function of the current output gap. In this case, the output gap constitutes the cyclical component of inflation. When the hybrid NKPC is used to describe inflation gap dynamics, the cyclical component is expressed by the summation term in (10) that is related to a weighted average of the current and past output gaps.

Figure 2 shows a graph of trend inflation estimates, inflation, and inflation gap measures. The top panel of Figure 2 depicts GDP inflation (solid line), NFB inflation (dashed line), Cogley and Sbordone's estimate of trend inflation (solid line with circles), its 90 percent credible sets (dash-dot lines), trend inflation from the UC model (line with crosses), and the trend component of GDP inflation based on the Hodrick- Prescot (HP) filter (dotted vertical line). The panel shows that GDP inflation is very tightly related to NFB inflation. The correlation between the inflation measures is 0.93. Cogley and Sbordone (2008)'s estimate shows rising trend in the 1960s and 1970s and declining trend in the 1980s. Trend inflation becomes quite flat in the 1990s and the early 2000s. The 90 percent credible sets of Cogley and Sbordone (CS)'s trend inflation reveals that trend inflation is measured with substantial uncertainty.[3] Therefore, it is natural to consider an alternative trend inflation measure for robustness check. The figure shows that the trend inflation estimates based on the UC model and the HP-filter are quite similar. The trend inflation estimates rose from below two percent to around six percent in the pre-1980 period and fell sharply in the early 1980s. The trend inflation estimates have become anchored around two percent since the mid-1990s. This

[3] Using the combined survey data of Blue Chip and Livingston for ten-year ahead CPI inflation expectations, we find that the correlation between the UC (CS) trend inflation and the survey inflation expectations is 0.97 (0.94). The Blue Chip survey for 10-year inflation expectations is available from 1979:4 to 1991:1. The Livingston survey for the same inflation expectations starts from 1990:2. We combine these two data set to compare the correlation between the trend inflation estimates and the survey expectations. The sample period considered here is 1979:4-2003:4.

Figure 2: Trend Inflation and Inflation Gap




Notes: The upper panel shows GDP inflation (solid line), NFB inflation (dashed line), trend inflation from the UC model (line with crosses), trend inflation based on the HP-filter (dotted vertical line), Cogley and Sbordone's estimate of trend inflation (line with circles), and its 90 percent credible sets (dash-dot lines). The bottom panel depicts the GDP inflation gap (solid line) and the NFB inflation gap (line with crosses) based on the UC trend inflation. It also plots the GDP inflation gap (dash-dot line) and the NFB inflation gap (line with circles) based on Cogely and Sbordone's trend inflation estimates.

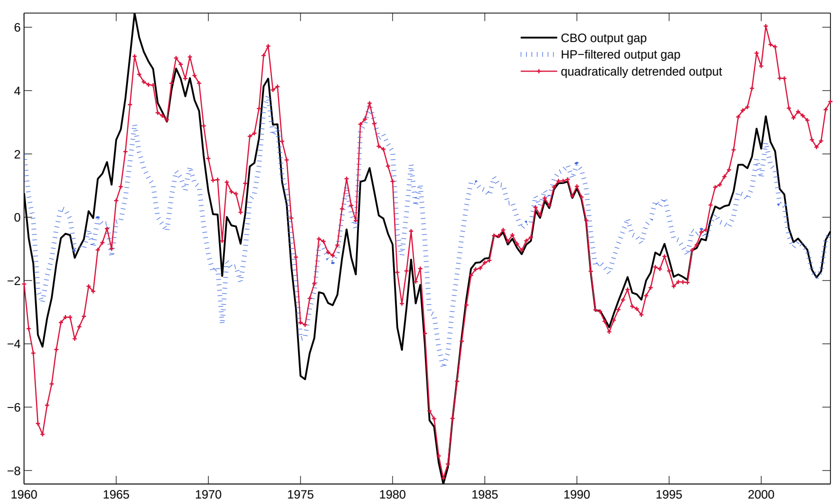
pattern is also observed with the CS inflation trend.

The bottom panel of Figure 2 reports the inflation gap measures based on the CS and UC trend inflation. The CS (UC) inflation gap is defined as the deviation of inflation from the CS (UC) trend inflation. We assume that the CS trend inflation is also common across the GDP and NFB inflation rates. The bottom panel depicts the GDP inflation gap (solid line) and the NFB inflation gap (line with crosses) based on the UC trend inflation. It also plots the GDP inflation gap (dash-dot line) and the NFB inflation gap (line with circles) based on Cogely and Sbordone's trend inflation estimates. These four inflation gap measures are considered to see whether our empirical results are sensitive.

Once inflation deviates from its trend, it reverts to its trend in the long run. This tendency is more frequently observed with the UC trend inflation than the CS trend inflation. In this respect, empirical analysis based on the UC inflation gap measures are less likely to overemphasize the importance of inflation gap persistence compared to those based on the CS inflation gap measures.

Figure 3 displays the output gap measures utilized in this article. We employ various output gap measures for empirical analysis in response to a concern that it may be subject to substantial measurement errors. The solid line depicts the Congressional Budget Office's (CBO's) estimate of the output gap. We also measure the output gap using the HP-filter and a quadratic time trend. In this article, HP denotes the HP-filtered output gap (dotted line) and Q represents the quadratically detrended log real GDP (line with crosses). As the figure shows, the output gap measures show very similar patterns even though its level is different.

Figure 3: Output Gap Measures




Notes: This figure displays output gap measures. The solid line depicts the CBO output gap and the dotted line plots the output gap measure based on the HP-filter. The line with crosses depicts the quadratically detrended output.

# 4 Empirical Results for Inflation Gap Persistence

## 4.1 Estimation Results

This section studies the importance of firm's backward-looking behavior with respect to the presence of the persistent inflationary effect of the output gap on the inflation gap. To investigate the issues, we do employ (13) which is obtained from iterating (9) backward $K$ times.

$$\pi_t - \pi_t^* = \tilde{\gamma} \sum_{k=0}^{K} \rho^k y_{t-k} + \rho^{K+1}(\pi_{t-(K+1)} - \pi_{t-(K+1)}^*) + \epsilon_t^\pi \tag{13}$$

We attach the residual term $\epsilon_t^\pi$ to (10). As in Smets and Wouters (2007), we assume it follows an ARMA(1,1) process, $\epsilon_t^\pi = \delta_{AR}\epsilon_{t-1}^\pi + \nu_t - \theta_{MA}\nu_{t-1}$, where $\nu_t$ is independent and identically distributed (i.i.d.) with mean zero and standard deviation $\sigma^\pi$.

We consider (13) instead of (10) to deal with the infinite number of the terms for past output gaps and $K$ is set to be 12 since the estimated correlation between $y_{t-(K+1)}$ and $\pi_t - \pi_t^*$ is close to zero around $K = 12$.[4] We estimate (13) using the method of Maximum Likelihood Estimation (MLE) to see whether the parameter $\rho$ is statistically different from zero. Table 2 summarizes estimation results. Panel A (B) report the results based on the GDP (NFB) inflation gaps. We also consider the cases in which trend inflation is fixed at zero (that is $\pi_t^* = 0$ ) and report estimation results in both panels.

The estimates of $\rho$ are always significantly different from zero across the output and inflation gap measures at the one percent significance level. The estimates range from 0.805 to 0.892 for the GDP inflation gap and from 0.751 to 0.877 for the NFB inflation gap. It suggests that the impact of the output gap on the inflation gap vanishes after around three years. The persistent inflationary impact reveals that a large fraction of the variation of inflation gap is explained by past output gaps.[5]

Interestingly, we find that imposing $\pi_t^* = 0$ on (13) does not change our results. Notice that the estimates of $\rho$ are between 0.835 and 0.879. The estimates of $\rho$ are not sensitive to detrending the low frequency component from the inflation rate. The findings indicate that observed inflation persistence is ascribed to firm's backward-looking behavior.

The parameter $\delta_{AR}$ is estimated to be higher once we impose the restriction $\pi_t^* = 0$ on (13). The estimates of $\delta_{AR}$ are distributed between 0.950 and 1.003 under the restriction of $\pi_t^* = 0$ while it ranges from 0.534 to 0.934 without the restriction. These results reveal that the low frequency component of inflation is captured by the AR(1) process of the residual term rather than the cyclical component.

It is worth mentioning that the high and statistically significant estimates of $\rho$ should not be interpreted as evidence that backward-looking pricing behavior plays a dominant role in determining the inflation gap. For simplicity, suppose that the economy is described by both $(1 - \rho L)(\pi_t - \pi_t^*) = \tilde{\gamma} \sum_{k=0}^{\infty} E_t y_{t+k}$ and $y_t = \varphi y_{t-1} + \epsilon_t$. Then, inflation gap dynamics can be written as $\pi_t - \pi_t^* = \rho(\pi_{t-1} - \pi_{t-1}^*) + \frac{\tilde{\gamma}}{(1-\varphi)} y_t$ where $\rho = \frac{1-\theta}{\theta}$. When the estimate of $\rho$ is around 0.8 as shown in Table 2, the parameter $\theta$ is computed to be 0.56. The result indicates that the forward- and backward-looking components play an equally important role in accounting for inflation gap dynamics. The equations also suggest that if forward-looking behavior plays a dominant role, the degree of inflation persistence will be estimated to be very low. Notice that $\theta = 0.8$ results in $\rho = 0.25$.

[4]The parameter $\rho$ is estimated to be around 0.75 - 0.89 as shown in Table 2. The second term on the right hand side of (13) plays a negligible role in determining the inflation gap because the value of $\hat{\rho}^{K=12}$ is quite close to zero.

[5]We further investigate this issue in a subsequent section.

Table 2: Estimation Results: 1960-2003

A. Inflation (Gap) Measures Based on the GDP Deflator

| $\pi_t - \pi_t^*$ | $y_t$ | $\rho$ | $\tilde{\gamma}$ | $\theta_{MA}$ | $\delta_{AR}$ | $\sigma^\pi$ | likelihood |
|---|---|---|---|---|---|---|---|
| CS inflation gap | CBO | 0.854 (0.031) | 0.075 (0.035) | 0.435 (0.077) | 0.934 (0.042) | 1.120 (0.101) | -238.2 |
| | HP | 0.892 (0.019) | 0.149 (0.029) | 0.538 (0.080) | 0.909 (0.047) | 1.050 (0.097) | -232.7 |
| | Q | 0.848 (0.033) | 0.069 (0.035) | 0.433 (0.084) | 0.924 (0.048) | 1.111 (0.099) | -237.5 |
| UC inflation gap | CBO | 0.815 (0.049) | 0.036 (0.020)* | 0.401 (0.126) | 0.772 (0.093) | 0.849 (0.069) | -235.5 |
| | HP | 0.880 (0.020) | 0.111 (0.016) | 0.290 (0.205)[a] | 0.534 (0.187) | 0.774 (0.068) | -227.2 |
| | Q | 0.805 (0.057) | 0.032 (0.017)* | 0.366 (0.127) | 0.732 (0.093) | 0.841 (0.068) | -234.6 |
| inflation ($\pi_t^* = 0$) | CBO | 0.857 (0.028) | 0.086 (0.028) | 0.468 (0.061) | 0.968 (0.022) | 1.093 (0.096) | -258.4 |
| | HP | 0.879 (0.021) | 0.143 (0.027) | 0.551 (0.059) | 0.968 (0.020) | 1.056 (0.094) | -255.3 |
| | Q | 0.852 (0.029) | 0.082 (0.027) | 0.480 (0.061) | 0.967 (0.024) | 1.080 (0.093) | -257.4 |

B. Inflation (Gap) Measures Based on the NFB Deflator

| $\pi_t - \pi_t^*$ | $y_t$ | $\rho$ | $\tilde{\gamma}$ | $\theta_{MA}$ | $\delta_{AR}$ | $\sigma^\pi$ | likelihood |
|---|---|---|---|---|---|---|---|
| CS inflation gap | CBO | 0.826 (0.059) | 0.117 (0.050) | 0.437 (0.073) | 0.933 (0.038) | 2.076 (0.164) | -287.9 |
| | HP | 0.877 (0.027) | 0.217 (0.040) | 0.498 (0.095) | 0.889 (0.057) | 1.916 (0.164) | -281.1 |
| | Q | 0.811 (0.074) | 0.103 (0.052) | 0.428 (0.079) | 0.922 (0.045) | 2.074 (0.161) | -287.7 |
| UC inflation gap | CBO | 0.780 (0.117) | 0.075 (0.053)[a] | 0.345 (0.110) | 0.835 (0.071) | 1.424 (0.096) | -281.2 |
| | HP | 0.853 (0.035) | 0.176 (0.036) | 0.372 (0.139) | 0.763 (0.092) | 1.297 (0.092) | -272.8 |
| | Q | 0.751 (0.178) | 0.056 (0.050)[a] | 0.319 (0.113) | 0.811 (0.070) | 1.434 (0.094) | -281.8 |
| inflation ($\pi_t^* = 0$) | CBO | 0.844 (0.043) | 0.132 (0.040) | 0.439 (0.068) | 0.960 (0.026) | 2.012 (0.155) | -312.1 |
| | HP | 0.876 (0.027) | 0.220 (0.037) | 0.503 (0.077) | 0.950 (0.032) | 1.904 (0.151) | -307.0 |
| | Q | 0.835 (0.046) | 0.133 (0.035) | 0.463 (0.059) | 1.003 (0.014) | 2.033 (0.157) | -324.7 |

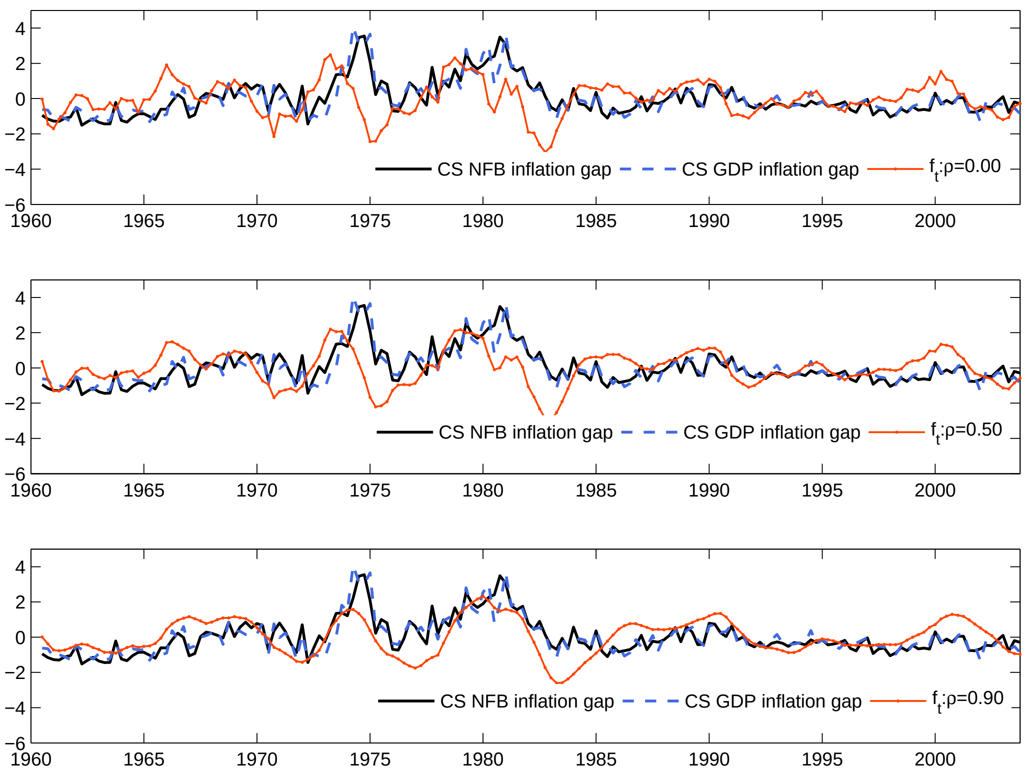
Notes: The sample period is 1960Q1-2003Q4. Asymptotic Newey-West standard errors are reported in parentheses. The superscript $*$ denotes significance at the ten percent level. The superscript letter $a$ denotes no signicance at the ten percent level. The case without $*$ or $a$ denotes statistical significance at the one or five percent level.

## 4.2 Firm's Backward-looking Behavior and Path of Inflation Gap

This subsection investigates how well the weighted average of current and past output gaps ($f_t \equiv \sum_{k=0}^{K=12} \rho^k y_{t-k}$ ) tracks the path of inflation gap. In particular, we study how $f_t$'s ability to track the path of the inflation gap varies with the degree of inflation gap persistence. Figure 4 plots the GDP inflation gap (dashed line) and the NFB inflation gap (solid line) computed using the CS trend inflation along with $f_t$ (line with crosses). The HP output gap is used as a measure of the output gap, $y_t$, to compute $f_t$. We consider three different values for the parameter $\rho$. The parameter $\rho$ is set to be 0, 0.5, and 0.9 in the top, middle, and bottom panel. Since the scale of $f_t$ depends on the value of $\rho$, each series is standardized to have a standard deviation of one and a mean of zero.

The figures show the tightness of linkage between the inflation gap measures and $f_t$ depends on the value of $\rho$. A higher value of $\rho$ is better able to account for inflation gap dynamics compared to a lower value. The $f_t$ does a poor job of tracking the inflation gap measures when the parameter $\rho$ is 0.0. We obtained very similar result for $\rho = 0.50$. In contrast to these results, the $f_t$ based on $\rho = 0.90$ works much better in tracking the ups and downs of the inflation gap measures, indicating that the persistent impact of output gap on inflation gap is necessary to describe the path of inflation gap.

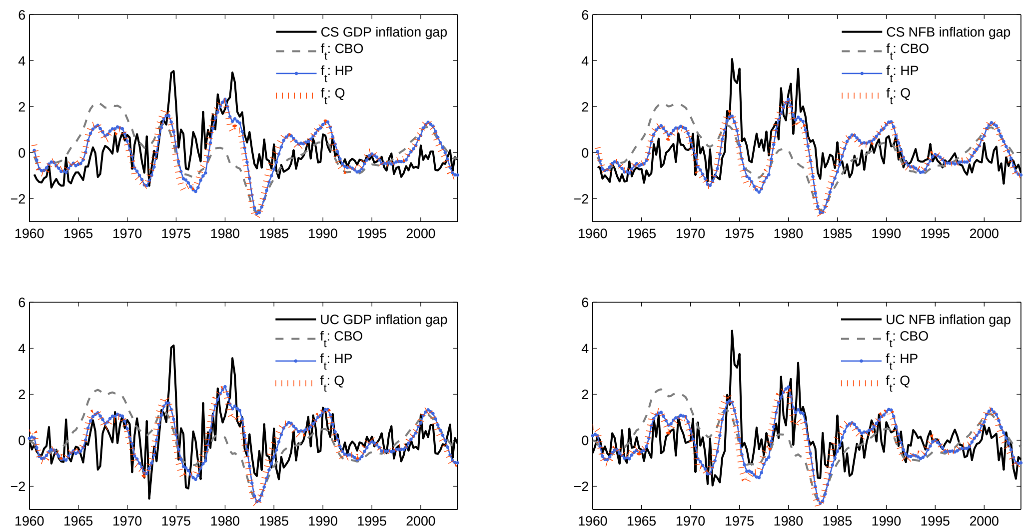
Figure 4: Degree of Inflation Persistence ($\rho$)and Inflation Gap Dynamics




Notes: This figure shows inflation gap measures and $f_t \equiv \sum_{k=0}^{K=12} \rho^k y_{t-k}$ computed using the HP output gap. The solid line represents the CS NFB inflation gap and the dashed line is the CS GDP inflation gap. The line with crosses depicts the $f_t$. The parameter $\rho$ is assumed to be 0.0, 0.5, and 0.9 to compute $f_t$ in each panel. Each series is normalized to have a one standard deviation and mean zero.

Figure 5 presents the four inflation gap measures along with the $f_t$ based on the output gap measures (CBO, HP, and Q) for the output gap $y_t$ and the estimates of $\rho$ reported from Table 2. In each panel, the dotted vertical line (blue line with dots) depicts the $f_t$ based on the Q (HP) output gap. The dashed line plots the $f_t$ based on the CBO output gap. The values of $f_t$ computed using the Q and HP output gap measures provide a satisfactory description of inflation gap even though a lagged inflation gap is absent from the Phillips curve. The $f_t$ based on the CBO output gap also moves in tandem with the inflation gap measures. However, there is still a considerable distance between the $f_t$ and the inflation gap measures in the 1970s. This result emerges from the fact that the CBO output gap has the most severe downward trend in the 1970s among the output gap series as shown in Figure 3 while the inflation gap measures do not show such a pattern.[6] Overall, we find that the $f_t$ emphasizing the persistent inflationary impact of the output gap does a reasonably good job of describing inflation gap dynamics and this result is consistently observed across various output and inflation gap measures.

Figure 5: Inflation Gap and a Weighted Average of Current and Past Output Gaps ($f_t$)




Notes: The figure presents inflation gap measures and $f_t = \sum_{k=0}^{K=12} \hat{\rho}^k y_{t-k}$ computed using various output gap measures (CBO, HP, and Q). The estimate of $\rho$ is plugged in $f_t$ to see whether $f_t$ works well in capturing the path of inflation gap.

One might be interested in the extent to which past output gaps have additional information not included in current output gap in predicting current inflation gap. A convenient way to investigate additional information content of past output gaps is to compare the correlation between $\pi_t - \pi_t^*$and $y_t$ to the one between $\pi_t - \pi_t^*$ and $f_t$.

The estimates of the correlation coefficients are reported in Table 3. It shows that the correlation between $\pi_t - \pi_t^*$ and $f_t$ is much higher than that between $\pi_t - \pi_t^*$and $y_t$. The contemporaneous correlation between $\pi_t - \pi_t^*$and $y_t$ is estimated to be slightly above zero. For the HP and Q output gaps, the correlation is between 0.05 and 0.14. When the weighted average $f_t$, instead of current output gap, is considered, the estimated correlation coefficient rises substantially by 0.33-0.40. As a consequence, the correlation ranges from 0.42 to 0.54. The contemporaneous correlation between the CBO output gap and inflation gap is often estimated to be negative. However, the observed negative correlation coefficients turn out to be positive once the current CBO output gap is replaced with $f_t$.[7] As the table shows, the

[6]Once we further eliminate the time trend component from the CBO output gap measure and use it to compute $f_t$, we find that the $f_t$ performs much better in tracking the inflation gap measures.

[7]Figure 3 shows that the CBO output gap has a downward trend in the 1970s and an upward trend in the 1980s and 1990s. When the trend component is further eliminated from the CBO output gap measure using the regression equation, $y_t = constant + \sum_{i=0}^{5} \alpha_i t^i$ where $t$ denotes time, the contemporaneous correlation

past output gaps also work to increase the correlation by 0.17-0.20.

These noticeable changes that are consistently observed across various inflation and output gap measures indicate past output gaps have an important contribution to inflation dynamics. In addition, the fact that the observed contemporaneous correlation between $y_t$ and $\pi_t - \pi_t^*$ is very close to zero cannot be reconciled with the forward-looking NKPC. Later in section 5, we show that the contemporaneous correlation between $y_t$ and $\pi_t - \pi_t^*$ should be greater than at least 0.50 once firm's forward-looking price-setting behavior plays a dominant role in explaining inflation gap dynamics. The result can also be conjectured from (6) in which the current output gap is a main driver of the inflation gap.

Table 3: Correlation between Inflation Gap and $f_t$

| | CS GDP inflation gap | | | UC GDP inflaton gap | | |
|---|---|---|---|---|---|---|
| correlation | CBO | HP | Q | CBO | HP | Q |
| $corr(\pi_t - \pi_t^*,\ y_t)$ | -0.12 | 0.12 | 0.09 | 0.05 | 0.14 | 0.15 |
| $corr(\pi_t - \pi_t^*,\ f_t)$ | 0.05 | 0.46 | 0.43 | 0.25 | 0.54 | 0.48 |
| | CS NFB inflation gap | | | UC NFB inflaton gap | | |
| correlation | CBO | HP | Q | CBO | HP | Q |
| $corr(\pi_t - \pi_t^*,\ y_t)$ | -0.13 | 0.11 | 0.05 | 0.00 | 0.14 | 0.05 |
| $corr(\pi_t - \pi_t^*,\ f_t)$ | 0.05 | 0.47 | 0.42 | 0.18 | 0.52 | 0.45 |

Notes: This table reports the correlation coefficient between $y_t$ and $\pi_t - \pi_t^*$, and the one between $\pi_t - \pi_t^*$ and $f_t$ based on the estimate of $\rho$ from Table 2.

## 4.3 Predictive Content of Current and Lagged Output Gaps for Inflation Gap Dynamics

This subsection investigates whether our empirical results for inflation gap persistence is robust to an alternative specification for the IS curve. Kim, Manopimoken, and Nelson (2014) consider the following model economy:

$$\pi_t - \pi_t^* = E_t(\pi_{t+1} - \pi_{t+1}^*) + \gamma y_t \tag{14}$$

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \epsilon_t^y \tag{15}$$

Once the economy is described by the above equations, the NKPC can be written as

$$\pi_t - \pi_t^* = \gamma E_t \sum_{k=0}^{\infty} y_{t+k} = \bar{\tau}_0 y_t + \bar{\tau}_1 y_{t-1} \tag{16}$$

where $\tau_0 = f(\phi_1,\ \phi_2)$ and $\tau_1 = g(\phi_1,\ \phi_2)$. The inflation gap can be predicted by the current and one-period lagged output gaps, $y_t$ and $y_{t-1}$, respectively when the output gap evolves

between the CBO output gap and inflation gap becomes positive. The correlation of $f_t$ with inflation gap becomes comparable to the cases associated with the other output gap measures. We find that the estimates of $\alpha_i$ for all integers $i \in [1,\ 5]$ are statistically different from zero at the five percent level.

over time according to an AR(2) process. Therefore, we can test the importance of the past output gap $y_{t-k}$ for integer $k \geq 2$. To this end, we consider an alternative test equation, which is given by

$$\pi_t - \pi_t^* = \tilde{\gamma} \left[ \tau_0 y_t + \tau_1 y_{t-1} + \sum_{k=2}^{K=12} \rho^k y_{t-k} + \rho^{K+1} (\pi_{t-(K+1)} - \pi_{t-(K+1)}^*) \right] + \epsilon_t^\pi \tag{17}$$

where $\tau_0 = \bar{\tau}_0 / \tilde{\gamma}$ and $\tau_1 = \bar{\tau}_1 / \tilde{\gamma}$. Since (17) nests the forward-looking Phillips curve (16), as a special case, we can assess the relevance of firm's backward-looking behavior in determining the inflation gap. If firms are purely forward-looking as shown in (16), the inflation gap is mostly determined by the current and lagged output gaps, $y_t$ and $y_{t-1}$, respectively. Otherwise, the inflation gap will be also influenced by the past output gap $y_{t-k}$ for integer $k \geq 2$.

Table 4.A reports estimation results of (17) with no restrictions on $\tau_0$ and $\tau_1$. Interestingly, the results reveal that the coefficients on $y_t$ and $y_{t-1}$ are not statistically different from zero. There is a negligible contribution of $y_t$ and $y_{t-1}$ to the inflation gap while the past output gap $y_{t-k}$ for integer $k \geq 2$ has a statistically important impact on the inflation gap. The parameter $\rho$ is significantly different from zero at the five percent level, except for the case associated with the UC NFB inflation gap among twelve cases. The estimates of $\rho$ are between 0.753 and 0.872 for the CS inflation gaps and between 0.585 and 0.858 for the UC inflation gaps. The degree of inflation gap persistence is estimated to be a bit higher with the CS inflation gaps than the UC inflation gaps. This result is conjectured from the fact that inflation reverts more often to the UC trend inflation than the CS trend inflation, as shown in Figure 2.

For robustness check, we reestimate (17) under some restrictions on $\tau_0$ and $\tau_1$ and report estimation results in Table 4.B. The restrictions that are imposed on (17) are as follows: 1) $\tau_0 = 1$ and $\tau_1 = \rho$ , 2) $\tau_0 = 0$ and $\tau_1 = 0$. The first case assumes that the most recent output gaps such as $y_t$ and $y_{t-1}$ have large impacts on the inflation gap while the second case assumes the current and past output gaps ($y_t$ and $y_{t-1}$) do not contribute to inflation gap at all. These two contrasting cases deliver interesting results. The second case always delivers a higher likelihood than the first across various inflation and output gap measures even though the estimate of $\rho$ is not sensitive to the restrictions. We also find that the slope of the Phillips curve that measures the magnitude of the impact of the output gap on the inflation gap is estimated to be higher with the restrictions of $\tau_0 = 0$ and $\tau_1 = 0$ compared to the other restrictions.

It might be natural to ask how much the past output gap, $y_{t-k}$, has predictive content for the inflation gap. To address this issue, we consider a simple regression equation in which

Table 4: Test of Presence of Backward-looking Behavior

Panel A. No Restrictions on $\tau_0$ and $\tau_1$

| gap | $y_t$ | $\rho$ | $\tilde{\gamma}$ | $\tau_0$ | $\tau_1$ | $\theta_{MA}$ | $\delta_{AR}$ | $\sigma^\pi$ | likelihood |
|---|---|---|---|---|---|---|---|---|---|
| CS GDP | CBO | 0.829 (0.043) | 0.140 (0.045) | -0.935 (0.775)$^a$ | 0.683 (0.724)$^a$ | 0.486 (0.080) | 0.943 (0.034) | 1.066 (0.101) | -234.2 |
| inflation | HP | 0.872 (0.024) | 0.198 (0.034) | -0.050 (0.495)$^a$ | 0.516 (0.472)$^a$ | 0.575 (0.086) | 0.924 (0.041) | 1.001 (0.094) | -229.0 |
| gap | Q | 0.805 (0.055) | 0.145 (0.055) | -1.027 (0.788)$^a$ | 0.669 (0.732)$^a$ | 0.481 (0.084) | 0.931 (0.042) | 1.057 (0.098) | -233.4 |
| CS NFB | CBO | 0.793 (0.080) | 0.212 (0.086) | -1.076 (0.912)$^a$ | 1.089 (0.986)$^a$ | 0.500 (0.085) | 0.942 (0.033) | 1.969 (0.150) | -283.6 |
| inflation | HP | 0.857 (0.033) | 0.278 (0.051) | -0.318 (0.510)$^a$ | 0.938 (0.678)$^a$ | 0.549 (0.102) | 0.907 (0.051) | 1.826 (0.152) | -277.3 |
| gap | Q | 0.753 (0.118) | 0.227 (0.132)* | -1.097 (1.033)$^a$ | 1.002 (1.077)$^a$ | 0.488 (0.090) | 0.930 (0.041) | 1.972 (0.146) | -283.6 |
| UC | CBO | 0.743 (0.115) | 0.114 (0.079)$^a$ | -1.311 (1.139)$^a$ | 0.508 (0.947)$^a$ | 0.330 (0.150) | 0.677 (0.133) | 0.814 (0.071) | -231.7 |
| GDP inflation | HP | 0.858 (0.025) | 0.154 (0.021) | -0.089 (0.510)$^a$ | 0.434 (0.511)$^a$ | 0.009 (0.281)$^a$ | 0.261 (0.280)$^a$ | 0.738 (0.066) | -223.0 |
| gap | Q | 0.640 (0.192) | 0.219 (0.271)$^a$ | -0.779 (1.088)$^a$ | 0.215 (0.685)$^a$ | 0.228 (0.165)$^a$ | 0.569 (0.153) | 0.800 (0.069) | -230.2 |
| UC | CBO | 0.667 (0.273) | 0.227 (0.315)$^a$ | -0.665 (1.235)$^a$ | 0.626 (1.310)$^a$ | 0.384 (0.118) | 0.829 (0.075) | 1.383 (0.094) | -278.6 |
| NFB inflation | HP | 0.828 (0.046) | 0.237 (0.054) | -0.083 (0.542)$^a$ | 0.782 (0.733)$^a$ | 0.397 (0.149) | 0.759 (0.100) | 1.259 (0.087) | -270.2 |
| gap | Q | 0.585 (0.357)$^a$ | 0.292 (0.567)$^a$ | -0.611 (1.418)$^a$ | 0.457 (1.163)$^a$ | 0.356 (0.123) | 0.799 (0.086) | 1.393 (0.091) | -279.1 |

Panel B. Restrictions on $\tau_0$ and $\tau_1$

| restriction on | $\tau_0$ and $\tau_1$ | $\tau_0 = 1$ and $\tau_1 = \rho$ | | | $\tau_0 = \tau_1 = 0$ | | |
|---|---|---|---|---|---|---|---|
| gap | $y_t$ | $\rho$ | $\tilde{\gamma}$ | likelihood | $\rho$ | $\tilde{\gamma}$ | likelihood |
| CS GDP | CBO | 0.854 (0.031) | 0.075 (0.035) | -238.2 | 0.832 (0.038) | 0.148 (0.042) | -235.0 |
| inflation | HP | 0.892 (0.019) | 0.149 (0.029) | -232.7 | 0.859 (0.027) | 0.216 (0.037) | -229.7 |
| gap | Q | 0.848 (0.033) | 0.069 (0.035) | -237.5 | 0.811 (0.048) | 0.146 (0.050) | -234.5 |
| CS NFB | CBO | 0.826 (0.059) | 0.117 (0.050) | -287.9 | 0.779 (0.084) | 0.260 (0.096) | -285.0 |
| inflation | HP | 0.877 (0.027) | 0.217 (0.040) | -281.1 | 0.830 (0.038) | 0.341 (0.060) | -279.0 |
| gap | Q | 0.811 (0.074) | 0.103 (0.052) | -287.7 | 0.741 (0.115) | 0.273 (0.143)* | -285.2 |
| UC | CBO | 0.815 (0.049) | 0.036 (0.020)* | -235.5 | 0.783 (0.079) | 0.081 (0.042) | -218.1 |
| GDP inflation | HP | 0.880 (0.020) | 0.111 (0.016) | -227.2 | 0.846 (0.029) | 0.170 (0.025) | -208.5 |
| gap | Q | 0.805 (0.057) | 0.032 (0.017)* | -234.6 | 0.746 (0.128) | 0.086 (0.066) | -217.6 |
| UC | CBO | 0.780 (0.117) | 0.075 (0.053)$^a$ | -281.2 | 0.658 (0.233) | 0.273 (0.314)$^a$ | -279.5 |
| NFB inflation | HP | 0.853 (0.035) | 0.176 (0.036) | -272.8 | 0.785 (0.052) | 0.317 (0.073) | -271.8 |
| gap | Q | 0.751 (0.178) | 0.056 (0.050)$^a$ | -281.8 | 0.595 (0.342)* | 0.281 (0.510)$^a$ | -280.3 |

Notes: The sample period is 1960Q1-2003Q4. Asymptotic Newey-West standard errors are reported in parentheses. The superscript $*$ denotes significance at the ten percent level. The superscript letter $a$ denotes no signicance at the ten percent level. The case without $*$ or $a$ denotes statistical significance at the one or five percent level.

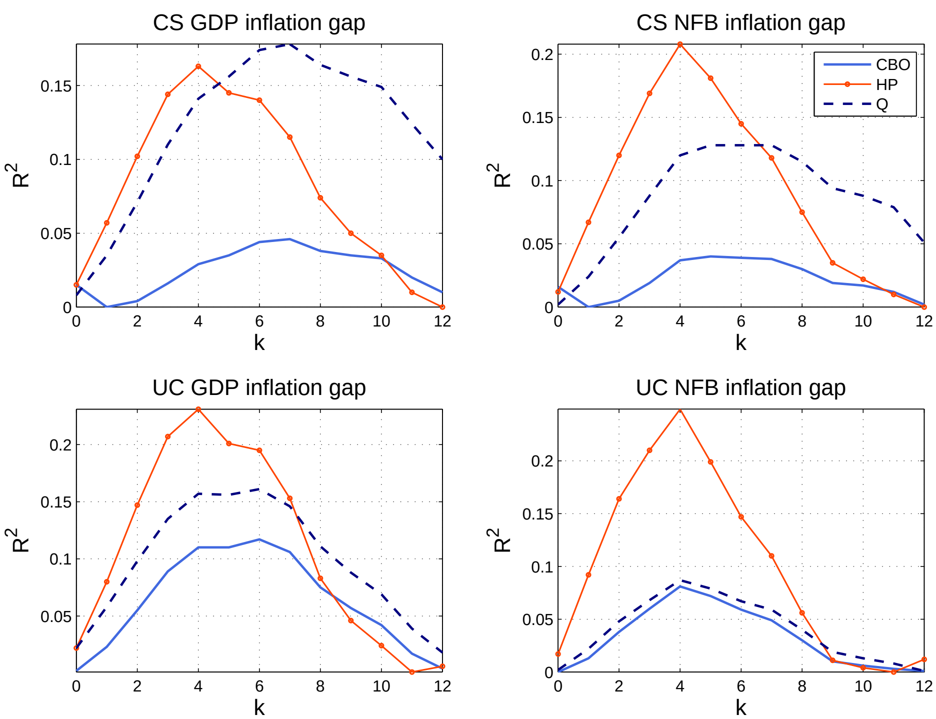
the inflation gap $\pi_t - \pi_t^*$ depends solely on $y_{t-k}$ as follows

$$\pi_t - \pi_t^* = constant + \delta_k y_{t-k} + u_t^\pi \tag{18}$$

where $u_t^\pi$ is a residual. The integer value of $k$ is from zero to twelve.

Figure 6 graphs $R^2$ at each value of $k$. The left (right) figures are for the GDP (NFB) inflation gaps. The three output gap measures are considered for $y_{t-k}$ in the regression. Not surprisingly, the goodness-of-fit is very low when the value of $k$ is close to zero. A rise in the value of $k$ results in an increase in $R^2$ for the first five values of $k$. The highest predictive power of $y_{t-k}$ for the inflation gap, $\pi_t - \pi_t^*$, is found at $k$ = 4 - 7. This evidence is not coherent with the predictions of the NKPC and reveals why the restrictions of $\tau_0 = \tau_1 = 0$ in (17) do not deteriorate the ability of the model to explain the dynamics of the inflation gap. In the next section, we will provide structural analysis showing why the current and most recent past output gaps have a very limited contribution to the inflation gap based on a conventional DSGE model framework.

Figure 6: Explanatory Power of Current and Past Output Gap for Inflation Gap




Notes: This figure shows $R^2$ at each integer value of $k$. The regression equation is $\pi_t - \pi_t^* = c + \delta_k y_{t-k} + u_t^\pi$. The $u_t^\pi$ denotes a residual. The left (right) panel reports $R^2$ based on the GDP (NFB) inflation gaps. Three output gap measures (CBO, HP, and Q) are considered for $y_{t-k}$ in the regression.

# 5 Backward-looking Firms, Trend Inflation, and Joint Dynamic Behavior between Output and Inflation Gaps

The survey papers by Schorfheide (2008) and Mavroeidis et al. (2014) show that estimation results of the hybrid NKPC are not conclusive about the importance of firm's backward-looking behavior. Hence, instead of re-estimating the hybrid NKPC, we attempt to evaluate the performance of the (hybrid) NKPC with respect to its ability to explain the standard deviations of key macroeconomic variables and the observed dynamic correlation between the output gap $y_t$ and the inflation gap $\pi_{t+k} - \pi^*_{t+k}$ where $k$ is an integer. These two criteria are as important as model fitting in evaluating the performance of the (hybrid) NKPC. Needless to say, a successful Phillips curve model is expected to help match the standard deviations of key macroeconomic variables. It is also quite natural to evaluate Phillips curve specifications with respect to the ability to explain the observed dynamic correlation between the output gap and inflation gap since it is designed to explain the dynamic relationship of the variables.

Galí and Gertler (1999) point out that the current output gap is positively linked to future inflation while it is negatively associated with past inflation. The hybrid NKPC is able to account for the reverse dynamic correlation between the output gap and inflation (Smets and Wouters, 2007). On the other hand, as shown in Chauvet, Hur, and Kim (2015), the NKPC cannot replicate the reverse dynamic correlation. It produces an abnormally high contemporaneous correlation between the two variables even though its estimate is very close to zero. Their results indicate the backward-looking component is an essential element in accounting for the observed low contemporaneous correlation and the reverse dynamic correlation between the output gap and inflation.

In this section, we further explore this issue. In particular, we are interested in whether the reverse dynamic correlation between the output gap and inflation is still observed even after the low frequency component is eliminated from inflation. Our work here responds to the previous studies that inflation persistence is mainly driven by the Fed's long run inflation target rather than firm's backward-looking price-setting behavior.

## 5.1 Firm's Backward-looking Behavior and Moments of Macroeconomic Variables

The DSGE model presented in section II is extended to incorporate interest rate smoothing in the monetary policy rule and a lagged output gap term in the IS curve. The model is given by

$$\pi_t - \pi^*_t = \theta E_t(\pi_{t+1} - \pi^*_{t+1}) + (1-\theta)(\pi_{t-1} - \pi^*_{t-1}) + \gamma y_t + \epsilon^{\pi}_t \tag{19}$$

$$y_t = \alpha E_t y_{t+1} + (1-\alpha) y_{t-1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1}) + \epsilon^{y}_t \tag{20}$$

$$i_t = \rho_i i_{t-1} + (1-\rho_i)[\pi_t^* + a_\pi(\pi_t - \pi_t^*) + a_y y_t] + \epsilon_t^i \quad (21)$$

$$\pi_t^* = \delta_{AR}^{\pi*}\pi_{t-1}^* + \epsilon_t^{\pi*} \quad (22)$$

We assume that the cost shock $\epsilon_t^\pi$ follows an ARMA(1,1) process as in Smets and Wouters (2007). The inflation target shock $\epsilon_t^{\pi*}$ is assumed to be a white noise process and the demand and monetary shocks ($\epsilon_t^y$ and $\epsilon_t^i$ ) follow an AR(1) process. The autoregressive parameters of the cost, demand, monetary, and inflation target shocks are $\delta_{AR}^\pi$ , $\delta_{AR}^y$, $\delta_{AR}^i$ , and $\delta_{AR}^{\pi*}$, respectively. The moving average parameter for the cost shock is $\theta_{MA}$. We fix the parameter $\delta_{AR}^{\pi*}$ at 0.999 to make inflation target follow a near random walk process. We also choose the value to make the rational expectations solution of the DSGE model stationary as in Ireland (2015).[8] The innovations to the target, cost, demand, and monetary shocks are independent and identically distributed with mean zero and standard deviation $\sigma_{\pi*}$, $\sigma_\pi$, $\sigma_y$, and $\sigma_i$, respectively.

We estimate the DSGE model parameters, holding the parameter $\theta$ fixed at a value which belongs to the set [0.1, 0.2,...,0.9, 1.0]. We have this strategy since the estimate of $\theta$ is not identified well as discussed before. To estimate the DSGE model, we use NFB inflation, HP output gap, the effective Federal Funds rate, estimated trend inflation by Cogely and Sbordone (2008).[9]

Panel A of Table 5 reports the prior distributions of the model parameters and their posterior modes. The bottom panel summarizes the observed second moments of the key variables (second column) and the DSGE model-implied moments (third to twelfth column). When $\theta \leq 0.50$, the autoregressive parameter $\delta_{AR}^\pi$ for the cost shock is estimated to be low around 0.04-0.29. The weak serial correlation implies that the cost shock has a limited contribution to inflation gap persistence. Regarding the IS curve, the estimate of $\alpha$ is around 0.85-0.86 and the autoregressive parameter $\delta_{AR}^y$ for the demand shock is estimated to be around 0.91-0.93. The dominant role of expected future output gap in the IS curve makes the current output gap, $y_t$, less persistent while the highly correlated demand shock operates in the opposite way. When $\theta > 0.50$, drastic changes occur in the parameters discussed above. The parameter $\delta_{AR}^\pi$ is estimated to be much higher around 0.95. The result shows that the role of the backward-looking component in matching inflation gap persistence is substituted by the highly serially autocorrelated cost-push shock. The estimates of $\alpha$ and $\delta_{AR}^y$ are around 0.05 and 0.38, respectively, which are much lower compared to the cases where $\theta \leq 0.50$. These results show that the parameter estimates regarding the IS curve and the cost shock are highly sensitive to the value of $\theta$ while the parameters regarding monetary policy are robustly estimated regardless of Phillips curve specifications.

[8] If we set the parameter $\delta_{AR}^{\pi*}$ to be one, the model cannot match the standard deviation of inflation at all since inflation becomes a nonstationary process.

[9] Our results are not sensitive to trend inflation estimates. Once we use the trend inflation estimate from the UC model, instead of CS trend inflation, we obtained very similar results.

The highest likelihood is observed when the parameter $\theta$ is set to one. Likelihood density favors the purely forward-looking NKPC as in Ireland (2007). However model fit comes at a cost of rising volatility of the key macroeconomic variables such as interest rates and the output gap as shown in panel B. When the forward-looking component of the hybrid NKPC plays a dominant role in explaining the inflation gap (including the case of $\theta = 1$), the DSGE model fails to explain the observed volatility of the output gap. In particular, when the purely forward-looking NKPC is employed for inflation gap dynamics, the model-implied output gap volatility is four times greater than an observed standard deviation of 1.56. In turn, the heightened output gap volatility makes interest rates more volatile than the observed one. Our simulation exercise shows that the hybrid NKPC in which forward- and backward-looking components are equally important delivers the best performance in matching the standard deviations of the output gap, and interest rates among the cases considered in this article.

In contrast to interest rates and the output gap, the second moments of inflation, trend inflation, and the inflation gap are less sensitive to the value of $\theta$. The NKPC corresponding to the case of $\theta = 1.0$ performs well in accounting for the second moments of inflation and inflation gap. The standard deviation of inflation is best matched when $\theta = 0.2$ and the standard deviation of inflation gap is best matched when $\theta = 0.5$.

## 5.2 Joint Dynamic Behavior of Output Gap and Inflation Gap

This section attempts to evaluate the ability of the (hybrid) NKPC to account for the observed dynamic correlation between $y_t$ and $\pi_{t+k} - \pi^*_{t+k}$ for integer $k \in [-15,\ 15]$. The dynamic correlation is computed using the four inflation gap measures and the three output gap measures (CBO, HP, and Q) and it is reported in panel A of Figure 7. The panel shows that current output gap is positively linked to future inflation gap, implying that output gap affects inflation gap with a substantial delay. It also exhibits that past inflation gap is negatively associated with current output gap. This negative correlation is likely to be associated with cost shocks since demand-side shocks cause the variables to move in the same direction. This reverse dynamic correlation is robustly observed across a variety of inflation and output gap measures even after the trend inflation component is eliminated from inflation.[10]

Our interest is in investigating whether the DSGE model allowing for trend inflation is able to replicate the observed dynamic correlation without relying on firm's backward-looking behavior. Therefore we presents the model-implied dynamic correlation between the output gap $y_t$ and the inflation gap $\pi_{t+k} - \pi^*_{t+k}$ in panel B of Figure 7. We picked the following five

[10] Although we do not report here, the reverse dynamic correlation is also observed when we adopt the inflation gap measure by Kim et al. (2014). The inflation gap measure was kindly provided by Manopimoken. We really appreciate for providing their trend inflation estimate.

Table 5: DSGE Model Estimates and Moments

A. DSGE Model Estimates with Restriction on the Parameter $\theta$

| | | restriction on parameter $\theta$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| prior dist. | parameter | $\theta = 0.10$ | $\theta = 0.20$ | $\theta = 0.30$ | $\theta = 0.40$ | $\theta = 0.50$ | $\theta = 0.60$ | $\theta = 0.70$ | $\theta = 0.80$ | $\theta = 0.90$ | $\theta = 1.00$ |
| Inv. Gamma (0.10, 0.05) | $\gamma$ | 0.223 | 0.156 | 0.135 | 0.094 | 0.067 | 0.035 | 0.038 | 0.039 | 0.041 | 0.042 |
| Beta (0.50, 0.15) | $\alpha$ | 0.862 | 0.862 | 0.849 | 0.850 | 0.857 | 0.045 | 0.046 | 0.048 | 0.049 | 0.050 |
| Normal (2.00, 0.50) | $\sigma$ | 6.247 | 6.311 | 6.274 | 6.322 | 6.342 | 6.033 | 5.964 | 5.890 | 5.825 | 5.771 |
| Beta (0.80, 0.05) | $\rho_i$ | 0.791 | 0.798 | 0.793 | 0.793 | 0.791 | 0.810 | 0.806 | 0.803 | 0.799 | 0.796 |
| Normal (1.75, 0.50) | $\alpha_\pi$ | 1.865 | 1.835 | 1.873 | 1.854 | 1.811 | 2.000 | 1.965 | 1.940 | 1.921 | 1.908 |
| Normal (0.25, 0.05) | $\alpha_y$ | 0.333 | 0.340 | 0.337 | 0.343 | 0.354 | 0.328 | 0.329 | 0.329 | 0.327 | 0.325 |
| Beta (0.50, 0.25) | $\theta_{MA}$ | 0.570 | 0.982 | 0.467 | 0.396 | 0.285 | 0.950 | 0.890 | 0.841 | 0.793 | 0.739 |
| Beta (0.50, 0.25) | $\delta^\pi_{AR}$ | 0.062 | 0.292 | 0.051 | 0.042 | 0.041 | 0.972 | 0.946 | 0.934 | 0.927 | 0.923 |
| Beta (0.50, 0.25) | $\delta^y_{AR}$ | 0.926 | 0.920 | 0.920 | 0.915 | 0.909 | 0.386 | 0.381 | 0.381 | 0.381 | 0.381 |
| Beta (0.50, 0.25) | $\delta^i_{AR}$ | 0.162 | 0.128 | 0.156 | 0.151 | 0.143 | 0.444 | 0.478 | 0.506 | 0.529 | 0.548 |
| Inv. Gamma (0.05, $\infty$) | $\sigma_{\pi*}$ | 0.051 | 0.051 | 0.051 | 0.051 | 0.051 | 0.051 | 0.051 | 0.051 | 0.051 | 0.051 |
| Inv. Gamma (0.50, $\infty$) | $\sigma_\pi$ | 1.302 | 1.131 | 1.177 | 1.114 | 1.054 | 0.914 | 0.838 | 0.768 | 0.701 | 0.634 |
| Inv. Gamma (0.50, $\infty$) | $\sigma_y$ | 0.193 | 0.192 | 0.196 | 0.197 | 0.196 | 0.847 | 0.858 | 0.865 | 0.871 | 0.876 |
| Inv. Gamma (0.50, $\infty$) | $\sigma_i$ | 1.138 | 1.132 | 1.139 | 1.136 | 1.137 | 1.083 | 1.086 | 1.090 | 1.095 | 1.099 |
| | Likelihood | -605.1 | -609.4 | -603.4 | -604.0 | -610.1 | -619.4 | -611.4 | -604.2 | -598.0 | -592.9 |

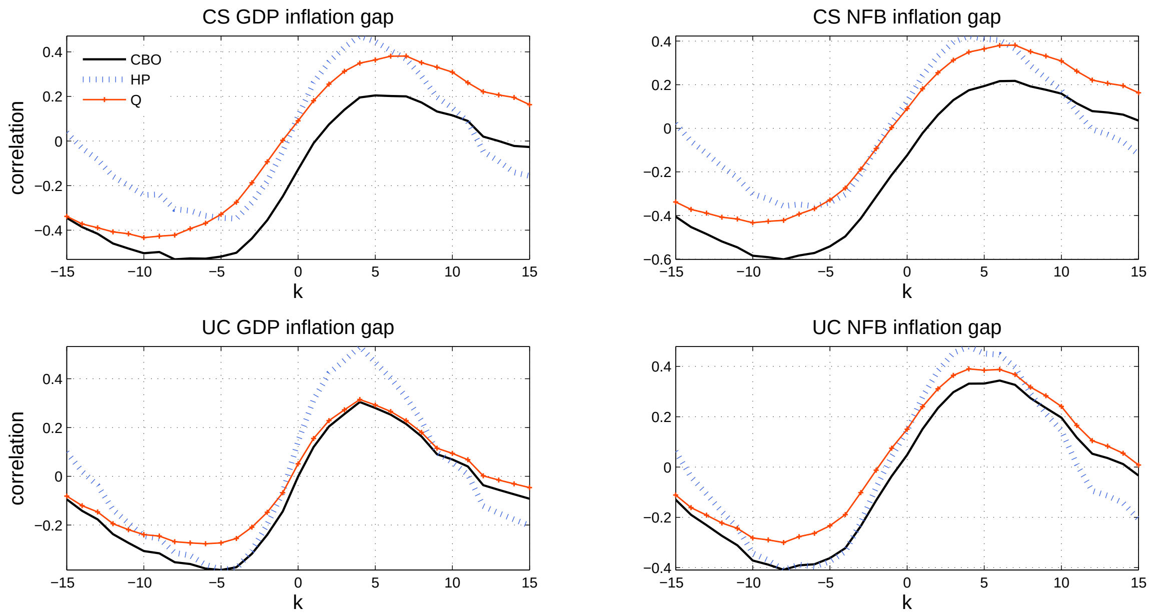
B. Moments: Data and Simulated Variables

| | | restriction on parameter $\theta$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | data | $\theta = 0.10$ | $\theta = 0.20$ | $\theta = 0.30$ | $\theta = 0.40$ | $\theta = 0.50$ | $\theta = 0.60$ | $\theta = 0.70$ | $\theta = 0.80$ | $\theta = 0.90$ | $\theta = 1.00$ |
| $\sigma(\pi_t)$ | 2.888 | 2.940 | 2.877 | 2.789 | 2.647 | 2.460 | 3.994 | 3.386 | 3.068 | 2.935 | 2.834 |
| $\sigma(\pi^*_t)$ | 0.971 | 0.706 | 0.695 | 0.706 | 0.706 | 0.706 | 0.695 | 0.706 | 0.695 | 0.706 | 0.706 |
| $\sigma(\pi_t - \pi^*_t)$ | 2.221 | 2.898 | 2.768 | 2.740 | 2.591 | 2.395 | 3.950 | 3.316 | 3.013 | 2.853 | 2.748 |
| $\sigma(y_t)$ | 1.557 | 2.159 | 2.140 | 2.135 | 2.103 | 2.070 | 5.436 | 5.475 | 5.699 | 5.879 | 6.086 |
| $\sigma(i_t)$ | 3.285 | 4.307 | 4.154 | 4.052 | 3.804 | 3.467 | 5.623 | 4.679 | 4.543 | 4.274 | 4.221 |

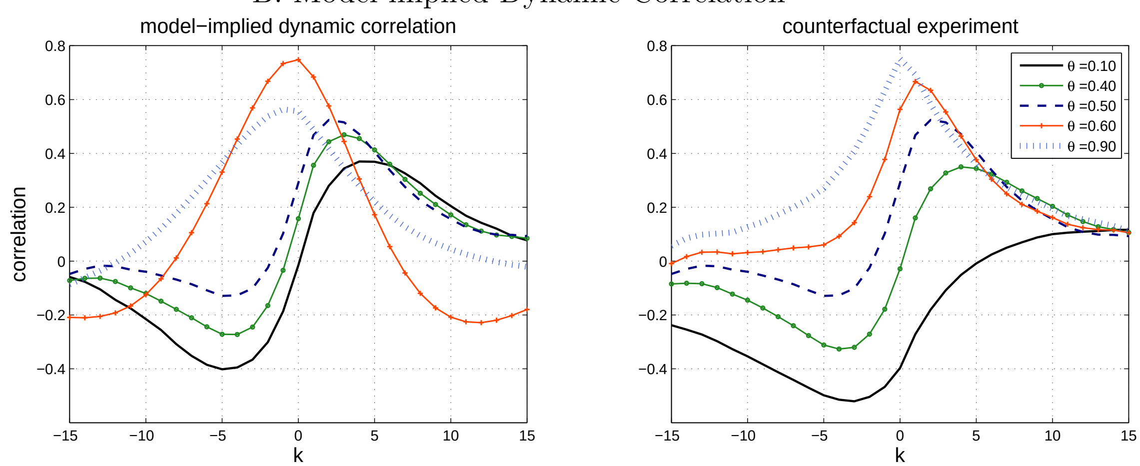
Notes: Panel A reports the prior distributions of the model parameters and its posterior mode. The DSGE model is estimated with the parameter $\theta$ fixed at a value which belongs to the set [0.1, 0.2,...,0.9, 1.0]. Panel B reports the second moments of the key model variables.

Figure 7: Dynamic Correlation between $y_t$ and $\pi_{t+k} - \pi^*_{t+k}$

A. Observed Dynamic Correlation from Data




B. Model-implied Dynamic Correlation




Notes: Panel A reports the observed dynamic correlation between the output gap $y_t$ and the inflation gap $\pi_{t+k} - \pi^*_{t+k}$ where $k \in [-15,\ 15]$. We consider four inflation gap measures and three output gap measures as denoted by CBO, HP, and Q. The left hand side of panel B displays the model-implied dynamic correlations based on the estimates of the DSGE model with the parameter $\theta$ fixed at 0.10, 0.40, 0.50, 0.60, and 0.90. The right hand side of panel B displays the results from a counterfactual exercise based on the estimated DSGE model parameters with $\theta$ fixed at 0.50. For this exercise, we simulate the DSGE model with the model parameters fixed at its estimate but replacing the estimate of $\theta$ with a counterfactual value such as $\theta = 0.10$, 0.40, 0.60, and 0.90.

values: $\theta = 0.10$, 0.40, 0.50, 0.60, and 0.90 for the hybrid NKPC. The values are chosen to keep approximately equal distance between the lines in panel B.

The left figure of panel B depicts the joint dynamic correlation which is generated based on the estimates of the DSGE model parameters presented in panel A of Table 5. The figure shows that the hybrid NKPC specifications with $\theta$ greater than or equal to 0.6 are not able to replicate the observed reverse dynamic correlation. In addition, the model-implied contemporaneous correlation is abnormally higher than the data. The same results are obtained for $\theta = 0.70$, 0.80, and 1.00 although we do not report here. The hybrid NKPC specifications with $\theta$ less than 0.5 perform remarkably well in matching the observed dynamic correlation between output gap and inflation gap. The backward-looking component causes the inflation gap to respond slowly in the face of demand and monetary shocks while the forward-looking component does not. The delayed response of the inflation gap to changes in the output gap associated with demand and monetary shocks causes the current output gap to be positively correlated to future inflation gap. The slow response of inflation gap leads to a low contemporaneous correlation between the output gap and inflation gap. Panel A shows that the peaks in the correlation between $y_t$ and $\pi_{t+k} - \pi^*_{t+k}$ are observed when $k$ is between four and seven. This pattern is consistent with the hybrid NKPC emphasizing the role of firm's backward-looking behavior.

The right figure of panel B displays the results from counterfactual exercises based on the DSGE model parameters estimated with $\theta$ fixed at 0.50. For these exercises, we simulate the estimated DSGE model with the parameter $\theta$ fixed at a counterfactual value such as $\theta = 0.10$, 0.40, 0.60, and 0.90. This counterfactual experiment is conducted to identify the pure contribution of firm's forward-looking behavior to the dynamic correlation between the output gap and the inflation gap. The panel shows that a drastic decline in the parameter $\theta$ from 0.90 to 0.10 significantly lowers the contemporaneous correlation between the two variables. Interestingly, the sign flips from a positive to a negative value when backward-looking behavior plays a dominant role in accounting for inflation gap dynamics. As the parameter $\theta$ declines, cost shocks become more influential in determining the contemporaneous correlation. When the parameter $\theta$ is close to zero, the inflation gap responds to changes in the output gap with a substantial delay in the face of demand shocks. On the other hand, the impact of cost (or supply) shocks on both the inflation and output gaps is immediate and substantial. Thus, the hybrid NKPC with a low value of $\theta$ generates a counterfactual negative contemporaneous correlation in response to both demand and supply shocks. By contrast, a high value of $\theta$ leads to a positive and unrealistically high contemporaneous correlation since demand shocks dominate supply shocks in determining the correlation.[11]

Taken together, our simulation exercises clarify that both backward- and forward-looking behaviors play an important role in determining the correlation between the output gap and the inflation gap. Our findings makes it evident that incorporating trend inflation into the DSGE model does not help produce the observed dynamic relationship between the output gap and the inflation gap if firm's forward-looking behavior plays a dominant role in

[11]When $\theta = 1$, the impact of changes in the output gap associated with demand shocks on the inflation gap is immediate and substantial becasue the inflation gap is determined by not only current output gap but also the stream of expected future output gaps as shown in (6). Thus, if demand shocks dominate supply shocks in business cycles, the DSGE model yields a high contemporaneous correlation between the output gap and the inflation gap

determining inflation.

## 6 Structural Change in Inflation Persistence

We now turn to sensitivity analysis. This section studies whether inflation gap persistence is sensitive to subsamples. The first sample starts from 1960 and ends in 1980 and the second sample period starts from 1981 and ends in 2003. Kim et al. (2014) document evidence on a structural change in the U.S. inflation dynamics in 1981. For a robustness check, we also consider a subsample that begins from 1984. We re-estimate (13) and report subsample estimates in Table 6. Panel A is for the CS GDP inflation gap and panel B is for the CS NFB inflation gap. The three output gap measures (CBO, HP, and Q) are used for subsample analysis.

The estimate of $\rho$ indicating the degree of inflation gap persistence is found between 0.782 and 0.911 in the first sample period and between 0.783 and 0.922 in the subsequent period. The estimates are statistically different from zero at the one percent level and the degree of the persistence is not sensitive to the output and inflation gap measures. These results indicate the speed that the impact of the output gap on the inflation gap dies out does not change across the subsamples. The slope of the Phillips curve is estimated to be lower in the post-1980 period as often reported in the literature (Coibion and Gorodnichenko, 2015). With a concern about the sensitivity of our results to the alternative inflation gap measures, we re-estimate (13) using the UC GDP/NFB inflation gap and report estimation results in Table 7. It shows that our results do not change with the alternative inflation gap measures.

Inflation persistence is often measured by the autoregressive coefficient under the assumption that inflation gap dynamics can be described by a first-order autoregressive (AR(1)) process. Cogley and Sbordone (2008) point out that there is no substantial changes in inflation persistence measured by the autoregressive coefficient before the Volcker disinflation and afterward while inflation gap becomes less persistent in the post-1980 period. In this regard, we investigate why the autoregressive coefficient is often estimated to be low in the post-1980 period. In particular, we are interested in whether a slight change in the AR(1) model specification for inflation gap dynamics leads to different results. A model specification considered here is given by

$$\pi_t - \pi_t^* = \rho_{AR}(\pi_{t-1} - \pi_{t-1}^*) + \epsilon_t^\pi - \theta_{MA}\epsilon_{t-1}^\pi \tag{23}$$

We consider the moving average process for the residual. The model is estimated by the method of maximum likelihood. For comparison, we also estimate (23) with a restriction of $\theta_{MA} = 0$. Our goal is to investigate whether the estimate of the autoregressive parameter $\rho_{AR}$ is affected by appending the moving average term to the AR(1) model.

Table 6: Subsample Analysis: CS Inflation Gap

A. GDP Inflation Gap

| period | $y_t$ | $\rho$ | $\tilde{\gamma}$ | $\theta_{MA}$ | $\delta_{AR}$ | $\sigma^{\pi}$ | likelihood |
|---|---|---|---|---|---|---|---|
| 1960-1980 | CBO | 0.810 (0.117) | 0.048 (0.098)$^a$ | 0.337 (0.166) | 0.924 (0.112) | 1.912 (0.323) | -120.9 |
| | HP | 0.911 (0.027) | 0.184 (0.050) | 0.604 (0.163) | 0.896 (0.112) | 1.741 (0.302) | -117.3 |
| | Q | 0.832 (0.081) | 0.070 (0.084)$^a$ | 0.355 (0.170) | 0.905 (0.112) | 1.884 (0.316) | -120.3 |
| 1981-2003 | CBO | 0.905 (0.020) | 0.083 (0.018) | 0.561 (0.089) | 0.986 (0.015) | 0.543 (0.098) | -103.6 |
| | HP | 0.922 (0.016) | 0.128 (0.023) | 0.591 (0.109) | 0.928 (0.044) | 0.519 (0.092) | -100.8 |
| | Q | 0.898 (0.022) | 0.072 (0.017) | 0.572 (0.095) | 0.991 (0.018) | 0.535 (0.099) | -103.1 |
| 1984-2003 | CBO | 0.873 (0.034) | 0.028 (0.015)$^*$ | 0.621 (0.160) | 0.862 (0.104) | 0.426 (0.081) | -79.6 |
| | HP | 0.872 (0.032) | 0.057 (0.027) | 0.590 (0.174) | 0.823 (0.128) | 0.424 (0.078) | -79.3 |
| | Q | 0.848 (0.043) | 0.012 (0.013)$^a$ | 0.590 (0.163) | 0.859 (0.106) | 0.432 (0.077) | -80.1 |

B. NFB Inflation Gap

| period | $y_t$ | $\rho$ | $\tilde{\gamma}$ | $\theta_{MA}$ | $\delta_{AR}$ | $\sigma^{\pi}$ | likelihood |
|---|---|---|---|---|---|---|---|
| 1960-1980 | CBO | 0.782 (0.218) | 0.091 (0.134)$^a$ | 0.365 (0.170) | 0.905 (0.108) | 3.448 (0.513) | -141.1 |
| | HP | 0.879 (0.050) | 0.257 (0.080) | 0.471 (0.209) | 0.855 (0.129) | 3.096 (0.509) | -137.2 |
| | Q | 0.801 (0.162) | 0.120 (0.117)$^a$ | 0.374 (0.174) | 0.884 (0.104) | 3.364 (0.513) | -140.2 |
| 1981-2003 | CBO | 0.873 (0.037) | 0.126 (0.029) | 0.513 (0.091) | 0.989 (0.014) | 1.003 (0.183) | -132.0 |
| | HP | 0.887 (0.030) | 0.187 (0.033) | 0.556 (0.104) | 0.960 (0.027) | 0.966 (0.179) | -129.6 |
| | Q | 0.863 (0.041) | 0.112 (0.028) | 0.517 (0.096) | 0.992 (0.016) | 0.994 (0.174) | -131.8 |
| 1984-2003 | CBO | 0.842 (0.059) | 0.041 (0.030)$^a$ | 0.669 (0.183) | 0.892 (0.115) | 0.683 (0.123) | -98.4 |
| | HP | 0.845 (0.061) | 0.093 (0.042) | 0.624 (0.222) | 0.825 (0.161) | 0.662 (0.123) | -97.1 |
| | Q | 0.783 (0.141) | 0.008 (0.024)$^a$ | 0.579 (0.199) | 0.846 (0.131) | 0.702 (0.121) | -99.5 |

Notes: Asymptotic Newey-West standard errors are reported in parentheses. The superscript $*$ denotes significance at the ten percent level. The superscript letter $a$ denotes no signicance at the ten percent level. The case without $*$ or $a$ denotes statistical significance at the one or five percent level.

Estimation results based on the CS/UC inflation gap measures are reported in Table 8. Imposing $\theta_{MA} = 0$ on (23) yields lower estimates of $\rho_{AR}$ than when the parameter $\theta_{MA}$ is not restricted to be zero. For the whole sample, the estimate of $\rho_{AR}$ ranges from 0.511 to 0.788 under the restriction. Eliminating the restriction leads to higher estimates which are between 0.743 and 0.908.

The most drastic drop in inflation gap persistence is observed for the 1984-2003 period. When the restriction of $\theta_{MA} = 0$ is imposed on (23), the estimates of inflation gap persistence are between 0.260 and 0.393. Interestingly, we find that embedding the moving average component to the AR(1) model raises the estimate $\rho_{AR}$ from 0.354 to 0.844 for the CS GDP inflation gap, from 0.343 to 0.821 for the CS NFB inflation gap, from 0.393 to 0.690 for the UC NFB inflation gap. These results suggest that there might be no structural changes in inflation gap persistence. For the UC GDP inflation gap, the estimates of $\rho_{AR}$ and $\theta_{MA}$ turn out to be statistically insignificant and the signs of the parameters are the opposite to conventional estimates. These results may emerge from a misspecification problem so that we also consider an alternative specification to investigate the degree of inflation gap persistence.

The misspecification problem of (23) may involve either a missing variable such as the

Table 7: Subsample Analysis: UC Inflation Gap

A. GDP Inflation Gap

| period | $y_t$ | $\rho$ | $\tilde{\gamma}$ | $\theta_{MA}$ | $\delta_{AR}$ | $\sigma^{\pi}$ | likelihood |
|---|---|---|---|---|---|---|---|
| 1960Q3-1980Q4 | CBO | 0.735 (0.293) | 0.019 (0.049) | 0.288 (0.196)$^a$ | 0.733 (0.150) | 1.317 (0.180) | -130.9 |
| | HP | 0.881 (0.034) | 0.135 (0.024) | -0.187 (0.491)$^a$ | 0.078 (0.485) | 1.151 (0.166) | -125.1 |
| | Q | 0.774 (0.156) | 0.036 (0.041) | 0.262 (0.205)$^a$ | 0.678 (0.163) | 1.290 (0.172) | -130.0 |
| 1981Q1-2003Q4 | CBO | 0.903 (0.020) | 0.052 (0.013) | 0.720 (0.106) | 0.954 (0.041) | 0.428 (0.077) | -91.9 |
| | HP | 0.916 (0.012) | 0.088 (0.013) | 0.648 (0.302) | 0.764 (0.253) | 0.389 (0.068) | -87.1 |
| | Q | 0.890 (0.024) | 0.044 (0.013) | 0.707 (0.109) | 0.974 (0.035) | 0.435 (0.080) | -92.9 |
| 1984Q1-2003Q4 | CBO | 0.874 (0.028) | 0.030 (0.009) | 0.676 (0.237) | 0.821 (0.185) | 0.357 (0.069) | -72.4 |
| | HP | 0.886 (0.026) | 0.060 (0.019) | 0.647 (0.226) | 0.803 (0.180) | 0.363 (0.068) | -73.1 |
| | Q | 0.844 (0.043) | 0.015 (0.009)* | 0.642 (0.195) | 0.837 (0.142) | 0.369 (0.067) | -73.7 |

B. NFB Inflation Gap

| period | $y_t$ | $\rho$ | $\tilde{\gamma}$ | $\theta_{MA}$ | $\delta_{AR}$ | $\sigma^{\pi}$ | likelihood |
|---|---|---|---|---|---|---|---|
| 1960Q3-1980Q4 | CBO | 0.726 (0.486)$^a$ | 0.066 (0.178)$^a$ | 0.262 (0.200)$^a$ | 0.812 (0.172) | 1.870 (0.208) | -145.8 |
| | HP | 0.833 (0.078) | 0.206 (0.100) | 0.322 (0.213)$^a$ | 0.753 (0.160) | 1.695 (0.186) | -141.5 |
| | Q | 0.726 (0.472) | 0.071 (0.151)$^a$ | 0.258 (0.191)$^a$ | 0.793 (0.145) | 1.844 (0.212) | -145.2 |
| 1981Q1-2003Q4 | CBO | 0.882 (0.039) | 0.095 (0.024) | 0.554 (0.110) | 0.971 (0.027) | 0.936 (0.143) | -132.0 |
| | HP | 0.899 (0.024) | 0.143 (0.025) | 0.467 (0.233) | 0.794 (0.126) | 0.868 (0.126) | -129.6 |
| | Q | 0.874 (0.041) | 0.079 (0.025) | 0.537 (0.123) | 0.981 (0.027) | 0.950 (0.143) | -131.8 |
| 1984Q1-2003Q4 | CBO | 0.848 (0.057) | 0.039 (0.024)$^a$ | 0.432 (0.264)$^a$ | 0.752 (0.200) | 0.713 (0.107) | -100.1 |
| | HP | 0.859 (0.055) | 0.100 (0.040) | 0.420 (0.300)$^a$ | 0.710 (0.239) | 0.687 (0.104) | -98.6 |
| | Q | 0.834 (0.072) | 0.008 (0.015)$^a$ | 0.409 (0.242)* | 0.756 (0.176) | 0.742 (0.114) | -101.7 |

Notes: Asymptotic Newey-West standard errors are reported in parentheses. The superscript $*$ denotes significance at the ten percent level. The superscript letter $a$ denotes no signicance at the ten percent level. The case without $*$ or $a$ denotes statistical significance at the one or five percent level.

Table 8: Persistence of Inflation Gap and MA Component

| | | GDP inflation gap | | | NFB inflation gap | | |
|---|---|---|---|---|---|---|---|
| | restriction on (23) | $\theta_{MA}=0$ | no restriction on $\theta_{MA}$ | | $\theta_{MA}=0$ | no restriction on $\theta_{MA}$ | |
| | parameter | $\rho_{AR}$ | $\rho_{AR}$ | $\theta_{MA}$ | $\rho_{AR}$ | $\rho_{AR}$ | $\theta_{MA}$ |
| CS | 1960-2003 | 0.788 (0.058) | 0.908 (0.056) | 0.352 (0.159) | 0.731 (0.066) | 0.892 (0.073) | 0.359 (0.234)$^a$ |
| | 1960-1980 | 0.829 (0.098) | 0.943 (0.079) | 0.161 (0.126)$^a$ | 0.744 (0.075) | 0.893 (0.106) | 0.335 (0.264)$^a$ |
| | 1981-2003 | 0.659 (0.038) | 0.789 (0.025) | 0.544 (0.092) | 0.664 (0.047) | 0.809 (0.054) | 0.615 (0.160) |
| | 1984-2003 | 0.354 (0.114) | 0.844 (0.110) | 0.589 (0.159) | 0.343 (0.122) | 0.821 (0.126) | 0.547 (0.158) |
| UC | 1960-2003 | 0.511 (0.095) | 0.743 (0.119) | 0.324 (0.225)$^a$ | 0.535 (0.099) | 0.775 (0.109) | 0.332 (0.264)$^a$ |
| | 1960-1980 | 0.552 (0.149) | 0.736 (0.154) | 0.277 (0.261)$^a$ | 0.544 (0.126) | 0.767 (0.134) | 0.307 (0.323)$^a$ |
| | 1981-2003 | 0.433 (0.056) | 0.654 (0.054) | 0.409 (0.076) | 0.494 (0.068) | 0.773 (0.109) | 0.515 (0.211) |
| | 1984-2003 | 0.260 (0.115) | -0.320 (0.262)$^a$ | -0.545 (0.217)$^a$ | 0.393 (0.115) | 0.690 (0.109) | 0.354 (0.167) |

Notes: Asymptotic Newey-West standard errors are reported in parentheses. The superscript $*$ denotes significance at the ten percent level. The superscript letter $a$ denotes no signicance at the ten percent level. The case without $*$ or $a$ denotes statistical significance at the one or five percent level.

output gap or the autoregressive component in residual. In this regard, we consider an alternative model specification given by

$$\pi_t - \pi_t^* = \rho_{AR}(\pi_{t-1} - \pi_{t-1}^*) + \gamma y_t + \epsilon_t^\pi \tag{24}$$

where the residual term $\epsilon_t^\pi$ is assumed to follow an ARMA(1,1) process, $\epsilon_t^\pi = \delta_{AR}\epsilon_{t-1}^\pi + \nu_t - \theta_{MA}\nu_{t-1}$. We use the HP output gap for $y_t$.[12] The model is estimated by the maximum likelihood method. The autoregressive component for the residual is taken into account as a competing source of inflation gap persistence to the lagged inflation gap term.

We estimate (24) with/without the restriction of $\delta_{AR} = 0$. Table 9 summarizes estimation results. The parameter $\rho_{AR}$ is estimated to be even higher compared to the previous specification (23). In particular, the estimate of $\rho_{AR}$ ranges from 0.753 to 0.950 over the 1984-2003 period, indicating there are no changes in inflation gap persistence. Our findings also indicate that inflation gap persistence is not sensitive to the restriction on $\delta_{AR} = 0$.

Table 9: Persistence of Inflation Gap and ARMA Component

| | restriction on (24) | $\delta_{AR} = 0$ | | no restriction on $\delta_{AR}$ | |
|---|---|---|---|---|---|
| inflation gap | period | $\rho_{AR}$ | $\theta_{MA}$ | $\rho_{AR}$ | $\theta_{MA}$ |
| CS GDP | 1960-2003 | 0.971 (0.015) | 0.625 (0.058) | 0.982 (0.015) | 0.793 (0.079) |
| | 1960-1980 | 1.008 (0.017) | 0.762 (0.073) | 1.014 (0.017) | 0.857 (0.097) |
| | 1981-2003 | 0.837 (0.026) | 0.605 (0.098) | 0.835 (0.026) | 0.571 (0.157) |
| | 1984-2003 | 0.911 (0.065) | 0.688 (0.127) | 0.904 (0.070) | 0.656 (0.186) |
| CS NFB | 1960-2003 | 0.950 (0.022) | 0.558 (0.061) | 0.937 (0.022) | 0.333 (0.108) |
| | 1960-1980 | 0.965 (0.037) | 0.550 (0.117) | 0.951 (0.036) | 0.217 (0.251)$^a$ |
| | 1981-2003 | 0.859 (0.033) | 0.717 (0.070) | 0.850 (0.039) | 0.586 (0.137) |
| | 1984-2003 | 0.825 (0.099) | 0.688 (0.127) | 0.753 (0.110) | 0.324 (0.247)$^a$ |
| UC GDP | 1960-2003 | 0.936 (0.028) | 0.831 (0.056) | 0.955 (0.031) | 1.074 (0.054) |
| | 1960-1980 | 0.973 (0.039) | 0.867 (0.080) | 0.983 (0.041) | 1.078 (0.085) |
| | 1981-2003 | 0.660 (0.068) | 0.463 (0.116) | 0.657 (0.067) | 0.398 (0.217)$^*$ |
| | 1984-2003 | 0.950 (0.034) | 0.600 (0.152) | 0.944 (0.038) | 0.976 (0.074) |
| UC NFB | 1960-2003 | 0.871 (0.041) | 0.451 (0.090) | 0.860 (0.038) | 0.275 (0.140)$^*$ |
| | 1960-1980 | 0.908 (0.056) | 0.437 (0.137) | 0.902 (0.052) | 0.155 (0.241)$^a$ |
| | 1981-2003 | 0.821 (0.068) | 0.531 (0.112) | 0.800 (0.072) | 0.328 (0.208)$^a$ |
| | 1984-2003 | 0.829 (0.097) | 0.457 (0.158) | 0.773 (0.104) | 0.058 (0.306)$^a$ |

Notes: Asymptotic Newey-West standard errors are reported in parentheses. The superscript $*$ denotes significance at the ten percent level. The superscript letter $a$ denotes no signicance at the ten percent level. The case without $*$ or $a$ denotes statistical significance at the one or five percent level.

# 4. Conclusion

In recent years, the view that inflation persistence results from the Federal Reserve's long-run inflation target has gained attention in the NKPC literature since the hybrid NKPC is not treated as structural, and the backward-looking mechanism is not likely to be invariant to

[12] We obtained very similar estimates of the parameter $\rho_{AR}$ for the alternative output gap measures we consider in this article even though we do not report here.

changes in monetary policy. In this perspective, the hypothesis that inflation persistence can be ascribed to drifts in the Federal Reserve's long-run inflation target was raised in the literature. However, our findings show that the backward-looking component is still of great importance even after the low frequency component is eliminated from inflation. In particular, we show that our empirical results are inconsistent with the purely forward-looking NKPC in that inflation gap is mostly driven by past output gaps. In addition, the second moments of the key macroeconomic variables and observed dynamic correlation between output gap and inflation gap can be best explained only when the backward-looking component matters a great deal. Taken together, our findings point out the presence of intrinsic inflation persistence.

## References


[1] Adam, Klaus and Mario Padula. "Inflation Dynamics And Subjective Expectations In The United States." Economic Inquiry, 2011, 49 (1), pp. 13–25.

[2] Barnes, Michelle L., Fabià Gumbau-Brisa, Denny Lie, and Giovanni P. Olivei. "Estimation of Forward-Looking Relationships in Closed Form: An Application to the New Keynesian Phillips Curve." Federal Reserve Bank of Boston Working Paper, No. 11-3, 2011.

[3] Calvo, Guillermo A. "Staggered Prices in a Utility-Maximizing Framework." Journal of Monetary Economics, 1983, 12 (3), pp. 383–98.

[4] Chauvet, Marcelle, Joonyoung Hur, and Insu Kim. "Can Model-Consistent Inflation Expectations Explain the Behavior of Inflation Dynamics?" Working Paper, 2015.

[5] Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy." Journal of Political Economy, 2005, 113 (1), pp. 1–45.

[6] Coibion, Olivier and Yuriy Gorodnichenko. "Is the Phillips Curve Alive and Well after All? Inflation Expectations and the Missing Disinflation," American Economic Journal: Macroeconomics, 2015, 7 (1), pp 197–232.

[7] Cogley, Timothy, Giorgio Primiceri, and Thomas J. Sargent. "Inflation-gap Persistence in the U.S." American Economic Journal: Macroeconomics, 2010, 2 (1), pp. 43–69

[8] Cogley, Timothy and Thomas J. Sargent. "Drifts and Volatilities: Monetary Policies and Outcomes in the Post WWII U.S." Review of Economic Dynamics, 2005, 8 (2), pp. 262–302.

[9] Cogley, Timothy and Argia Sbordone. "Trend Inflation, Indexation and Inflation Persistence in the New Keynesian Phillips Curve." American Economic Review, 2008, 98 (5), pp. 2101–2126.

[10] Fuhrer, Jeffery C. "Intrinsic and Inherited Inflation Persistence" International Journal of Central Banking, September 2006, pp. 49–86.

[11] Fuhrer, Jeffery C. "Real Expectations: Replacing Rational Expectations with Survey Expectations in Dynamic Macro Models." Federal Reserve Bank of Boston Working Paper, No. 12-19, 2012.

[12] Fuhrer, Jeffery C. "Inflation Persistence." Federal Reserve Bank of Boston Working Papers, No. 09-14, 2009.

[13] Fuhrer, Jeffery C. and George R. Moore. "Inflation Persistence." Quarterly Journal of Economics, 1995, 110 (1), pp. 127–59.

[14] Galí, Jordi and Gertler, Mark. "Inflation Dynamics: A Structural Econometric Analysis." Journal of Monetary Economics, 1999, 44 (2), pp. 195–222.

[15] Ireland, Peter N. "Changes in the Federal Reserve's Inflation Target: Causes and Consquences." Journal of Money, Credit, and Banking, 2007, 39 (8), pp. 1851-1882.

[16] Ireland, Peter N. "Monetary Policy, Bond Risk Premia, and the Economy." Working Paper, 2015.

[17] Kim, Chang-Jin, Pym Manopimoke, and Charles Nelson. "Trend Inflation and the Nature of Structural Breaks in the New Keynesian Phillips Curve." Journal of Money, Credit, and Banking, 2014, 46 (2-3), pp. 253–266.

[18] Lubik, Thomas and Frank Schorfheide. "Testing for Indeterminacy: An Application to U.S. Monetary Policy." American Economic Review, 2004, 94 (March), pp. 190–217.

[19] Mavroeidis, Sophocles, Mikkel Plagborg-Møller, and James H. Stock. "Empirical Evidence on Inflation Expectations in the New Keynesian Phillips Curve." Journal of Economic Literature, 2014, 52 (1), pp. 124-188.

[20] Nunes, Ricardo. "Inflation Dynamics: The Role of Expectations." Journal of Money, Credit, and Banking, 2010, 42 (6), pp. 1161–1172.

[21] Rabanal, Pau and Juan F. Rubio-Ramírez. "Comparing New Keynesian Models of the Business Cycle: A Bayesian Approach." Journal of Monetary Economics, 2005, 52 (6), pp. 1151–1166.

[22] Roberts, John M. "New Keynesian Economics and the Phillips Curve." Journal of Money, Credit, and Banking, 1995, 27, pp. 975–984.

[23] Roberts, John M. "Is inflation sticky?" Journal of Monetary Economics, 1997, 39, pp. 173–196.

[24] Rudd, Jeremy and Karl Whelan. "New Tests of the New-Keynesian Phillips Curve." Journal of Monetary Economics, 2005a, 52 (6), pp. 1167–1181.

[25] Rudd, Jeremy and Karl Whelan. "Does Labor's Share Drive Inflation?" Journal of Money, Credit, and Banking, 2005b, 37 (2), pp. 297–312.

[26] Rudd, Jeremy and Karl Whelan. "Can Rational Expectations Sticky-Price Models Explain Inflation Dynamics?" American Economic Review, 2006, 96, pp. 303–320.

[27] Rudd, Jeremy and Karl Whelan. "Modeling Inflation Dynamics: A Critical Review of Recent Research." Journal of Money, Credit, and Banking, 2007, 39 (1), pp. 155–170.

[28] Rudebusch, Gleen D. "Assessing Nominal Income Rules for Monetary Policy with Model and Data Uncertainty." Economic Journal, 2002, 112 (479), pp. 402–432.

[29] Sahuc, Jean-Guillaume. "Partial Indexation, Trend Inflation, and the Hybrid Phillips Curve." Economic Letters, 2006, 90, pp. 42–50.

[30] Sbordone, Argia M. "Do Expected Future Marginal Costs Drive Inflation Dynamics?" Journal of Monetary Economics, 2005, 52 (6), pp. 1183–1197.

[31] Sbordone, Argia M. "Inflation Persistence: Alternative Interpretations and Policy Implications." Journal of Monetary Economics, 2007, 54, pp. 1311–1339.

[32] Schorfheide, Frank. "DSGE Model-Based Estimation of the New Keynesian Phillips Curve." Economic Quarterly, 2008, 94 (4), pp. 397– 433

[33] Steinsson, Jón. "Optimal Monetary Policy in an Economy with Inflation Presistence.", Journal of Monetary Economics, 2003, 50 (7), pp. 1425–1456.

[34] Thomas, Lloyd B., Jr. "Survey Measures of Expected U.S. Inflation." Journal of Economic Perspectives, 1999, 13 (4), pp. 125–144.

[35] Woodford, Michael. Interest and Prices. Princeton University Press, 2003.

[36] Woodford, Michael. "How Important Is Money in the Conduct of Monetary Policy?" Journal of Money, Credit, and Banking, 2008, 48 (8), pp. 1561–1598.