The Role of Reputation in Firm's Voluntary Disclosure*

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Abstract

In this paper, we analyze the effect of reputation on the CEO's disclosure strategies by comparing the private disclosure channel with the public disclosure channel. It is shown that when the reputation effect is large, the public disclosure channel can reveal more information; however, when this effect is small, the private disclosure channel can reveal more information. Two players may prefer different disclosure channels. First, when the reputation effect is large, the CEO prefers the public disclosure channel. When this effect is small, he prefers the private disclosure channel if only the conflict parameter between the CEO and the investor decreases, and at the same time the reputation effect approaches the reference. Otherwise, the CEO prefers the public disclosure channel. On the other hand, the investor always prefers a disclosure channel in which more information is disclosed. Specifically, when the reputation effect is large, the investor prefers the public disclosure channel, and when this effect is small, she prefers the private disclosure channel. Thus, the channel that the CEO prefers does not always coincide with the one that the investor prefers.

Keywords: voluntary disclosure, communication game, private disclosure, public disclosure, partial disclosure equilibrium

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1 Introduction

A public firm is required to disclose certain information. Moreover, it sometimes discloses information voluntarily through public channels and/or through private channels. Public channels include xxx, yyy, zzz, and private channels include xxx, yyy, zzz. Using the information delivered through one of the channels, investors decides whether they invest in the public firm and how much to invest. For example, a CEO of a firm may possess an information on the required investment for a new project. Given that a CEO usually wants to attract more investment than necessary, the CEO will seek to obfuscate the investors to invest more by distorting the information, e.g., even if the information transmitted is verifiable with a hard evidence, the CEO may simply hide the information. However, there is a negative side of transmitting misleading information: hiding information or providing unclear information may be interpreted that the CEO is a less able one, so that her reputation could be marred.

To avoid undermining her reputation, the CEO may prefer the private channel to disclose inside information. For example, Bourjade and Jullien (2011) consider the environment in which a CEO may send verifiable information or *not* send any information at all, and the CEO is concerned about his reputation too (not sending any information might be interpreted that the CEO is a less able in receiving information). By transmitting information through a private channel, the CEO does not have to worry about the reputation. Thus, when the information channel is private, the CEO chooses not to send any information more often, compared to the case when the information channel is public. Given that the information sent is verifiable, the transmission of the information is always welfare improving; thus, the public channel is always better in terms of welfare. The CEO also prefers the public channel because [**Check this and FILL IN LATER**].

[**Motivation For Nonverifiability**] We complement Bourjade and Jullien (2011) by considering unverifiable signaling by a CEO.

[**Reference, e.g., how many percentage of disclosure by the firms in Down Jones is about future profit, past profit, qualitative aspect, and so on?**]

[**CHECK this later w.r.t. English and everything**] Beyer at. al.(2010) emphasize the fact that management forecasts on future profit provide, on average, approximately 55% of accounting-based information. In addition, earnings pre-announcements contribute an additional 11% of the accounting-based information. Thus, for the average firm in their study, approximately 66% of the accounting-based information is provided by voluntary disclosures. Furthermore, many voluntary disclosures are (i) qualitative in nature and (ii) represent management's communication of there unverifiable, unaudited and forward-looking beliefs about value-relevant events. So our model applies to financial disclosures that permit some degree of managerial intension and discretion, such as Management Discussion and Analysis, and to voluntary disclosures, such as disclosure of the optimal investment level and earning forecasts, etc, that need not be precise.

[OTHER REAL WORLD EXAMPLES THAT FIT IN THE MODEL: need to check later]

Notice that although the main focus of this article relates to firm's voluntary disclosure, the insights will be relevant for other contexts as well. For example, suppose that an uninformed decision maker is considering reforming a tax policy and an biased expert who has some information about the optimal tax rate and cares about her reputation on expertise. The goal of decision maker is maximizing a social welfare. So in order to obtain the information, the decision maker visits a informed expert for advise. In this situation, what are the incentives of the expert to hide a some information to decision maker although he can deliver the false advise? In particular, the decision maker believes that there is a positive probability that the advisor has poor expertise. In this case, reputation concerns will give the expert an incentive not to conceal information. Furthermore, if reputation concerns are sufficiently large, the expert never conceal information.

[OUR MODEL] We consider the following environment: a CEO considers starting a new project, the CEO receives a noisy signal about the required investment for the project, the CEO transmits an unverifiable signal to an investor, then the investor decides how much to invest, or not to invest at all. The details are following. With a positive probability, the CEO receives a noisy signal about the required investment; however, the CEO receives an empty signal with the other positive probability. There are two types of CEOs: a high type and a low type. The probability of receiving no signal is lower for the high type CEO, and more for the low type. We assume that even the CEO does not know which type she is, as was assumed in XXX, YYY, ZZZ.[1] The CEO may choose not to disclose

[1]More realistic assumption would be that the CEO knows her type to a certain extent. However, our simplifying assumption makes it possible to focus on the signaling of the required investment, separated from the additional layer of

the signal to investors if it is of lower investment: since the CEO's preference is biased toward a larger investment, the CEO may try to obfuscate the investors to invest more than necessary by hiding the signal. Additionally, the signal that the CEO transmit is unverifiable. Thus, even if the CEO decides to transmits the signal, he may want to exaggerate the signal she has received, again since the CEO's preference is biased toward a larger investment. After listening to what the CEO has to say, investors will decide how much to invest, or not to invest at all.

When the CEO received the signal of lower investment, the CEO faces the tradeoff between his reputation and lower investment by the investors: (i) the market for CEO may interpret the CEO's no report that the CEO has not received any signal, i.e., the higher probability that the CEO is a low type, (ii) by not reporting at all, the investor will invest the average amount of investment (after updating the distribution of the investment given the no report). Thus if private channel for information transmission is available (e.g., xxx, yyy, zzz), the CEO will ex-ante prefer the private channel as the market will not learn whether the CEO has reported a signal or not. This was one of the findings of J&B (200?).

However, given that

However, the CEO is not expected to reveal all the information.

[**FINDINGS**]

In this paper, we analyze the situations in which the CEO who wants to enhance his reputation in the CEO labor market confronts an issue involving which disclosure channel he should disclose information through. Which disclosure channel is preferred by the CEO who considers his reputation very seriously. On the contrary, which disclosure channel is preferred by the CEO who has a greater incentive to make investors choose investments that he wants rather than reputation. And which disclosure channel is preferred by the investor who does not have any information. In this paper, we would like to answer these questions through the communication game model.

It is often seen that firms voluntarily disclose useful information for investors' decision making on the investments choice in the financial market. Investors strongly depend on the information that firms disclose when they make a decision because their opportunities to gain information about the optimal investments are limited compared to firms. However, firms do not always completely disclose their

signaling – that is, type signaling in the sense of Spence (XXXX) and YYY.

private information truthfully. Therefore, serious attention is required for how to use the information that firms disclose. The reason that firms do not always completely disclose their private information truthfully is as follows. Information disclosure of the firms is carried out mainly by the CEOs who have the incentives to make investors invest more money than the optimal level of investments. Thus, CEOs intend to reveal information about the optimal investments after much magnification and distortion. Additionally, in real world, investors are uncertain whether the CEO actually possesses the information about the optimal investments. Due to this uncertainty, when the CEO acquires information about low optimal investments, he has an incentive not to disclose information in order to make investors choose investments that he wants. But, the CEO's behavior of hiding information hurts his reputation in the CEO labor market. Here, the CEO with reduced reputation receives lower payoff since the CEO labor market pays higher payoff to the CEO with a better reputation. Therefore, when the CEO observes low optimal investments, he decides whether he should reveal information by comparing the benefits that can be obtained by inducing higher investments from investors with the losses that can be incurred by the decline of reputation due to hiding information.

In this paper, we analyze firms' voluntary disclosure in a situation where a conflict between the CEO and the investor and the reputation effect for the CEO's competency exist. Specifically, we analyze the amount of information and players' ex ante expected payoffs by comparing the public disclosure channel with the private disclosure channel. Here, the reputation effect occurs because the CEO's behavior of disclosing information provides information about his competency for the CEO labor market under the uncertain situation about his competency.

The public disclosure channel includes IR, earnings forecast disclosure, reporting via internet website and media, publishing business proposals, etc. And the information disclosed through this channel can be observed by all participants in the market. Moreover, the public disclosure channel is a one-sided information disclosure channel in which the CEO discloses company-related information to the public through official documents.

Meanwhile, the private disclosure channel includes closed conference call, CEO-investor face-to-face meeting, briefing sessions targeting specific persons such as securities analysts or fund managers, etc. And unlike the public disclosure channel, it can disclose information by selectively separating participants in the market. For example, if information receivers are investors and the CEO labor market, the private disclosure channel can separate investors from the CEO labor market to communicate only with the investors.

In this paper, asymmetry of information exists only between the CEO and investors. That is, all investors are identical, and hence, there is no asymmetry of information among them. Therefore, in this paper, the information receiver is consisted of representative investor and the CEO labor market.

In each model, as part of the perfect Bayesian equilibrium, we focus on various information disclosure strategies of the CEO. First, we show that the full disclosure equilibrium does not exist in both the private disclosure model and the public disclosure model. Subsequently, we show that in the two models there is an equilibrium in which low optimal investments are not disclosed, and the remaining intervals are disclosed after being divided. Meanwhile, we show that the partial disclosure equilibrium disclosing the optimal investment by dividing it into intervals without hiding any interval exists uniquely in the public disclosure model. we also show that in the two models the non-disclosure equilibrium always exists in which the CEO does not disclose any information. And based on the equilibria, we analyze the amount of information and disclosure channels preferred by players. It is shown that when the reputation effect is large, the public disclosure channel can reveal more information, but when this effect is small, the private disclosure channel can reveal more information. Two players may prefer different disclosure channels. First, when the reputation effect is large, the CEO prefers the public disclosure channel. When this effect is small, he prefers the private disclosure channel only if the conflict parameter between the CEO and the investor decreases, and at the same time the reputation effect approaches the reference. Otherwise, the CEO prefers the public disclosure channel. On the other hand, the investor always prefers a disclosure channel in which more information is disclosed. Specifically, when the reputation effect is large, the investor prefers the public disclosure channel, and when this effect is small, she prefers the private disclosure channel. Thus, the channel that the CEO prefers does not always coincide with the one that the investor prefers.

[WHY DOES A CEO NOT KNOW WHO HE IS: to eliminate the signaling of a CEO's quality in order to see the pure interaction between the reputation effect and the bias in the two setups (private signaling and public signaling)]

[WHY DO WE NOT CONSIDER THE CHOICE OF A CEO ON WHICH CHANNEL TO CHOOSE (unlike F&R): (i) first of all, it is trivial (with certain parameter value, we can characterize which channel a CEO prefers), (ii) second, if we assume the CEO does not know who he is, we can model a non-trivial case; yet, the signaling of the CEO's quality will make it unclear to see the interaction between the reputation effect and the bias]

This paper is organized as follows. First, in section II, previous research related to this paper is discussed. In section III, the common basic compositions of the private and public disclosure models are examined. In section , the private disclosure model is analyzed while in section , the public disclosure model is analyzed. Based on these, in section , the private disclosure model is compared with the public disclosure model. Finally, section contains conclusion and discussion.

2 Literature Review

Ferreira and Rezende (2007) considers two periods model in which a CEO receives a noisy signal on the optimal investment amount in the first period, and the precise information on the optimal investment in the second period. [FILL IN LATER!] [DIFFERENCE WITH OURS]

Theoretical studies about firms' voluntary disclosure can be divided into two categories depending on whether studies assume that information that the CEO discloses is verifiable or not. First, unlike this paper, studies assuming that information is verifiable come from those of Grossman(1981) and Milgrom(1981). Grossman(1981) and Milgrom(1981) showed that full disclosure equilibrium always exists because the CEO cannot disclose false information.

Since these two studies, there has been a lot of extension in many ways. First, different from Grossman(1981) and Milgrom(1981), Dye(1985), Jung and Kwon(1988), and Shin(1994, 1998, 2003) dealt with the situation where uncertainty about whether the CEO possesses private information or not exists. According to their studies, using the uncertainty the CEO hides bad information because when the investor does not receive any information from the CEO, she is not sure if the CEO does not have any information or he has information but is hiding it. That is, they showed that full disclosure equilibrium does not exist.

On the other hand, Wolinsky(2003) dealt with the situation where uncertainty about CEO's incentive or bias exists. He suggested that the amount of information disclosed by the CEO decreases when investors are uncertain whether the CEO has a positive or negative bias. However, unlike this paper, the aforementioned studies do not consider the reputation effect.

Similar to this study , Bourjade and Jullien(2011) analyze the effect that is produced when the biased CEO considers reputation about his competency in the CEO labor market under the assumption that information is verifiable. According to their result, the amount of information disclosed by the CEO increases due to the presence of the reputation effect . Different from Bourjade and Jullien(2011), we show that although the reputation effect exists, the amount of information disclosed by the CEO does not always increase.

Subsequently, similar to this study, studies assuming that information is non-verifiable result from Crawford and Sobel(1982). They obtained the result that when a conflict between the CEO and the investor exists, the CEO discloses his private information by dividing it into intervals. That is, they showed that full disclosure equilibrium does not exist while partial disclosure equilibrium do exist.

In studies of this category, extension has been made in various ways like other studies assuming that information is verifiable. Austen-Smith(1994) showed that when it is uncertain whether the CEO possesses private information or not, using this uncertainty the CEO hides bad information. And from this fact, the CEO can disclose good news credibly. Also through this result, Austen-Smith(1994) showed that useful information can be disclosed for a wider range of parameter values than when the investor is sure/certain the CEO possesses private information. However, different from this paper, Austen-Smith(1994) neither considers the reputation effect nor focuses on the players' welfare.

Unlike Austen-Smith(1994), Morgan and Stocken(2003) deal with the situation where the investor is uncertain about the CEO's bias. The result is that the CEO could disclose bad information credibly while he could not disclose good information credibly. However, Morgan and Stocken(2003) also do not consider the reputation effect.

Like this paper, studies considering the reputation effect include Ottaviani and Sorensen(2001, 2006a, 2006b), Sobel(1985), Benabou and Laroque(1992), Morris(2001) etc. Ottaviani and Sorensen(2001, 2006a, 2006b) consider the reputation about the CEO's competency. On the other hand, Sobel(1985), Benabou and Laroque(1992), and Morris(2001) deal with the reputation effect for the CEO's incentive.

While we assumed that the information disclosure channel is given exogenously, Ferreira and Rezende(2007) deal with the issue of CEO's choice for a disclosure channel. As a result, they obtained that the CEO chooses the disclosure channel maximizing the investors' payoffs. That is, the disclosure channel that the CEO prefers always coincides with the one that the investor prefers. However, Ferreira and Rezende(2007)'s result is not always satisfied in this paper which does not consider the issue of choice for a disclosure channel. In other words, in this paper, there is a case that the disclosure channel preferred by the CEO does not coincide with the one that the investor prefers.

3 Model

We consider two environments: one having a public channel only, and the other having a private channel only. In both the environments, there are two players: a CEO and a representative investor.

A CEO is planning a project that requires a certain level of investment. The investor does not know the optimal level of investment $t \in [0,1]$, while the CEO can observe optimal investment level t with probability p, and null information (denoted by ϕ) with probability $1-p$. Hence, a CEO's information set is $[0,1] \cup \{\phi\}$. We assume that the investment level t is uniformly distributed on $[0,1]$, which is a common knowledge for all the players. There are two types of CEO, high type and low type, $\{h, \ell\}$. The high type CEO's probability is $p = \bar{p}$, and the low type CEO's is $p = \underline{p}$. We assume that the CEO's type is unknown to all the players, including the CEO himself. The probability that a CEO is of high type is π, and low type $1-\pi$. The ex-ante probability that the CEO observes the optimal investment level is $p_e = \pi\bar{p} + (1-\pi)\underline{p}$.

After learning the information $t \in [0,1]$ or ϕ, the CEO sends a message $m \in \Omega \cup \{\phi\}$ where Ω is the set of intervals, $\Omega = \{S \subset [0,1] : S \text{ is an interval}\}$.[2] For example, the CEO may send a closed interval contained in $[0,1]$ as a message or no message ($m = \phi$). However we assume that the CEO who has received signal ϕ has to report $m = \phi$.[3] In other words, we focus only on the strategic information. See also Austen-Smith (1994) and Bourjade and Jullien (2011).) transmission of the CEO who has observed non-null information.

After listening to the CEO's message m (*either* through a private channel or a public channel),

[2] **[Crawford and Sobel (1982) demonstrated that assuming the set of intervals as the message space was without loss of generality in terms of payoffs.]**

[3] In many situation, the CEO needs to show information source together with the information being disclosed. That is, the fact that the CEO obtained the useful information is verifiable. Austen-Smith (1994) and Bourjade and Jullien (2011) assume similarly.

the investor choose investment $a \in [0,1]$ that is not necessarily same to t or m.[4] Finally, the labor market for CEOs updates the belief on the type of the CEO using available information I. If the information transmission is through a public channel, the available information is $I \in \{\phi, \phi^C\}$, where ϕ means no information disclosed, and ϕ^C means a certain information disclosed. If the information disclosure is through a private channel, the labor market receives no additional information; thus, I is an empty set, $I = \{\}$. Note again that we consider two separate environments of private and public information transmission, and we do not consider a CEO's strategic choice of the channel. This is to isolate the effect of strategic information transmission on efficiency (?) from the signaling effect of choosing an information transmission channel. (Analysis of the environment with the two effects, the strategic information transmission under the presence of reputation concern *and* the strategic choice of information transmission channel, is a promising future research topic.)

The following summarizes the timing of the game.

1. Nature chooses CEO's type, h or ℓ.

2. The CEO learns optimal investment level $t \in [0,1]$ with probability p, or nothing with probability $1-p$ ($p = \bar{p}$ for h and $p = \underline{p}$ for ℓ).

3. The CEO sends message $m \in \Omega \cup \{\phi\}$ to the investor, privately or publicly.

4. After observing m, the investor chooses investment $a \in [0,1]$. The labor market for CEOs updates the belief on the CEO's type with available information.

The payoffs of the CEO and the investor are:

$$\text{Investor's payoff}: \quad u(a,t) = -|a-t|, \tag{1}$$

$$\text{CEO's payoff}: \quad v(a,t,b,R) = -|a-(t+b)| + R \cdot \Pr(p = \bar{p}|I) \tag{2}$$

where $b > 0$ represents a conflict of interests between the CEO and the investor (i.e., a CEO wants to exaggerate the required investment), and $R > 0$ is the (marginal) future benefit when the CEO is the high type rather than a low type; thus, $R \cdot \Pr(p = \bar{p}|I)$ is the expected future benefit of the CEO where $\Pr(p = \bar{p}|I)$ is the updated probability that the CEO is of high type, given information I

[4] The investor will never have incentive to invest more than 1.

Given optimal investment level t, the investor wants to choose investment a to be equivalent to t as shown in the investor's payoff. However, the CEO wants the investment to be larger than t, to be exact, $t + b$, as shown in the CEO's payoff. Assuming $b > 0$, parameter b represents how severe the conflict of interests between the players is. Due to the presence of $b > 0$, the informed CEO wants to exaggerate the required investment to induce a higher than t and closer to $t + b$; hence, the minimizes $-|a - (t + b)|$. However, if the CEO sends message as a strictly increasing function of t, then the investor can deduce what the optimal investment level is. Thus, the message cannot be a strictly increasing function (if the CEO succeeds in exaggerating the optimal investment level in equilibrium), rather a function having flat area (or a step function). Another possible way to distort the information on the optimal investment is by *not* reporting at all: if a CEO observed low investment amount t, then he may simply report nothing ($m = \phi$) to make the investor to believe that the expected value of t is approximately average ($\mathbf{E}(t) \approx \frac{1}{2}$), even at the cost of gaining the reputation of being low type in the environment with public channel. (However, this implies that the investor receiving $m = \phi$ may suspect that the message comes from low type; hence, the expected value of t may be lower than the CEO has expected. Thus only the CEO receiving very low t will play this strategy in the environment with public channel only; hence the expected t will be lower than $\frac{1}{2}$.)

The CEO's future prospect depends on the CEO market's assessment of the CEO's expertise. Thus, the CEO has an incentive to establish a good reputation if the information transmission is through a public channel. On the other hand, the CEO will not have incentive to do so under the private channel information transmission. We model the benefit from establishing a good reputation as a linear function of the (updated) probability of the CEO being high type (Ottaviani and Sorensen [2001, 2006a, 2006b] and Bourjade and Jullien [2011] choose the same modeling).

In section 3.1, we consider the aforementioned environment with private channel: the CEO can report m only through the private channel. Thus, the labor market for CEOs does not observe whether the CEO has reported $m = \phi$ or not; hence, the labor market cannot update the probability of the CEO being high type. Without the update, the CEO receiving low t may report $m = \phi$ without fearing the downgrade of one's reputation. In section 3.2, we consider the aforementioned environment with public channel. With the public channel, the labor market can update its belief on the probability of the CEO being high type. In other words, the CEO who received t and reported $m = \phi$ in the model with private channel may not report $m = \phi$ with the public channel, fearing the labor market's reassessment of his type using information $m = \phi$. We will compare how much information is transmitted from the CEO to the investor in the two models; hence, the comparison of efficiency under the the two information transmission structures.

3.1 Private Disclosure Model

To distinguish the messages and the implemented investments in the two different models, we use subscript $k = \{\text{pr}, \text{pu}\}$ for message and the investment, m_{pr} and m_{pu}, and a_{pr} and a_{pu}. The disclosure strategy is a mapping from $[0, 1]$ to $\Omega \cup \{\phi\}$, i.e., $m_{\text{pr}}(t)$. The investment strategy is a mapping from $\Omega \cup \{\phi\}$ to $[0, 1]$, i.e., $a_{\text{pr}}(m_{\text{pr}})$. The information that the CEO labor market has is null in private disclosure model, $I = \{\}$.

[this has to go to Introduction]We analyze the private disclosure model similarly to the abstract communication model without reputation effect of Austen-Smith (1994).

The perfect Bayesian equilibrium is $\langle m^*_{\text{pr}}(t), a^*(m_{\text{pr}}), \mu(t|m_{\text{pr}}), \Pr(p = \bar{p}|I)\rangle$, where $\mu(t|m_{\text{pr}})$ is the investor's belief on optimal investment level t, and $\Pr(p = \bar{p}|I = \{\})$ is the labor market's belief on the CEO being high type. Since $I = \{\}$, there is no update over the prior on CEO's type. (On the other hand, $I = \{\phi, \phi^c\}$ is the case in the model with public disclosure channel.)

Without loss of generality, we consider only players' pure strategies.[5] For given CEO's information disclosure strategy $m^*_{\text{pr}}(t)$, we derive investor's belief $\mu(t|m_{\text{pr}})$ and the CEO labor market's assessment of the CEO's type. Then we calculate the investor's best response $a^*(m_{\text{pr}})$ under these beliefs. Finally, I confirm whether the CEO's equilibrium information disclosure strategy $m^*_{\text{pr}}(t)$ is the best response to the given investment strategy and beliefs.

After observing message m_{pr} sent privately by the CEO, the investor updates her belief on the optimal investment t, $\mu(t|m_{\text{pr}})$. With the updated belief, she chooses investment strategy as a function

[5] Aside from notational convenience, the benefit of pure strategies is that the equilibrium messages may be interpreted literally, unlike mixed strategy equivalents. As noted in Crawford and Sobel (1982), when equilibrium messages are defined this way, it is necessary to specify how the investor interprets off the equilibrium messages. As they also point out, any set of beliefs which does not expand the set of investor's best response actions supports the equilibrium.

of m_{pr}:

$$a^*(m_{\text{pr}}) = \underset{a}{\text{argmax}} \int_{T(m_{\text{pr}})} -|a-t|\mu(t|m_{\text{pr}})dt \tag{3}$$

where $T(m_{\text{pr}}) = \{t \in [0,1] | m^*_{\text{pr}}(t) = m_{\text{pr}}\}$. Since the CEO labor market does not have access to m_{pr}, the CEO market's belief is identical to prior belief π.

Given the investor's strategy $a^*(m_{\text{pr}})$ and the CEO market's belief π, the CEO chooses the private disclosure strategy $m_{\text{pr}}(t)$ to maximize his expected payoff:

$$m^*_{\text{pr}} \in \underset{m_{\text{pr}}}{\text{argmax}}[-|a^*(m_{\text{pr}}) - (t+b)| + R\pi] \tag{4}$$

Lastly, the investor's equilibrium belief when $T(m_{\text{pr}}) \neq \emptyset$ is as follows.

$$\mu(t|m_{\text{pr}}) = \begin{cases} \frac{f(t)}{\int_{T(m_{\text{pr}})} f(\tau)d\tau} & \text{if } m^*_{\text{pr}}(t) = m_{\text{pr}} \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

When $T(m_{\text{pr}}) = \emptyset$, we can assign beliefs arbitrarily.

Definition 1 *$\langle m^*_{pr}(t), a^*(m_{pr}), \mu(t|m_{pr}), \Pr(p = \bar{p}|I = \{\}) = \pi\rangle$ is the perfect Bayesian equilibrium in the model with private signal if and only if $m^*_{pr}(t)$, $a^*(m_{pr})$, and $\mu(t|m_{pr})$ are given by equations (3), (4), and (5).*

First, we obtain the following result from equation (3).

Lemma 2 *The investor's optimal investment strategy $a^*(m_{pr})$ is as follows.*

$$a^*(m_{pr}) = \textbf{Median}(t|m_{pr}) \tag{6}$$

Proof. See Appendix XXX ∎

The equilibrium investment strategy is equivalent to the conditional median of the optimal investment level, given message m_{pr}. It results from the fact that the investor's payoff function is given as a form of linear loss function. Note that the CEO's private message does not directly enter the CEO's payoff function, but it indirectly influences the CEO's payoff through the investor's investment level. This is a typical feature of the costless signaling model.

In the first best equilibrium (if it exists), the exact information on the optimal investment level will be always disclosed, i.e., *full disclosure equilibrium*. Given that the signal is costless, any full disclosure equilibrium with $m_k(t) = f(t)$ (where $k \in \{\mathrm{pr}, \mathrm{pu}\}$) with strictly monotone $f(\cdot)$ is equivalent to another full disclosure equilibrium with $m_k(t) = t$. So we define the full disclosure equilibrium as follows without loss of generality:

Definition 3 *Full disclosure equilibrium is an equilibrium in which the CEO's disclosure strategy is $m_k^*(t) = t$ ($k \in \{pr, pu\}$).*

In order to examine the existence of the full disclosure equilibrium, suppose that the CEO discloses optimal investment level t truthfully, $m_{\mathrm{pr}}(t) = t$. Then, the belief for the optimal investment level with private message t is $\mu(t|t) = 1$. Based on this belief, the investor chooses investment $a^*(t) = Median(t|t) = t$. However, the CEO will deviate from $m_{\mathrm{pr}}^*(t) = t$ with this belief and the investor's strategy $a^*(t) = t$: the CEO achieves $-b + R\pi$ with the truthful disclosure $m = t$, while the CEO achieves payoff $R\pi$ by sending $m = t + b$, i.e., untruthful disclosure. (Note that the CEO labor market does not play any role in the context of private information disclosure as the market does not update belief based on the message.) Thus, the full disclosure equilibrium does not exist in private model, and this equilibrium does not exist under public disclosure model either by a similar reasoning. It is a typical result in the communication game based on the costless signaling model.

Proposition 4 *Full disclosure equilibrium does not exist in the private disclosure model.*

Proof. Follows from the above discussion. ∎

Although full disclosure equilibrium does not exist, the other extreme equilibrium, that is *non-disclosure equilibrium* in which no information is disclosed at all, exists.

Definition 5 *Non-disclosure equilibrium is an equilibrium in which the CEO's strategy is to send null information, $m_k^*(t) = \phi$ for all $t \in [0, 1]$, $k \in \{pr, pu\}$.*

With belief $\mu(t|\phi) = \mu(t|m)$ for $m \neq \phi$ and $m_{\mathrm{pr}}(t) = \phi$, the investor chooses $a^*(t) = Median(t|\phi) = \frac{1}{2}$ in the non-disclosure equilibrium. In addition, the CEO labor market's belief on high type is identical to prior π. With these belief and strategies, the CEO cannot make any difference by sending message different from ϕ. In short, if the investor ignores the CEO's message, the non-disclosure strategy is the CEO's best response. Also, if the CEO does not send any message, the investor has no option but to depend on the prior distribution on CEO's characteristic.[6] On the other hand, non-disclosure equilibrium may not exist with public information disclosure, which will be shown later.

Under non-disclosure equilibrium, the payoffs of the investor and the CEO are:

$$U_{\mathrm{pr}} = \int_0^1 -|\frac{1}{2} - t|dt = -\frac{1}{4} \tag{7}$$

$$V_{\mathrm{pr}} = \int_0^1 -|\frac{1}{2} - t - b|dt + R\pi = \begin{cases} -\frac{1}{4} - b^2 + R\pi & \text{if } b < \frac{1}{2} \\ \int_0^1 (\frac{1}{2} - t - b)dt + R\pi = -b + R\pi & \text{otherwise} \end{cases} \tag{8}$$

Note that if $b < \frac{1}{2}$, the V_{pr} is derived by:

$$\int_0^{\frac{1}{2}-b} -(\frac{1}{2} - t - b)dt + \int_{\frac{1}{2}-b}^1 (\frac{1}{2} - t - b)dt = -\frac{1}{4} - b^2 + R\pi.$$

Proposition 6 summarizes the above findings.

Proposition 6 *Non-disclosure equilibrium always exists with private information disclosure. The equilibrium (non-)disclosure strategy of the CEO, the investor's equilibrium investment strategy, and the equilibrium payoffs are:*

(i) $m^*_{pr}(t) = \phi$ *for all* $t \in [0, 1]$ *and* $a^*_{pr}(m_{pr}) = \frac{1}{2}$ *for all* m_{pr}*,*

(ii) $U_{pr} = -\frac{1}{4}$, $V_{pr} = -\frac{1}{4} - b^2 + R\pi$ *if* $b < \frac{1}{2}$; $-b + R\pi$ *otherwise.*

Proof. Follows from the above discussion. ∎

We will show later that only non-disclosure equilibrium may exist if $b \geq \frac{1}{2}$.

Although we show that non-disclosure equilibrium exists, our interest is on the equilibrium in which at least *some* information is transmitted from the CEO to the investor. We call such equilibrium *partial disclosure equilibrium*. As a first candidate, we consider the following N_k-partial disclosure equilibrium due to Crawford and Sobel (1982).

[6] Austen-Smith (1994) focuses on only the informative equilibrium in which some information is disclosed. He did not show the existence of non-disclosure equilibrium, however, a similar equilibrium exists in his model by the same reason.

Definition 7 *N_k-partial disclosure equilibrium is an equilibrium in which there are $N_k \geq 2$ closed intervals, $\{T_i\}_{1\leq i\leq N_k}$ such that $m_k^*(t) = T_i$ for all $t \in T_i = [t_{i-1}, t_i]$, $i = 1, \ldots, N_k$ and $m_k^*(\phi) = \phi$, $k \in \{pr, pu\}$.*

In a N_k-partial disclosure equilibrium, the CEO discloses an interval containing the optimal investment level t, and t_is are the boundary types between intervals T_i and T_{i+1} with $0 = t_0 < t_1 < \ldots < t_{N_k}$ without loss of generality. First note that there are finite number of investment levels $\{t_1, \ldots, t_{N_k-1}\}$ such that $m^*(t_j)$ is not well-defined, but it is an event with probability zero. Also we can easily modify the interval into half-closed and half-open to redefine the equilibrium. Second, $m_k^*(t) = T_i$ implies that the optimal investment level t is distributed uniformly in interval $T_i = [t_{i-1}, t_i]$ for $k \in \{\text{pr}, \text{pu}\}$.

We show that N_{pr}-partial disclosure equilibrium does not exist in private disclosure model. If such equilibrium exists, then the CEO with low t will deviate to $m = \phi$, so that the implemented investment becomes $a = \frac{1}{2}$ that is higher than the median of the interval that t belongs to. Formally, suppose such equilibrium exists. First we have $a^*(T_i) = \frac{t_{i-1}+t_i}{2}$, $i = 1, 2, \ldots, N_{pr}$ and $a^*(\phi) = \frac{1}{2}$ from Lemma 2. The CEO's payoff from reputation is $R\pi$ regardless of the messages that the CEO reveals.

Any CEO with $t \leq \frac{1}{2} - b$ receives payoff $-|\frac{t_{i-1}+t_i}{2} - (t+b)| + R\pi = -(t + b - \frac{t_{i-1}+t_i}{2}) + R\pi$ through disclosing the intervals. However, if the CEO with $t \leq \frac{1}{2} - b$ sends message $m = \phi$, his payoff becomes $-|\frac{1}{2} - (t+b)| + R\pi = -(\frac{1}{2} - (t+b)) + R\pi$. Thus, the CEO of type t satisfying $t + b - \frac{t_{i-1}+t_i}{2} > \frac{1}{2} - (t+b)$ prefers to send message ϕ to message T_i. This implies that there is no N_{pr}-partial disclosure equilibrium. Intuitively, if the CEO has received the signal of very low optimal investment, then he may simply pretend not having received any signal, hoping that the investor's expectation of the optimal investment higher than the actual optimal investment.

Proposition 8 *N_k-partial disclosure equilibrium does not exist with private disclosure.*

Proof. Follows from the above discussion. ∎

By showing the non-existence of N_{pr}-partial disclosure equilibrium in the private disclosure model, we learned that the CEO with lower optimal investment t had incentive to hide the fact that he had observed low t. By hiding the observation, the CEO wants to induce the investor to implement higher investment than the optimal investment. Thus by allowing the possibility of sending message $m = \phi$ to the CEO with lower optimal investment, we may construct an equilibrium in which some information is disclosed by the CEO.

Definition 9 *(ϕ, N_k)-partial disclosure equilibrium is an equilibrium in which there are $N_k \geq 2$ closed intervals, $\{T_i\}_{1\leq i\leq N_k}$ such that $m_k^*(t) = \phi$ for all $t \in T_1 = [0, t_1]$, and $m_k^*(t) = T_i$ for all $t \in T_i = [t_{i-1}, t_i]$, $i = 2, \ldots, N_k$ and $m_k^*(\phi) = \phi$, $k \in \{pr, pu\}$*

Note that $m_k^*(t) = \phi$ for $t \in [0, t_1]$ means that the CEO hides the lowest interval on optimal investment, and that $m_k^*(t) = T_i$ means that the optimal investment level t is distributed uniformly in the interval $T_i = [t_{i-1}, t_i]$ conditional to the fact that T_i is the signal.

Example 1 [INSERT A GRAPH LATER: read the paragraphs after this example]

We first illustrate $(\phi, 3)$-partial disclosure equilibrium. Suppose the CEO who observed $t \in T_1 = [0, t_1]$ sends a message ϕ while the CEO who observed $t \in T_i = [t_{i-1}, t_i], i = 2, 3$ discloses interval T_i. Receiving message ϕ implies that (i) with probability $\frac{p_e t_1}{(1-p_e)+p_e t_1}$, the CEO has obtained information and optimal investment level is uniformly distributed on $[0, t_1]$, or (ii) with probability $1 - \frac{p_e t_1}{(1-p_e)+p_e t_1}$, the CEO did not obtain any information (so that optimal investment level must be uniformly distributed on $[0, 1]$). The investor's optimal action is $a^*(\phi) = \frac{p_e {t_1}^2+1-p_e}{2(p_e t_1+1-p_e)}$ with this belief.[7] *Receiving message T_i, $i = 2, 3$, implies that the optimal investment level is uniformly distributed on $[t_{i-1}, t_i], i = 2, 3$, so the investor's optimal action is $a^*(T_i) = \frac{t_{i-1}+t_i}{2}, i = 2, 3$.*

For the incentive compatibility constraints, the CEO who observed $t \in T_1 = [0, t_1]$ must prefer sending message ϕ to the other messages ($\{T_2, T_3\}$), and the CEO who observed $t \in T_2$ or $t \in T_3$ respectively must prefer sending T_2 or T_3 respectively to the other messages.

The closer to $t + b$ the induced investment is, the better off the CEO becomes. For boundary type t_2, the investment induced by message T_2 and the investment by T_3 must be equally away – but in the opposite direction – from $t_2 + b$. Once this is satisfied, we do not have to check all the other types $t \in (t_2, t_3 = 1]$ by construction. For boundary type t_1, the investment induced by message T_2 and the investment by ϕ must be equally away – but in the opposite direction – from $t_1 + b$. Moreover, we need to have $a^*(\phi) \leq a^*(T_2)$. Otherwise, we will have $t - a^*(\phi) < t - a^*(T_2)$ for all $t \in T_2$, which contradicts

[7] Note that the total probability that ϕ is sent is $p_e t_1 + (1-p_e)$, i.e., the probability that the CEO receives low optimal investment level. Thus the optimal action is given by $a^*(\phi) = \frac{t_1}{2}\frac{p_e t_1}{p_e t_1+(1-p_e)} + \frac{1}{2}\frac{1-p_e}{p_e t_1+(1-p_e)} = \frac{p_e t_1^2+1-p_e}{2(p_e t_1+1-p_e)}$

the construction of the equilibrium. Then, we again do not have to check the incentive compatibility constraints for all the other types $t \in [0, t_1)$ and $t \in [t_1, t_2)$ by construction. The reason that we need this additional constraint is message ϕ is sent both by $t \in T_1$ and those who did not observe the optimal investment level (XXX).

Formally, we need:

$$(t_1 + b) - a^*(T_2) = a^*(\phi) - (t_1 + b), \tag{9}$$

$$(t_2 + b) - a^*(T_3) = a^*(T_2) - (t_2 + b), \tag{10}$$

$$a^*(\phi) \leq a^*(T_2) \tag{11}$$

where $a^*(\phi) = \frac{p_e {t_1}^2 + 1 - p_e}{2(p_e t_1 + 1 - p_e)}$ *and* $a^*(T_i) = \frac{t_{i-1} + t_i}{2}$.

Solving the system of equations (9) and (10), we can obtain the boundary type t_1 and t_2, and the rest of the values can be calculated from the above.[8] *Finally, $a^*(T_2) \geq a^*(\phi)$ is equivalent to the condition that b must be less than $\frac{7 - \sqrt{49 - 48p_e}}{48p_e}$.*[9]

The first thing to note in Example 1 is that, the interval T_{i+1} to the right of T_i is wider by $4b$, as seen in equation (10) that is equivalent to $(t_3 - t_2) = (t_2 - t_1) + 4b$. The crucial reason is that the CEO prefers $t + b$ the most for given optimal investment t. For the boundary type t_i to be indifferent, $t + b$ must be equally away from the median values of T_i and T_{i+1}. If T_{t+1} is wider than T_i by *less* than (or *more* than) $4b$, then the boundary type strictly prefers $m_{pr} = T_{i+1}$ (or $m_{pr} = T_i$).

The second to note is: the first observation, however, does not apply to the first and the second intervals. When the investor receives message ϕ, the investor can not distinguish as to whether the message is sent by an CEO who observed low optimal investment level or who did not at all. This uncertainty makes $a^*(\phi)$ become larger than the median of T_1. That is,

$$a^*(\phi) = \frac{t_1}{2} \frac{p_e t_1}{p_e t_1 + (1 - p_e)} + \frac{1}{2} \frac{1 - p_e}{p_e t_1 + (1 - p_e)} > \frac{t_1}{2} = \textbf{Median}[t | t \in T_1].$$

So, the relation between the first and the second intervals is not clear.

[8] The boundary types are:

$$t_1 = \frac{-5 + (6 - 12b)p_e + \sqrt{25 + 24p_e(-1 + b(-1 + 6bp_e))}}{6p_e}, t_2 = \frac{-5 + (12 - 36b)p_e + \sqrt{25 + 24p_e(-1 + b(-1 + 6bp_e))}}{12p_e}.$$

[9] Plugging the computed t_1 and t_2 into the $a^*(T_i)$ and $a^*(\phi)$, we can easily compute this.

The third to note is: $a^*(T_2)$ must be greater than or equal to $a^*(\phi)$. Since $\frac{t_1}{2} < a^*(\phi) < \frac{1}{2}$, $a^*(\phi)$ might be larger than $a^*(T_2) = \frac{t_1+t_2}{2}$. However, if $a^*(\phi)$ is larger than $a^*(T_2)$, then the CEO who observes $t > t_1$ deviate to sending the message ϕ (and the CEO who observes $t < t_1$ deviate to sending the message T_2), which is a contradiction. So, $a^*(\phi)$ must be less than or equal to $a^*(T_2)$ in (ϕ, N_{pr})-partial disclosure equilibrium.

From the above illustration, we find the conditions for (ϕ, N_{pr})-partial disclosure equilibrium:

$$a^*(\phi) \leq a^*(T_2), \tag{12}$$

$$v(a^*(\phi), t_1, b) = v(a^*(T_2), t_1, b) \;\Leftrightarrow\; |a^*(\phi) - (t_1 + b)| = |a^*(T_2) - (t_1 + b)|, \tag{13}$$

$$v(a^*(T_i), t_i, b) = v(a^*(T_{i+1}), t_i, b) \;\Leftrightarrow\; |a^*(T_i) - (t_i + b)| = |a^*(T_{i+1}) - (t_i + b)|, \forall i \geq 2 \tag{14}$$

where $a^*(T_i) = Median[t|T_i] = \frac{t_{i-1}+t_i}{2}, i = 2, 3, \ldots, N_{\text{pr}}$ and $a^*(\phi) = Median[t|\phi] = H(T_1|\phi) * \frac{t_1}{2} + (1 - H(T_1|\phi)) * \frac{1}{2} = \frac{p_e t_1^2 + 1 - p_e}{2(p_e t_1 + 1 - p_e)}$ with $H(T_1|\phi)$ being the conditional probability that CEO had observed t in the lowest interval, given that the CEO sent message ϕ (thus, $H(\phi|T_1) = \frac{p_e t_1}{(1-p_e)+p_e t_1}$).

Equations (13) and (14) ensure that boundary type t_1 must be indifferent between sending message T_2 and ϕ, and that boundary types $\{t_2, \ldots, t_N\}$ must be indifferent between sending the neighboring intervals as a message. (Equations (13) and (14) are called "No arbitrage" condition in the literature.) Equation (12) guarantee that $a^*(\phi)$ be less than $a^*(T_2)$ in (ϕ, N_{pr})-partial disclosure equilibrium.

If there is no upper bound for the number of intervals for (ϕ, N_{pr})-partial disclosure equilibrium for given bias b and ex-ante probability of being uninformed p_e, we can simply set $N_{\text{pr}} = \infty$ to derive the most informative equilibrium. This is congruent to the model of Bourjade and Jullien (2011), in which the CEO send either truthful optimal investment level or simply hide information. However, the non-verifiability of message in our model implies that the CEO does not report the true optimal investment level generically. Thus, we expect that there has to be the upper bound for N_{pr}, by the same reason that full disclosure equilibrium does not exist.

Fixing N_{pr}, we can compute boundary types t_is by equation (13) and (14), and the calculation shows that $a^*(T_2) - a^*(\phi)$ decreases in N_{pr}.[10] Thus, if N_{pr} becomes too large, $a^*(T_2) - a^*(\phi)$ may become negative, which should not happen in an equilibrium. In other words, the maximum number of intervals (denoted by $\bar{N}_{\text{pr}}(b, p_e)$ for given b and p_e) is the largest integer value to guarantee $a^*(T_2) \geq a^*(\phi)$.

[10]See Appendix XXX

Note again that our model includes an uninformed CEO who does not exist in Crawford and Sobel (1982), which is the essential reason that we need condition (12).

Finally, the following proposition summarizes (ϕ, N_{pr})-partial disclosure equilibrium together with an additional result on bias b for existence.

Proposition 10 *For all $b < \frac{1}{2}$, there exists a positive integer $\bar{N}_{pr}(b, p_e)$ such that $N_{pr} \in \{2, 3, \ldots, \bar{N}_{pr}(b, p_e)\}$ is associated with unique (ϕ, N_{pr})-partial disclosure equilibrium. In (ϕ, N_{pr})-partial disclosure equilibrium, the CEO's equilibrium disclosure strategy, the investor's equilibrium investment strategy, and the players' equilibrium (expected) payoffs are:*

(i) For all $t \in [0, t_1)$, $m^*_{pr}(t) = \phi$ and for all $t \in [t_{i-1}, t_i)$, $m^*_{pr}(t) = T_i$, $i = 2, 3, \ldots, N_{pr} - 1$ and for all $t \in [t_{N_{pr}-1}, 1]$, $m^*_{pr}(t) = T_{N_{pr}}$,

(ii) $a^*(\phi) = \frac{p_e t_1^2 + 1 - p_e}{2(p_e t_1 + 1 - p_e)}$, $a^*(T_i) = \frac{t_{i-1} + t_i}{2}$, $i = 2, 3, \ldots, N_{pr}$,

where $t_1 = \frac{2((b+1)p_e - 1)N_{pr} - 2bp_e N_{pr}^2 + 1 + \sqrt{x}}{2p_e N_{pr}}$,

$x = 4p_e(1 - p_e)((2b + 1)N_{pr} - 2bN_{pr}^2)N_{pr} + (2(b + 1)p_e - 1)N_{pr} - 2bp_e N_{pr}^2 + 1)^2$,

$t_i = (\frac{N_{pr}-i}{N_{pr}-1})t_1 - 2b(i - 1)(N_{pr} - i) + \frac{i-1}{N_{pr}-i}$, $i = 2, \ldots, N_{pr}$,

(iii) $\bar{N}_{pr}(b, p_e)$ is the largest integer N_{pr} satisfying inequality (12).

(iv) $U_{pr} = -\frac{(1-p_e)(1-t_1)t_1}{2(1-p_e(1-t_1))} - \frac{4b^2 N_{pr}(N_{pr}-1)^2 + 3(1-t_1)^2}{12(N_{pr}-1)}$,

(v) $V_{pr} = -\frac{(2b(1-p_e)(1-t_1)(p_e t_1^2) - (1-p_e))^2}{4(1-p_e(1-t_1))^2} - \frac{4b^2 N_{pr}(N_{pr}^2 - 2N_{pr} + 3)(N_{pr}-1)^2 + 3(1-t_1)^2}{12(N_{pr}-1)} + R\pi$.

Proof. See Appendix **??** ∎

From Proposition 8 and 10, we see that when the CEO discloses information to the investor using the private disclosure channel, he does not disclose the lowest interval of optimal investment levels but disclose higher optimal investment by dividing the remaining interval into several intervals. In the presence of the possibility that the CEO would be uninformed, the CEO who observed low optimal investment levels pretends to not have obtained the information. This leaves the investor unsure as to whether the message is sent by a CEO who obtained low optimal investment levels or an uninformed CEO. In the end, the investor's uncertainty leads to a relatively higher investment, as compared to that induced when the investor knows for sure that the CEO obtained the optimal investment levels in the lowest interval.

We compute $\bar{N}_{\text{pr}}(b, p_e)$.

Corollary 11 *The maximum number of intervals disclosed in (ϕ, N_{pr})-partial disclosure equilibrium is* $\bar{N}_{pr}(b, p_e) = \lceil \frac{1+bp_e-\sqrt{1-p_e+b^2p_e^2}}{2bp_e} \rceil$. *where $\lceil \ell \rceil$ is the smallest integer greater than or equal to ℓ.*

Proof. Follows from condition (iii) in Proposition 10) ∎

The maximum number of intervals, $\bar{N}_{pr}(b, p_e)$, increases in bias b and decreases in the ex-ante probability of receiving signal p_e. In other words, information disclosure becomes more precise (i) as the conflict of interest between the CEO and the investor decreases, and (ii) as the information acquisition becomes more likely. Also $\bar{N}_{pr}(b, p_e)$ is finite for all b and p_e, but it approaches infinity as b approaches zero and p_e approaches one.

Austen-Smith (1994) considers $(\phi, 2)$-partial disclosure equilibrium without the concern for the reputation (that does not exist in the current model with private disclosure). He shows that there is no such equilibrium if and only if b is large enough. We generalize his findings to characterize the maximum number of intervals for given bias b and ex-ante probability of being informed p_e.

[WORK MORE ON THIS!] From Example 1, we can know that with the possibility of being uninformed, the maximum number of intervals is less than that of Crawford and Sobel (1982) with the same parameters (especially b). **[In leading example of their model, for same parameter value as example 1 $b = 1/25$, there exists 4-partition equilibrium. But there is no $(\phi, 4)$-partial disclosure equilibrium in our private disclosure model. The reason is that the room divided is shrunk from $[0, 1]$ to $[t_1, 1]$ in our private disclosure model compared that of Crawford and Sobel (1982).]**

[WORK MORE ON THIS AFTER WORKING ON THE NEXT SECTION!] Even with the possibility for being uninformed, we show that the CEO's and investor's equilibrium expected payoff increases as the equilibrium number of partition N_{pr} increase as in Crawford and Sobel (1982). Thus, there is no doubt that the most informative $(\phi, \bar{N}_{\text{pr}}(b, p_e))$ equilibrium should be our focus. However, we will consider public disclosure model and show that the finding (that the investor's equilibrium payoffs increases in N_{pr}) may not be true anymore. The reason is that the CEO may care about his reputation more than the relation with investor, so that the CEO gives up the accuracy of information for the his reputation gain.

3.2 Public Disclosure Model

In this subsection, the CEO discloses information to the investor, using the public disclosure channel (IR, earnings forecast disclosure, reporting via internet web-site and media, publishing business proposals, etc.). Since the CEO discloses information through the public disclosure channel, both the investor and the CEO labor market observe the information disclosed by the CEO. We analyze how the CEO's disclosure strategy is affected by the CEO market's observation of the disclosed information. We characterize equilibria in the presence of public disclosure, then we compare them with the equilibria under private disclosure.

The CEO discloses the information on the optimal investment level t through the public disclosure channel observable to everyone. After observing the disclosed information, the investor chooses investments level a affecting the payoffs of the both players. At the same time, the CEO labor market evaluates the competency of the CEO, using the disclosed information.

The public disclosure model can be analyzed similarly to the private disclosure model in the previous subsection. The perfect Bayesian equilibrium of the public disclosure model is

$$\langle m^*_{\text{pu}}(t), a^*(m_{\text{pu}}), \mu(t|m_{\text{pu}}), \nu(I)\rangle,$$

where $\mu(t|m_{\text{pu}})$ is the investor's belief on optimal investment level t, and $\nu(I)$ is the labor market's belief on the CEO being high type, that is, $\nu(I) = \Pr(p = \bar{p}|I)$.

After observing message m_{pu} sent publicly by the CEO, the investor updates her belief on the optimal investment t, $\mu(t|m_{\text{pu}})$. With the updated belief, she chooses investment strategy as a function of m_{pu}:

$$a^*(m_{\text{pu}}) = \underset{a}{\operatorname{argmax}} \int_{T(m_{\text{pu}})} -|a-t|\mu(t|m_{\text{pu}})dt \tag{15}$$

where $T(m_{\text{pu}}) = \{t \in [0,1] | m^*_{\text{pu}}(t) = m_{\text{pu}}\}$.

Given the investor's strategy $a^*(m_{\text{pr}})$ and the CEO market's belief $\nu(I)$, the CEO chooses the private disclosure strategy $m_{\text{pu}}(t)$ to maximize his expected payoff:

$$m^*_{\text{pu}} \in \underset{m_{\text{pu}}}{\operatorname{argmax}}[-|a^*(m_{\text{pu}}) - (t+b)| + R\nu(I)] \tag{16}$$

The investor's equilibrium belief when $T(m_{\text{pu}}) \neq \emptyset$ is as follows.

$$\mu(t|m_{\text{pu}}) = \begin{cases} \frac{f(t)}{\int_{T(m_{\text{pu}})} f(\tau)d\tau} & \text{if } m^*_{\text{pu}}(t) = m_{\text{pu}} \\ 0 & \text{otherwise} \end{cases} \tag{17}$$

When $T(m_{\text{pu}}) = \emptyset$, we can assign beliefs arbitrarily. Finally, the labor market's belief on the CEO being high type is as follows.

$$\nu(I) = \begin{cases} \frac{\pi[\bar{p}t_1+(1-\bar{p})]}{p_e t_1+(1-p_e)}^{11} & \text{if } I = \phi \\ \frac{\pi\bar{p}}{p_e} & \text{otherwise} \end{cases} \tag{18}$$

The equations from (15) to (17) form the perfect Bayesian equilibrium.

We will show that, unlike the model with private disclosure, there exists N_{pu}-partial equilibrium in definition 7 for certain parameter values. However, this does not necessarily imply that (ϕ, N_{pu})-partial equilibrium does not exists; both of the equilibria may exist for certain parameter values.

For the existence of N_{pu}-partial equilibrium, we can similarly characterize the boundary types who are indifferent between sending the intervals on the right-hand and the left-hand sides, i.e., $v(a^*(T_i), t_i, b) = v(a^*(T_{i+1}), t_i, b)$. Moreover, nobody who has observed t should not prefer to send ϕ, i.e., $v(a^*(T_i), t, b) > v(a^*(\phi), t, b)$. Given that $v(a^*(T_i), t_i, b) = -|a^*(m_{\text{pu}}) - (t+b)| + R\nu(I)$ for all t, the two condition become:

$$|a^*(T_i) - (t_i + b)| = |a^*(T_{i+1}) - (t_i + b)|, \forall i \geq 1, \tag{19}$$

$$\Delta_0 > \max_t [|a^*(\phi) - (t+b)| - |a^*(T_i) - (t+b)|], \forall T_i \tag{20}$$

where $a^*(T_i) = Median[t|T_i] = \frac{t_{i-1}+t_i}{2}, i = 1, 2, \ldots, N_{pu}$, $a^*(\phi) = Median[t|\phi] = \frac{1}{2}$, and $\Delta_0 = R[\nu(\phi^c) - \nu(\phi)] = \frac{R(1-\pi)\pi(\bar{p}-\underline{p})}{p_e(1-p_e)}$.[12]

Proposition 12 summarizes N_{pu}-partial disclosure equilibrium with public disclosure.

Proposition 12 *For all $b < \Delta_0 - \frac{1}{4}$, there exists a positive integer $\bar{N}_{pu}(b, \Delta_0, p_e)$ such that $N_{pu} \in \{1, 2, \ldots, \bar{N}_{pu}(b, \Delta_0, p_e)\}$ is associated with unique N_{pu}-partial disclosure equilibrium. In an N_{pu}-partial disclosure equilibrium, the CEO's equilibrium disclosure strategy $m^*_{pu}(t)$, the investor's equilibrium investment strategy $a^*(m_{pu})$, and the players' equilibrium expected payoffs are as follows:*

[12]Note the following: $\nu(\phi^c) = \frac{\pi\bar{p}}{p_e}, \nu(\phi) = \frac{\pi(1-\bar{p})+\bar{p}\Pr(T(\phi))}{1-p_e+p_e\Pr(T(\phi))}$.

(i) For all $t \in [t_{i-1}, t_i)$, $m^*_{pu}(t) = [t_{i-1}, t_i)$, $i = 1, 2, \ldots, N_{pu} - 1$ *and for* $t \in [t_{N_{pu}-1}, 1]$, $m^*_{pu}(t) = [t_{N_{pu}-1}, 1]$,

(ii) $a^*(T_i) = \frac{t_{i-1}+t_i}{2}$, *where* $t_i = \frac{i}{N_{pu}} - 2bi(N_{pu} - i), i = 1, 2, \ldots, N_{pu}$.

(iii) $\bar{N}_{pu}(b, \Delta_0, p_e)$ *is the largest integer* N_{pu} *satisfying inequality (19).*

(iv) $U_{pu} = -\frac{1}{4N_{pu}} - \frac{b^2 N_{pu}(N_{pu}^2-1)}{3}$,

(v) $V_{pr} = U_{pu} - b^2 N_{pu} + \frac{\pi\bar{p}}{p_e}$ *if* $b < \frac{1}{2N_{pu}^2}$

$V_{pr} = U_{pu} - b^2 N_{pu} + \frac{1}{4N_{pu}^2} + b^2 N_{pu}^2 - b + \frac{\pi\bar{p}}{p_e}$ *otherwise*

Unlike the model with private information disclosure, there exists N_{pu}-partial disclosure equilibrium in which the informed CEO never hides the fact that he has received a certain signal (with high enough reputation effect).

Corollary 13 *The maximum number of intervals disclosed in* N_{pu}*-partial disclosure equilibrium is* $\bar{N}_{pu}(b, \Delta_0, p_e) = \lceil \frac{1+b-\sqrt{b^2+2b}}{2b} \rceil$, *i.e., not a function in* Δ_0 *or* p_e.

Thus we simply denote the maximum number of intervals by $\bar{N}_{pu}(b)$. The maximum number of intervals, $\bar{N}_{pu}(b)$ decreases in conflict parameter b.

However, with the small reputation effect, there may exist an equilibrium in which the CEO hides the interval of low optimal investment levels. That is, (ϕ, N_{pu})-partial disclosure equilibrium exists.

We denote by $\Delta = R[\nu(T_i) - \nu(\phi)] = \frac{R(1-\pi)\pi(\bar{p}-p)}{p_e(1-p_e+p_e t_1)}$ the reputation gain of sending message T_i (instead of ϕ) for the CEO who has observed $t_i \in T_i$. Then, (ϕ, N_{pu})-partial disclosure equilibrium of the public disclosure model consists of $m^*_{pu}(t)$ and $a^*(m_{pu})$ satisfying the following equations.

$$a^*(\phi) < a^*(T_2) \tag{21}$$

$$v(a^*(\phi), t_1, b) = v(a^*(T_2), t_1, b) \;\Leftrightarrow\; |a^*(\phi) - (t_1 + b)| = |a^*(T_2) - (t_1 + b)| + \underbrace{R(\nu(T_2) - \nu(\phi))}_{\text{reputation gain}}, \tag{22}$$

$$v(a^*(T_i), t_i, b) = v(a^*(T_{i+1}), t_i, b), \;\Leftrightarrow\; |a^*(T_i) - (t_i + b)| = |a^*(T_{i+1}) - (t_i + b)|, \forall i \geq 2. \tag{23}$$

where $\nu(T_2) = Pr(p = \bar{p}|T_2) = \frac{\pi\bar{p}}{p_e}$, $\nu(\phi) = Pr(p = \bar{p}|\phi) = \frac{\pi(1-\bar{p})}{1-p_e+p_e t_1}$.

Note that other the *reputation gain* in equation (22), the three equations are identical to those in the private disclosure model. The next proposition characterizes (ϕ, N_{pu})-partial disclosure equilibrium by solving (21), (22), and (23).

Proposition 14 *For all $b < \frac{1-\Delta}{2}$, there exists a positive integer $\bar{N}_{pu}(b, \Delta, p_e)$ such that $N_{pu} \in \{2, 3, \ldots, \bar{N}_{pu}(b, \Delta, p_e)\}$ is associated with unique (ϕ, N_{pu})-partial disclosure equilibrium. In (ϕ, N_{pu})-partial disclosure equilibrium, the CEO's equilibrium disclosure strategy $m^*_{pu}(t)$, the investor's equilibrium investment strategy $a^*(m_{pu})$, and the players' equilibrium (expected) payoffs are:*
(i) For all $t \in [0, t_1)$, $m^*_{pu}(t) = \phi$, for all $t \in [t_{i-1}, t_i)$, $m^*_{pu}(t) = T_i$, $i = 2, 3, \ldots, N_{pu} - 1$ and for all $t \in [t_{N_{pu}-1}, 1]$, $m^*_{pu}(t) = T_{N_{pu}}$,
(ii) $a^*(\phi) = \frac{p_e t_1^2 + 1 - p_e}{2(p_e t_1 + 1 - p_e)}$, $a^*(T_i) = \frac{t_{i-1}+t_i}{2}$, $i = 2, 3, \ldots, N_{pu}$, where $t_1 = \frac{2((b-\Delta+1)p_e - 1)N_{pu} - 2bp_e N_{pu}^2 + 2\Delta p_e + 1 + \sqrt{y}}{2p_e N_{pu}}$,
$y = 4p_e(1-p_e)((2(b-\Delta)+1)N_{pu} - 2bN_{pu}^2)N_{pu} + (2(b-\Delta+1)p_e - 1)N_{pu} - 2bp_e N_{pu}^2 + 2\Delta p_e + 1)^2$,
$t_i = (\frac{N_{pu}-i}{N_{pu}-1})t_1 - 2b(i-1)(N_{pu}-i) + \frac{i-1}{N_{pu}-i}$, $i = 2, \ldots, N_{pu}$.
(iii) $\bar{N}_{pu}(b, \Delta, p_e)$ is the largest integer N_{pu} satisfying inequality (21).
(iv) $U_{pu} = -\frac{(1-p_e)(1-t_1)t_1}{2(1-p_e(1-t_1))} - \frac{4b^2 N_{pu}(N_{pu}-1)^2(N_{pu}-2) + 3(1-t_1)^2}{12(N_{pu}-1)}$,
(v) $V_{pu} = -\frac{(2b(1-p_e)(1-t_1)(p_e t_1^2) - (1-p_e))^2}{4(1-p_e(1-t_1))^2} - \frac{4b^2 N_{pu}(N_{pu}^2 - 2N_{pu} + 3)(N_{pu}-1)^2 + 3(1-t_1)^2}{12(N_{pu}-1)} + \frac{\pi(\bar{p}(1-p_e-t_1) + p_e(t_1 + \pi t_1 - \pi t_1^2))}{p_e(1-p_e+p_e t_1)} R.$

Even with the reputation effect, (ϕ, N_{pu})-partial disclosure equilibrium may exist if the reputation gain is small enough. More specifically, the CEO who has received the signal of low optimal investment level will send signal ϕ. However, note that the *hidden interval* in the public disclosure model (i.e., the interval of CEO's type in which the CEO sends message ϕ) is smaller than that in the private disclosure model. This is because, at the margin, the CEO is concerned about his reputation in the CEO labor market. That is, as long as $\Delta > 0$, the size of the hidden interval in the private disclosure model is always greater. Moreover, the difference of the hidden intervals in the two models becomes zero when Δ becomes zero.

Corollary 15 *The maximum number of intervals disclosed in (ϕ, N_{pu})-partial disclosure equilibrium is $\bar{N}_{pu}(b, \Delta, p_e) = \lceil \frac{\sqrt{(2b^2+\Delta^2)p_e^2 - 2b(\sqrt{(b^2 p_e - 1)p_e + 1} - 1)p_e} - \Delta p_e}{2bp_e} \rceil$.*

Note that unlike the N_{pu}-partial disclosure equilibrium, the maximum number of intervals in (ϕ, N_{pu})-partial disclosure equilibrium depends not only on the parameter b but also on Δ and p_e. That is, for given b, the greater the reputation effect Δ and p_e is, the fewer the number of intervals of the optimal investment level divided by the CEO is. On the contrary, the smaller the Δ and p_e are, the greater the number of intervals is. Additionally, for given b and p_e, as Δ approaches 0, $\bar{N}_{pu}(b, \Delta, p_e)$ in the model with public disclosure converges to $\bar{N}_{pr}(b, p_e)$ in the model with private disclosure.

Corollary 16 *When $\Delta_0 \leq \frac{1}{4}$, only (ϕ, N_{pu})-partial disclosure equilibrium exists in the public disclosure model.*

From Proposition 12, when $\Delta_0 \leq \frac{1}{4}$, $b \leq 0$ should be satisfied for the existence of N_{pu}-partial disclosure equilibrium. However, since $b > 0$ is assumed, N_{pu}-partial disclosure equilibrium cannot exist. In other words, when the reputation effect Δ_0 is small, the CEO who observed lower optimal investment level hides the observed level since the cost of giving up reputation is lower than the benefit of the increased investor's investment.

Corollary 17 *When $\Delta \geq 1$, only N_{pu}-partial disclosure equilibrium exists in the public disclosure model.*

From proposition 14, when $\Delta \geq 1$, to make (ϕ, N_{pu})-partial disclosure equilibrium exist, $b \leq 0$ should be satisfied. However, this is again contradictory to the assumption of $b > 0$. Thus, (ϕ, N_{pu})-partial disclosure equilibrium does not exist. In other words, when the reputation effect is large, the CEO never hides information to enhance his reputation in the CEO labor market.

4 Comparison of disclosure channels

We examine which disclosure channel reveals more information, and which disclosure channels are preferred by players. When multiple equilibria exist, we consider only the most informative equilibrium.

4.1 Disclosure channel in which more information is disclosed

We can show that with $\Delta_0 > \frac{1}{4}$ and $b < \min\{\Delta_0 - \frac{1}{4}, \frac{1}{4}\}$, N_{pu}-partial disclosure euqilibrium is superior to (ϕ, N_{pu})-partial disclosure equilibrium in the public disclosure model.

Lemma 18 *When $\Delta_0 > \frac{1}{4}$ and $b < \min\{\Delta_0 - \frac{1}{4}, \frac{1}{4}\}$, N_{pu}-partial disclosure equilibrium is superior to (ϕ, N_{pu})-partial disclosure equilibrium in the public disclosure model.*

Proof. From Corollary 13 we know that $\bar{N}_{pu}(b, \Delta_0, p_e) = \lceil \frac{1+b-\sqrt{b^2+2b}}{2b} \rceil$, and from Corollary 15 that $\bar{N}_{pu}(b, \Delta, p_e) = \lceil \frac{\sqrt{(2b^2+\Delta^2)p_e^2 - 2b(\sqrt{(b^2 p_e - 1)p_e + 1} - 1)p_e} - \Delta p_e}{2bp_e} \rceil$. When $\Delta_0 > \frac{1}{4}$ and $b < \min\{\Delta_0 - \frac{1}{4}, \frac{1}{4}\}$, we can derive that $\bar{N}_{pu}(b, \Delta_0, p_e) \geq \bar{N}_{pu}(b, \Delta, p_e)$. ■

Lemma 19 *When $\Delta_0 > \frac{1}{4}$ and $\min\{\Delta_0 - \frac{1}{4}, \frac{1}{4}\} \leq b < \frac{1}{2}$, (ϕ, N_{pu})-partial disclosure equilibrium is superior to N_{pu}-partial disclosure equilibrium in the public disclosure model.*

Proof. Similary to the Lemma 18, when $\Delta_0 > \frac{1}{4}$ and $b < \min\{\Delta_0 - \frac{1}{4}, \frac{1}{4}\}$, we can derive that $\bar{N}_{pu}(b, \Delta_0, p_e) \leq \bar{N}_{pu}(b, \Delta, p_e)$. ■

Comparing N_{pu}-partial disclosure equilibrium in the public disclosure model with (ϕ, N_{pr})-partial disclosure equilibrium in the private disclosure model, we derive the following result.

Proposition 20 *With $\Delta_0 > \frac{1}{4}$ and $b < \min\{\Delta_0 - \frac{1}{4}, \frac{1}{4}\}$, more information is disclosed in the public disclosure model than in the private disclosure model.*

Proof. In the range of given parameters, $\bar{N}_{pu}(b, \Delta_0, p_e) \geq \bar{N}_{pr}(b, \Delta, p_e)$ is always true from Lemma 18, Corollary 13 and Corollary 11. ■

On the other hand, we can show that with $\Delta_0 \leq \frac{1}{4}$ and $\min\{\Delta_0 - \frac{1}{4}, \frac{1}{4}\} \leq b < \frac{1}{2}$, reverse is true.

Proposition 21 *When $\Delta_0 \leq \frac{1}{4}$ or $\min\{\Delta_0 - \frac{1}{4}, \frac{1}{4}\} \leq b < \frac{1}{2}$, more information is disclosed in the private disclosure model than in the public disclosure model.*

Proof. First, when $\Delta_0 \leq \frac{1}{4}$, the only equilibrium in the public disclosure model is (ϕ, N_{pu})-partial disclosure equilibrium. Therefore, from Corollary 11 and Corollary 13, we can see $\bar{N}_{pr}(b, \Delta, p_e) \geq \bar{N}_{pu}(b, \Delta, p_e)$. On the other hand, when $\{\min \Delta_0 - \frac{1}{4}, \frac{1}{4}\} \leq b < \frac{1}{2}$, from Lemma 19, Corollary 11 and Corollary 15, we can see $N_{pr}(b, \Delta, p_e) \geq N_{pu}(b, \Delta, p_e)$. ■

4.2 Disclosure channel that players prefer

Now, based on earlier Lemmas and Propositions, let us examine disclosure channels that players prefer. Players will prefer the disclosure channel maximizing their ex ante expected payoffs.

In the private disclosure model, the CEO cannot reveal his competency in the market because the CEO labor market cannot observe information that the CEO discloses, while he can gain higher reputation through disclosure in the public disclosure model. Therefore, in the public disclosure model, higher expected payoff can be obtained through the reputation gain. Due to this, if the CEO wants to obtain higher expected payoff in the private disclosure model than in the public disclosure model, the expected payoff increased by disclosing more information to the investor should be greater than the one increased by the reputation effect. Lets look at the disclosure channel that the CEO prefers for each parameter in below Propositions. First, let us look at the case where the reputation effect Δ_0 is large.

Proposition 22 *Suppose that $\Delta_0 > \frac{1}{4}$ or $b \geq \frac{1}{2}$. Then the CEO prefers the public disclosure channel.*

The expected payoff due to the reputation effect is always high in the public disclosure model because he can enhance his reputation through disclosing information in this model. And when $b < \frac{1}{4}$, from Proposition 20, more information can be disclosed in the public disclosure model. Thus, when $b < \frac{1}{4}$, we can clearly see that the CEO prefers the public disclosure model. Meanwhile, when $\{\min \Delta_0 - \frac{1}{4}, \frac{1}{4}\} \leq b < \frac{1}{2}$, more information is disclosed in the private disclosure model. However, even in this case, the effect of the increased amount of information on the CEO's expected payoff in the private disclosure model is not significant, because the difference in the amount of information disclosed in the two models is not large. Therefore, for parameter values given in the proposition, the CEO has higher expected payoff in the public disclosure model. That is, from Proposition 22, we can see that when the reputation effect is large, the CEO prefers the public disclosure channel that can enhance his reputation through disclosure.

Now let us look at the case where the reputation effect is small.

Proposition 23 *Suppose that $\Delta_0 \leq \frac{1}{4}$ and $b < \frac{1}{2}$. Then, when Δ_0 approaches 0, or b approaches $\frac{1}{2}$. That is, when $\Delta_0 \to 0$ or $b \to \frac{1}{2}$, the CEO prefers the public disclosure channel.*

As the reputation effect gets smaller, the CEOs payoff is affected by two aspects. First, similar to Proposition 22, the expected payoff due to the reputation is always higher in the public disclosure model because the CEO can enhance his reputation through disclosure in the public disclosure model. But, as the reputation effect gets smaller, the difference of expected payoffs in the two models becomes smaller. Second, the expected payoff due to the amount of information appears higher in the private disclosure model because the amount of information disclosed is greater in the private disclosure model. However, as the reputation effect gets smaller or b larger, the amount of information disclosed becomes similar in the two models, making the difference of expected payoffs negligible. But as the reputation effect gets smaller or larger, we have a higher expected payoff in the public disclosure model than in the private disclosure model because the first effect is always greater than the second effect. Thus, the CEO prefers the public disclosure channel for parameter values given in Proposition 23.

Proposition 24 *Suppose that $\Delta_0 \leq \frac{1}{4}$ and $b < \frac{1}{2}$. Then, when Δ_0 approaches $\frac{1}{4}$, and b approaches 0. That is when $\Delta_0 \to \frac{1}{4}$ and $b \to 0$, the CEO prefers the private disclosure channel.*

For parameter values given in Proposition 24, when Δ_0 approaches $\frac{1}{4}$, and b approaches 0, from Corollary 15, the amount of information disclosed in the public disclosure model is finite. On the other hand, the amount of information disclosed in the private disclosure model increases infinitely. Therefore, the CEO prefers the private disclosure channel through which he can disclose more information because in the situation where the future benefit is small, the difference of the expected payoffs due to the amount of information in the two models is large. The reason the amount of information does not increase like this although b approaches 0 in the public disclosure channel is that it should be limited in order to make the CEO who observed the optimal investment level $t = \frac{1}{2} - b$ prefer disclosing information over hiding it.

Proposition 25 *The investor prefers the disclosure channel through which more information is disclosed.*

The investor emphasizes the amount of information disclosed because unlike the CEO, she does not care about the reputation and only cares about the optimal investment level. Thus, the investor prefers the disclosure channel through which more information is disclosed.

Corollary 26 *Suppose that $\Delta_0 > \frac{1}{4}$ and $b < \{\min \Delta_0 - \frac{1}{4}, \frac{1}{4}\}$. Then, the investor prefers the public disclosure channel.*

When parameter values are given as Corollary 26, from Proposition 20, we can see that more information is disclosed in the public disclosure model, and from this, that the investor prefers the public disclosure model. In other words, when the reputation effect is large, and the conflict between the CEO and the investor is small, the investor prefers the public disclosure channel.

Corollary 27 *Suppose that* $\Delta_0 \leq \frac{1}{4}$ *or* $\{\min \Delta_0 - \frac{1}{4}, \frac{1}{4}\} \leq b < \frac{1}{2}$. *Then, the investor prefers the private disclosure channel.*

Meanwhile, when the parameter values are given as Corollary4 27, from Proposition 21, we can see that more information is disclosed in the private disclosure model and from this, that the investor prefers the private disclosure channel. That is, when the reputation effect is small or the conflict between the CEO and the investor is large, the investor prefers the private disclosure channel.

Corollary 28 *The disclosure channel that the CEO prefers does not always coincide with the one that the investor prefers.*

From Proposition 22 to **??**, and Corollary 26 and 27, the channel preferred by the CEO is different from the one preferred by the investor depending on the parameter values. That is, the disclosure channel preferred by the CEO does not always coincide with the one preferred by the investor. This is not consistent with the Ferreira and Rezende(2007)s result suggesting that the disclosure channel preferred by the CEO coincides with the one preferred by the investor.

5 Conclusion and Discussion

In this paper, we analyze firms voluntary disclosure under the assumption that information that the CEO discloses is non-verifiable and in the situation where the reputation effect exists by presuming the representative investor and the CEO labor market as information receivers. Specifically, we analyze the amount of information disclosed and the disclosure channels preferred by players by comparing the private disclosure channel with the public disclosure channel. The difference between the private disclosure model and the public disclosure model is whether the CEO can communicate by selectively separating information receivers. In the private disclosure model, the CEO can disclose information about the optimal investment level only to the investor, while in the public disclosure model, not only the investor but also the CEO labor market can observe the information disclosed by the CEO.

According to the analysis results, when the reputation effect is large, more information is disclosed through the public disclosure channel while more information is disclosed through the private disclosure channel when the reputation effect is small. Meanwhile, in terms of the disclosure channels preferred by players, inconsistency exists between two players. First, when the reputation effect is large, the CEO prefers the public disclosure channel. And when the reputation effect is small, for given reputation effect, as the conflict between the CEO and the investor decreases or for given conflict between the CEO and the investor as the reputation effect decreases, the private disclosure channel is preferred. On the contrary, when the conflict between the CEO and the investor decreases, and the reputation effect increases, the CEO prefers the private disclosure channel.

On the other hand, the investor prefers a disclosure channel where more information is disclosed. Specifically, when the reputation effect is large, the public disclosure channel is preferred while when it is small, the private disclosure channel is preferred. Therefore, the disclosure channels preferred by the two players do not always coincide.

Several models not discussed in this paper can be considered as future study topics, such as a model in which the dual disclosure channel is allowed where both the public disclosure channel and the private disclosure channel can be used and a model in which the CEO can directly choose a disclosure channel endogenously. Additionally, a model in which the CEO knows what his type is can also be considered as a future study topic.

A Appendix

Proof for Lemma 2: The first-order condition is

$$\frac{d}{da}\int_{T(m_{pr})} -|a-t|\mu(t|m_{pr})dt = 0. \tag{24}$$

Rearrangement of the integrand gives

$$\frac{d}{da}[\int_{t_{i-1}}^{a} -|a-t|\mu(t|m_{pr})dt + \int_{a}^{t_i}(a-t)\mu(t|m_{pr})dt] = 0. \tag{25}$$

or

$$\frac{d}{da}[-aF(a|m_{pr}) + \int_{t_{i-1}}^{a} t\mu(t|m_{pr})dt + a(1-F(a+m_{pr})) + \int_{a}^{t_i} t\mu(t|m_{pr})dt] = 0 \tag{26}$$

where $F(t|m_{pr})$ is cumulative posterior probability given message m_{pr}. Differentiation gives

$$[-F(a|m_{pr}) - a\mu(t|m_{pr}) + a\mu(t|m_{pr}) + 1 - F(a|m_{pr}) + a(1-\mu(t|m_{pr})) + a(1-\mu(t|m_{pr}))] = 0 \quad (27)$$

or

$$1 - 2F(a|m_{pr}) = 0 \quad (28)$$

and the solution is

$$F(a|m_{pr}) = \frac{1}{2} \quad (29)$$

or the median of the posterior distribution. ∎

Proof for Proposition 10: From (14), we must have:

$$t_{i+1} = 2t_i - t_{i-1} + 4b, i = 2, 3, \ldots, N_{pr}. \quad (30)$$

That is, all upper intervals are of longer length of 4b than all lower intervals. Therefore:

$$t_i = \frac{N_{pr} - i}{N_{pr} - 1} t_1 - 2b(i-1)(N_{pr} - i) + \frac{i-1}{N_{pr} - 1}, i = 2, 3, \ldots, N_{pr}. \quad (31)$$

And from (13), we have the following.

$$t_i = \frac{1}{4p_e}(-3 + 3p_e - 4bp_e + p_e t_2 + \sqrt{(8(p_e-1)p_e(-1+4b-t_2) + (3 + (4b-3)p_e - p_e t_2)^2)}). \quad (32)$$

From (31), we know that

$$t_2 = \frac{1 - 2b(N_{pr}^2 - 3N_{pr} + 2) + (N_{pr} - 2)t_1}{N_{pr} - 1}. \quad (33)$$

Substitute (33) to (32), then solve it for t_1, then we obtain the following.

$$t_1 = \frac{2((b+1)p_e - 1)N_{pr} - 2bp_e N_{pr}^2 + 1 + \sqrt{x}}{2p_e N_{pr}}. \quad (34)$$

Next, the investors expected payoff is the following.

$$\begin{aligned} U_{pr} &= \int_0^{t_1} -|\frac{p_e t_1^2 + 1 - p_e}{2(p_e t_1 + 1 - p_e)} - t|dt + \sum_{i=2}^{N_{pr}} \int_{t_{i-1}}^{t_i} -|\frac{t_{i-1} - t_i}{2} - t|dt \\ &= -\frac{(1-p_e)(1-t_1)t_1}{2(1-p_e(1-t_1))} - \frac{4b^2 N_{pr}(N_{pr}-1)^2(N_{pr}-2) + 3(1-t_1)^2}{12(N_{pr}-1)}. \end{aligned}$$

And the CEOs expected payoff is the following.

$$V_{pr} = \int_0^{t_1} -|\frac{p_e t_1^2 + 1 - p_e}{2(p_e t_1 + 1 - p_e)} - t - b|dt + \sum_{i=2}^{N_{pr}} \int_{t_{i-1}}^{t_i} -|\frac{t_{i-1} - t_i}{2} - t - b|dt + R\pi$$
$$= -\frac{2b(1 - p_e(1 - t_1)) - p_e t_1^2 - (1 - p_e))^2}{4(1 - p_e(1 - t_1))^2} - \frac{4b^2(N_{pr}^2 - 2N_{pr} + 3)(N_{pr} - 1)^2 + 3(1 - t_1)^2}{12(N_{pr} - 1)} + R\pi.$$

∎

Proof for Corollary 11: From (iii) in Proposition 10, for (ϕ, N_{pr})-partial disclosure equilibrium to exist, the following condition must be satisfied.

$$a^*(\phi) < t_1 + b.$$

Substituting $a^*(\phi)$ and t_1 in Proposition 10 and rearranging this inequality for N_{pr} gives us the following.

$$N_{pr}(b, p_e) = \lceil \frac{1 + bp_e - \sqrt{1 - p_e + b^2 p_e^2}}{2bp_e} \rceil.$$

∎

Proof for Proposition 12: From (19), we must have:

$$t_{i+1} = 2t_i - t_{i-1} + 4b, i = 1, 2, \ldots, N_{pu}. \tag{35}$$

That is, all upper intervals are of longer length of 4b than all lower intervals. Therefore:

$$t_i = \frac{i}{N_{pr}} - 2bi(N_{pu} - i), i = 1, 2, \ldots, N_{pu}. \tag{36}$$

And from (20), the following condition is necessary and sufficient for informative N_{pu}-partial disclosure equilibrium to exist:

$$-|\frac{t_{N_{pu}-1} + t_{N_{pu}}}{2} - (\frac{1}{2} - b + b)| + \Delta_0 > 0 for N_{pu} = 2.$$

That is, the CEO that observes $t = \frac{1}{2} - b$ prefers disclosing that information to hiding. Using 36, this condition become:

$$b < \Delta_o - \frac{1}{4}.$$

$\blacksquare$

Proof for Proposition 14: From (23), we must have:

$$t_{i+1} = 2t_i - t_{i-1} + 4b, i = 2, 3, \ldots, N_{pu}. \tag{37}$$

That is, all upper intervals are of longer length of 4b than all lower intervals. Therefore:

$$t_i = \frac{N_{pu} - i}{N_{pu} - 1} t_1 - 2b(i-1)(N_{pu} - i) + \frac{i-1}{N_{pu} - 1}, i = 2, 3, \ldots, N_{pu}. \tag{38}$$

And from (23), we have the following.

$$t_i = \frac{1}{4p_e}(-3 + 3p_e - 4bp_e - 2\Delta p_e + p_e t_2 + \sqrt{(8(p_e - 1)p_e(-1 + 4b + 2\Delta - t_2) + (3 + (4b + 2\Delta - 3)p_e - p_e t_2)^2)}). \tag{39}$$

From (38), we know that

$$t_2 = \frac{1 - 2b(N_{pu}^2 - 3N_{pu} + 2) + (N_{pu} - 2)t_1}{N_{pu} - 1}. \tag{40}$$

Substitute (40) to (39), then solve it for t_1, then we obtain the following.

$$t_1 = \frac{2((b+1)p_e - 1)N_{pu} - 2bp_e N_{pu}^2 + 1 + \sqrt{y}}{2p_e N_{pu}}. \tag{41}$$

where $y = 4p_e(1-p_e)(2\Delta(N_{pu}-1)+(2b(N_{pu}-1)-1))N_{pu}+(2((b-\Delta+1)p_e-1)N_{pu}-2bp_eN_{pu}^2+2\Delta p_e+1)^2$.

Next, the investors expected payoff is the following.

$$\begin{aligned} U_{pu} &= \int_0^{t_1} -|\frac{p_e t_1^2 + 1 - p_e}{2(p_e t_1 + 1 - p_e)} - t|dt + \sum_{i=2}^{N_{pr}} \int_{t_{i-1}}^{t_i} -|\frac{t_{i-1} - t_i}{2} - t|dt \\ &= -\frac{(1-p_e)(1-t_1)t_1}{2(1 - p_e(1-t_1))} - \frac{4b^2 N_{pr}(N_{pr}-1)^2(N_{pr}-2) + 3(1-t_1)^2}{12(N_{pr}-1)}. \end{aligned}$$

And the CEOs expected payoff is the following.

$$\begin{aligned} V_{pr} &= \int_0^{t_1} -|\frac{p_e t_1^2 + 1 - p_e}{2(p_e t_1 + 1 - p_e)} - t - b|dt + \sum_{i=2}^{N_{pr}} \int_{t_{i-1}}^{t_i} -|\frac{t_{i-1} - t_i}{2} - t - b|dt + R\pi \\ &= -\frac{2b(1 - p_e(1-t_1)) - p_e t_1^2 - (1-p_e))^2}{4(1 - p_e(1-t_1))^2} - \frac{4b^2(N_{pr}^2 - 2N_{pr} + 3)(N_{pr}-1)^2 + 3(1-t_1)^2}{12(N_{pr}-1)} \\ &+ R\pi[\frac{\bar{p}(1 - p_e + p_e t_1)t_1 + p_e(1-\bar{p})(1-t_1)}{p_e(1 - p_e + p_e t_1)}. \end{aligned}$$

$\blacksquare$