Central Bank Digital Currency, Tax Evasion, and Central Bank Independence

Ohik Kwon ¹  Seungduck Lee ²  Jaevin Park ³

¹ Bank of Korea
² Sungkyunkwan University
³ University of Mississippi

Department of Economics
Sungkyunkwan University
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Recently, Central Bank Digital Currency (CBDC) has inspired extensive research among central banks and international organizations.

- In particular, Swiss National Bank, Bank of Canada and People’s Bank of China have been actively exploring the possibility of issuing CBDC and launched some pilot programs.

Our understanding about CBDC is still limited despite of its relevance to economic activities and macroeconomic policies.

- Existing papers have so far provided limited insight on it.
- There are still many questions that need to be addressed in the perspectives of i) monetary policy, ii) financial stability, and iii) payment and settlement.
CBDC AS DIGITAL CURRENCY

CBDC, like other digital currencies, necessarily has electronic ledgers (and its record keeper), where the currency status and changes of ownership are electronically recorded.

1. A CBDC digital ledger can be shared with the fiscal authority for tax collection if necessary.

2. CBDC can bear positive or negative interest.

3. Paying CBDC implies that Payment = Clearing = Settlement.

<table>
<thead>
<tr>
<th>Central Bank B/S</th>
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<tbody>
<tr>
<td>Assets</td>
</tr>
<tr>
<td>Reserves</td>
</tr>
<tr>
<td>Cash in circulation</td>
</tr>
<tr>
<td>CBDC</td>
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<tr>
<td>Other liability</td>
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<td></td>
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CBDC for retail

Source: BIS (2017)
This paper aims to examine how CBDC can affect welfare in an economy in which tax evasion exists in cash transactions. We answer the following questions by constructing a parsimonious monetary search model with cash and CBDC as a medium of exchange (MOE):

- What is the condition for the coexistence of cash and CBDC in equilibrium?
- How does monetary policy affect economic activities such as trade volume by MOE type?
- How does CBDC affect welfare? What is the optimal strategy of monetary policy?
- How does the central bank independence affect this effect of CBDC?
MAIN RESULTS

- CBDC is endogenously used as a MOE at least when its real rate of return is greater than that of cash.

- It depends on **central bank independence** whether introducing CBDC is welfare-improving.

  - When the central bank is **not independent**, CBDC does **not necessarily help to achieve higher welfare**. Even in an economy with only cash, efficient consumption allocations can be achieved by collecting inflation tax for government spending.

  - When the central bank is **independent**, CBDC **improves welfare**. CBDC with a positive interest can mitigate a distortion in the relative MU between tax-payed and tax-evaded consumption that sales tax can generate.

- Removing cash does not necessarily expand the feasible allocation set.

Dual Currency and Seigniorage: Zhang (2014)


CONTENTS

1 Model

2 Stationary Equilibria

3 Welfare Comparison

4 Concluding Remarks
MODEL ENVIRONMENT

- Infinite horizon model \((t = 0, 1, \ldots \infty)\) with two sub-periods in each period: Centralized Market (CM) and Decentralized Market (DM)

- Population: Buyers and Sellers (unit mass)

- Preferences
  - Buyers: \(E_0 \sum_{t=0}^{\infty} \beta^t [u(x_t) - H_t]\)
  - Sellers: \(E_0 \sum_{t=0}^{\infty} \beta^t [-h_t + X_t]\), where \(x_t\) is consumption of DM goods and \(H_t\) is the labor supply in the CM production. \(h_t\) is the labor supply in DM production and \(X_t\) is consumption of CM goods.

- Production Technology: one unit of labor produces one unit of perishable consumption goods
MODEL ENVIRONMENT

Centralized Market (CM)

- A competitive Walrasian market

- Agents adjust their portfolios of cash and CBDC among all agents at their real prices of $\phi_t$ and $\psi_t$ in terms of CM goods in period $t$, respectively

- The central bank injects (withdraws) money through lump-sum transfers to (taxes from) agents and the government

- Government provides public goods such as national defense and social infrastructure
Decentralized Market (DM)

- Because of anonymity and limited commitment, a MOE is necessary for a trade to occur
  - Cash and CBDC serve as MOE
- Each buyer makes a take-it-or-leave-it offer to the counterpart to determine terms of trade in a pairwise meeting
- A fraction $\rho$ of meetings are not monitored, but the rest $1 - \rho$ are monitored by the fiscal authority
  - In monitored meetings, the fiscal authority imposes a proportional sales tax ($\tau$), regardless of MOE type
  - In non-monitored meetings, it depends on MOE type. Only when buyers use CBDC as a MOE, the fiscal authority can use its record-keeping technology to impose the sales tax
Central Bank supplies two types of currencies: cash \((C_t)\) and CBDC \((D_t)\)

\[C_{t+1} = \mu_c C_t \quad \text{and} \quad D_{t+1} = \mu_d D_t\]

\[\mu_i \geq \beta \quad \text{for} \quad \{c, d\} \quad \text{for existence of monetary equilibria}\]

The central bank budget constraints are given by

\[T_0 = S_0 = \phi_0 C_0 + \psi_0 D_0,\]
\[T_t \leq S_t = \phi_t (C_t - C_{t-1}) + \psi_t (D_t - D_{t-1}),\]

where \(S_t\) is seigniorage revenue and \(T_t\) is a transfer to Government.

Government finances its expenditure \((G_t)\) through the sales tax \((\tau)\) and a transfer from the central bank \((T_t)\),

\[G_t = \rho \tau x_t^n \mathbb{I}_{\{d_t^n > 0\}} + (1 - \rho) \tau x_t^m + T_t\]
Figure: Market Timing
MAXIMIZATION PROBLEM

In non-monitored meetings,

$$\max_{x_t^n, c_t^n, d_t^n} u(x_t^n) - c_t^n - (1 + \tau)d_t^n,$$

subject to the seller’s participation constraint given by

$$\frac{\beta \phi_{t+1}}{\phi_t} c_t^n + \frac{\beta \psi_{t+1}}{\psi_t} d_t^n - x_t^n \geq 0,$$

$$x_t^n \geq 0, c_t^n \geq 0 \text{ and } d_t^n \geq 0$$
In monitored meetings,

$$\max_{x_t^m, c_t^m, d_t^m} u(x_t^m) - (1 + \tau)c_t^m - (1 + \tau)d_t^m,$$

subject to the seller’s participation constraint given by

$$\frac{\beta \phi_{t+1}}{\phi_t} c_t^m + \frac{\beta \psi_{t+1}}{\psi_t} d_t^m - x_t^m \geq 0,$$

$$x_t^m \geq 0, c_t^m \geq 0 \text{ and } d_t^m \geq 0$$
SOLUTIONS TO BUYER’S PROBLEM

Lemma

Given the rates of return on cash and CBDC, \( \frac{\phi_{t+1}}{\phi_t} \) and \( \frac{\psi_{t+1}}{\psi_t} \), and the sales tax rate, \( \tau \), in non-monitored meetings,

i) If \( \frac{1}{(1+\tau)} \frac{\psi_{t+1}}{\psi_t} < \frac{\phi_{t+1}}{\phi_t} \), then \( x^n_t = f \left( \frac{1}{\beta} \frac{\phi_t}{\phi_{t+1}} \right) \), \( c^n_t = \frac{1}{\beta} \frac{\phi_t}{\phi_{t+1}} f \left( \frac{1}{\beta} \frac{\phi_t}{\phi_{t+1}} \right) \), and \( d^n_t = 0 \);

ii) If \( \frac{1}{(1+\tau)} \frac{\psi_{t+1}}{\psi_t} > \frac{\phi_{t+1}}{\phi_t} \), then \( x^n_t = f \left( \frac{(1+\tau)}{\beta} \frac{\psi_t}{\psi_{t+1}} \right) \), \( c^n_t = 0 \), and \( d^n_t = \frac{1}{\beta} \frac{\psi_t}{\psi_{t+1}} f \left( \frac{(1+\tau)}{\beta} \frac{\psi_t}{\psi_{t+1}} \right) \)

Here, \( f(\cdot) \equiv u'^{-1}(\cdot) \). Next, in monitored meetings,

i) If \( \frac{\psi_{t+1}}{\psi_t} < \frac{\phi_{t+1}}{\phi_t} \), then \( x^m_t = f \left( \frac{(1+\tau)}{\beta} \frac{\phi_t}{\phi_{t+1}} \right) \), \( c^m_t = \frac{1}{\beta} \frac{\phi_t}{\phi_{t+1}} f \left( \frac{(1+\tau)}{\beta} \frac{\phi_t}{\phi_{t+1}} \right) \), and \( d^m_t = 0 \);

ii) If \( \frac{\psi_{t+1}}{\psi_t} > \frac{\phi_{t+1}}{\phi_t} \), then \( x^m_t = f \left( \frac{(1+\tau)}{\beta} \frac{\psi_t}{\psi_{t+1}} \right) \), \( c^m_t = 0 \), and \( d^m_t = \frac{1}{\beta} \frac{\psi_t}{\psi_{t+1}} f \left( \frac{(1+\tau)}{\beta} \frac{\psi_t}{\psi_{t+1}} \right) \)
We focus on a steady state equilibrium where all real quantities are constant: $\phi_t C_t = \phi_{t+1} C_{t+1}$ and $\psi_t D_t = \psi_{t+1} D_{t+1}$

1. The decision rules of a representative agent optimally solves her/his individual maximization problem, taking the prices, the gov’t spending, and the sales tax rate as given

2. Markets clear

$$\rho c^n + (1 - \rho)(1 + \tau)c^m = \phi_t C_t,$$
$$\rho(1 + \tau)d^n + (1 - \rho)(1 + \tau)d^m = \psi_t D_t,$$

for all $t \geq 1$
3 The government and central bank budget constraints hold as follows

\[ G = \rho \tau \frac{\beta}{\mu_d} d^n + (1 - \rho) \tau \left( \frac{\beta}{\mu_c} c^m + \frac{\beta}{\mu_d} d^m \right) + T, \]

\[ T \leq S = \left( 1 - \frac{1}{\mu_c} \right) \left[ \rho c^n + (1 - \rho)(1 + \tau) c^m \right] \]

\[ + \left( 1 - \frac{1}{\mu_d} \right) \left[ \rho (1 + \tau) d^n + (1 - \rho)(1 + \tau) d^m \right] \]
Corollary

*Cash and CBDC coexist in equilibrium when* $\mu_d \leq \mu_c \leq (1 + \tau)\mu_d$.

- When $\tau > 0$, there exist five different types of equilibria which are feasible for monetary policy combinations of $(\mu_c, \mu_d)$

<table>
<thead>
<tr>
<th>Non-monitored</th>
<th>Monitored</th>
<th>Cash only $(\mu_c &lt; \mu_d)$</th>
<th>CBDC only $(\mu_c &gt; \mu_d)$</th>
<th>Both $(\mu_c = \mu_d)$</th>
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<tbody>
<tr>
<td>Cash only $(\mu_c &lt; \mu_d(1 + \tau))$</td>
<td>O (P)</td>
<td>O (S)</td>
<td>O (PP)</td>
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</tr>
<tr>
<td>CBDC only $(\mu_c &gt; \mu_d(1 + \tau))$</td>
<td>X</td>
<td>O (P)</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Both $(\mu_c = \mu_d(1 + \tau))$</td>
<td>X</td>
<td>O (PP)</td>
<td>X</td>
<td></td>
</tr>
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Notes: P, PP and S represent Pooling, Partially Pooling and Separating, respectively.
Sales Tax Revenue

\[ T^s \equiv \rho \tau f\left(\frac{\mu_d(1 + \tau)}{\beta}\right) + (1 - \rho)\tau f\left(\frac{\mu_j(1 + \tau)}{\beta}\right) \]

for \( j \in \{c, d\} \).

Lemma (Laffer curve for sales tax collection)

Given \( \mu_j \) for \( j \in \{c, d\} \), \( \frac{\partial T^s}{\partial \tau} > 0 \) holds in \( \tau \in [0, \bar{\tau}) \).
Inflation Tax Revenue

\[ S = \rho \left( \frac{\mu_i - 1}{\beta} \right) f \left( \frac{\mu_i}{\beta} \right) + (1 - \rho) \left( \frac{\mu_j - 1}{\beta} \right) (1 + \tau) f \left( \frac{\mu_j (1 + \tau)}{\beta} \right) \geq 0 \]

for \( i, j \in \{c, d\} \).

Lemma

Given \( \tau \) and \( \mu_j \), \( \frac{\partial S}{\partial \mu_i} > 0 \) holds in \( \mu_i \in [\beta, \bar{\mu}_i) \). Similarly, given \( \tau \) and \( \mu_i \), \( \frac{\partial S}{\partial \mu_j} > 0 \) holds in \( \mu_j \in [\beta, \bar{\mu}_j) \).
WELFARE COMPARISON

- Welfare is defined as
  \[ W = \rho \{ u(x^n) - x^n \} + (1 - \rho) \{ u(x^m) - x^m \} \]

- We define \( \zeta \equiv \frac{\mu_d}{\mu_c} \). Then, \( \frac{1}{\zeta} - 1 \) can be interpreted as an interest rate on CBDC

- The central bank independence is defined as \( T \leq 0 \). Given \( G > 0 \), since it is not efficient to collect the sales tax to make a transfer from the fiscal authority to the central bank, the transfer is zero, i.e. \( T = 0 \), in equilibrium
Without the Central Bank Independence

The consolidated government budget constrain is

\[
G = \rho \left( u'(x^n) - \frac{1}{\beta} \right) x^n \\
+ (1 - \rho) \left( u'(x^m) - \frac{1}{\beta} \right) x^m - (1 - \rho) \frac{1}{\beta} \tau x^m + (1 - \rho) \tau x^m,
\]

where \( \tau = \frac{u'(x^m)}{\zeta u'(x^n)} - 1 \) by \( u'(x^m) = \frac{\mu_c(1+\tau)}{\beta} \) and \( u'(x^n) = \frac{\mu_c}{\beta} \).
In the economy without CBI

Proposition

1. *In a cash-only economy without the CBI, the optimal policy mix is* \( \tau = 0 \) and \( \mu_c = \hat{\mu}_c > 1 \), where \( \hat{\mu}_c \) solves \( G = \left( \frac{\hat{\mu}_c - 1}{\beta} \right) f \left( \frac{\hat{\mu}_c}{\beta} \right) \).

2. *In a cash and CBDC economy without CBI, the optimal policy mix is* \( \tau = 0 \), \( \zeta = 1 \) and \( \mu_c = \mu_d = \hat{\mu}_c > 1 \).

3. *In a CBDC-only economy without CBI, the optimal policy mix is* \( \tau = 0 \), \( \zeta = 1 \) and \( \mu_d = \hat{\mu}_c > 1 \).
WELFARE COMPARISON

Two types of inefficiency: distortion in the relative MU in transactions with and without tax evasion, and tax revenues losses

The optimal policy mix, $\tau = 0$ and $\mu_c = \hat{\mu}_c > 1$, is consistent with the Ramsey’s principle of optimal taxation

At $B$, $\mu_d = \mu_c < \mu_d(1 + \tau) \Rightarrow$ At $A$, $\mu_d = \mu_c = \hat{\mu}_c$ and $\tau = 0$
WELFARE COMPARISON

With Central Bank Independence

- The gov’t and central bank budget constrains are

\[ G = T^s = (1 - \rho)\tau f \left( \frac{\zeta \mu_c (1 + \tau)}{\beta} \right), \]

\[ S = \rho \left( \frac{\mu_c - 1}{\beta} \right) f \left( \frac{\mu_c}{\beta} \right) + (1 - \rho) \left( \frac{\zeta \mu_c - 1}{\beta} \right) (1 + \tau) f \left( \frac{\zeta \mu_c (1 + \tau)}{\beta} \right) \]

- The non-negative seigniorage condition is given by

\[ \frac{G}{\beta} = \rho \{ u'(x^n) - \frac{1}{\beta} \} x^n + (1 - \rho) x^m \{ u'(x^m) - \frac{1}{\beta} \} x^m \]
In the economy with CBI

Proposition

4. In a cash-only economy with CBI, the optimal monetary and fiscal policy mix is $\mu_c = 1$, and $\tau = \hat{\tau} > 0$, where $\hat{\tau}$ solves 
$$G = (1 - \rho)\hat{\tau} f\left(\frac{1+\hat{\tau}}{\beta}\right).$$

5. In a cash and CBDC economy with CBI, if $G$ and $\rho$ is not so large, the optimal policy mix is $\zeta = \frac{1}{1+\tau^*}$, $\mu_c = \mu_c^* > 1$, and $\tau = \tau^* > 0$, where $\mu_c^*$ solves $G = (\mu_c^* - 1) f\left(\frac{\mu_c^*}{\beta}\right)$ and $\tau^*$ solves 
$$G = (1 - \rho)\tau^* f\left(\frac{\mu_c^*}{\beta}\right).$$

6. In a CBDC-only economy with CBI, the optimal policy is $\tau = \tau^\# < \tau^*$, $\zeta = 1$ and $\mu_d = 1 < \mu_c^*$, where $\tau^\#$ solves 
$$G = \tau^\# f\left(\frac{1+\tau^\#}{\beta}\right).$$
Paying a strictly positive interest on CBDC can move the equilibrium allocation from $E$ to $F$ at which welfare is higher.

At $E$, $\mu_d = \mu_c < \mu_d(1 + \tau)$ \Rightarrow At $F$, $\mu_d < \mu_c = \mu_d(1 + \tau)$ (Partially Pooling)
CONCLUDING REMARKS

- When the central bank is not independent, CBDC does not necessarily improve welfare more than cash does.

- When the central bank is independent, CBDC improves welfare.
  - CBDC can mitigate the inefficiency that the sales tax can generate: misalignment in the marginal rate of substitution in transactions with and without tax evasion.

- Tax evasion in the economy with CBI can rationalize the introduction of CBDC with a positive interest rate for higher welfare.

- Eliminating cash does not necessarily improve welfare further, compared with the cash and CBDC economy.