Institutional Changes and CEO Pay Inequality

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Abstract

Inequality within top incomes in the U.S. had been continuously increasing until the Great Recession since the 1980s, and then showed a modest decline in recent years after the recession. Inequality among the top executives, who form the largest group at the top of the income distribution, has shown a similar trend in their pay distribution while the inequality in the firm size distribution has been relatively constant (Zipf's law). This is puzzling if CEO pays are assumed to be tied to the firm size. Using Compustat and Execucomp data since 1992, we first document the puzzle by showing changes in CEO pay distribution and its relationship to firm size distribution, revisiting Gabaix and Landier (2008) and Roberts' law. We then propose a theoretical model to show that institutional environments surrounding CEO compensation can affect CEO pay inequality, while the firm size distribution remains stable. In particular, our theoretical model predicts that the CEO pay distribution becomes more equal when it becomes more difficult to raise a CEO pay in the pay bargaining. This result is consistent with the recent decline in inequality following institutional changes to limit executive compensations after the Great Recession.

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1. Introduction

Rising income inequality has been one of the much-debated issues in many developed countries, and the top 1% income group hsde. In particular, Piketty and Saez (2003, 2013), Kaplan and Rauh (2008, 2013), Bakija, Cole, Heim et al. (2012), and Kaplan (2012) document that top 1% are taking a greater share of income than ever before. It would be misleading, however, to think that compensations of all the top 1% earners have increased proportionally. In the United States, while the top 1% income share[1] has increased from 7.8% in 1970 to 18.39% in 2015, the top 0.1% income share has grown more rapidly from 1.94% to 7.86% over the same period. (World Inequality Database, 2016) Ironically, while the top 1% has become better off over time as a group, a majority of the top 1% themselves may not feel much richer than before if they compare themselves to their upper-income neighbors.

Taking a closer look at the composition of occupations in the top 1% income group, Bakija, Cole, Heim et al. (2012)[2] show that executives, managers, and supervisors who work outside of finance form the biggest group both in the top 1% and in the top 0.1%, accounting for about a third of the top 1% and more than 40% of the top 0.1% in the U.S. since 1979. This suggests that studying executive compensations can be helpful to understand the top income dynamics in the U.S. This paper focuses on executive compensations to explain rising income inequality at the top of the income distribution.

During the last three decades before the global financial crisis, executives had experienced a dramatic increase in their top income inequality. According to Bakija, Cole, Heim et al. (2012), during 1979-2005, real income of executives at the top 0.1% grew seven times faster than that of executives in p99-99.5. For the other occupations, the real income growth rate for top 0.1% was 2.4 times of that for p99-99.5 on average. This indicates a great divergence in top executive compensation. The increased inequality in CEO compensation is puzzling because the firm size distribution has been quite stable following Zipf's law and a CEO compensation is proportional to its firm size. (Axtell (2001), Luttmer (2007), Gabaix and Landier (2008), Mizuno, Ohnishi, Watanabe et al. (2016) ,Gabaix, Landier and Sauvagnat (2014))

The model we present in this paper shows that the board of directors' weakened

[1] Income here is defined as gross income excluding capital gains and before individual taxes.

[2] The estimates in Bakija, Cole, Heim et al. (2012) are based on individual income tax return data.

(strengthened) bargaining power can be a channel through which the CEO pay distribution becomes more unequal (equal). Assuming a Pareto distribution for firm productivities, a firm and a CEO are matched and then they bargain over the CEO pay. In our model, firms lower their bargaining power if the bargaining costs of the board to control the bargaining power increase. This will raise CEO pays disproportionately more for high talent CEOs than low talent CEOs, which results in the fatter tail of the CEO pay distribution. Our result implies that institutional changes in compensation bargaining can explain why the CEO pay distribution has become more unequal or more equal while firm sizes exhibit the relatively stable Zipf's law.

2. Related Literature

As well documented in Devers, Cannella, Reilly, Yoder et al. (2007), Frydman and Jenter (2010), and Edmans and Gabaix (2015), there exist several sets of theories to model how a CEO compensation is determined. One set of studies (Boyd (1994), Core, Holthausen and Larcker (1999), Chhaochharia and Grinstein (2009)) explains that executive compensation is largely determined by managerial rent extraction behaviors. Some other studies take competitive assignment models in which CEOs and firms try to be matched optimally with a partner that gives them the greatest payoffs (Rosen (1982), Tervio (2008) and Gabaix and Landier (2008)). As noted by Frydman and Jenter (2010), while both rent-extraction and optimal contracting are important determinants of CEO compensation, neither provides an fully consistent explanation with emprical data. Our model takes a bargaining framework that can integrate both rent-extraction and competitive assignment views.

In the rent-extraction theories, the weaker the board's monitoring power is, the greater share of the profit a CEO receives for his or her compensation. A number of researchers have studied how the board structure or monitoring power and a CEO pay are related. According to Boyd (1994), the degree of board control has a dominating effect on CEO remuneration compared to other widely considered factors such as firm size and profitability. Moreover, by analyzing data for 205 firms in the U.S., Core, Holthausen and Larcker (1999) showed that CEO pay is an increasing function of board inefficiency. Chhaochharia and Grinstein (2009) directly showed that enhanced board monitoring negatively affects CEO pay by comparing executives' compensation before and after the enforcement of the Sarbanes-Oxley Act (SOX) and other regulations on the boards established in 2002.

[c] On the other hand, competitive assignment models view the CEO-firm match and CEO pay as a result of the optimal contract between a firm and a CEO. In these models, CEOs with different levels of talent and firms with heterogeneous productivity levels are matched with a partner who gives them the greatest utility. As a consequence, a CEO compensation is directly tied to the CEO's managerial ability and the matched firm's productivity. According to Rosen (1982), such choices result in assortative matching. Therefore, top CEOs face much greater compensation gap than their talent differences. This is also relevant to the 'Superstar' effect in **?**. Gabaix and Landier (2008) propose a calibratable assignment model in which a CEO pay increases with the CEO talent and firm size. They show that recent increases in the CEO pay level are fully explained by the increases in the aggregate firm size. Tervio (2008) also presents an assignment model that can infer the underlying CEO talent distribution and its economic impact. He shows that that the differences between firms rather than differences in managerial ability can explain much of the compensation gap between CEOs.

It is a well-documented regularity in the empirical literature on executive compensation that CEO compensation is proportional to a power function of firm size, $w \sim (\text{firm size})^{\phi}$. It is sometimes referred to as 'Roberts' law', and empirical estimates for ϕ, the cross-sectional elasticity of CEO pays, are around $1/3$. (Roberts (1956), Baker and Hall (1998), Frydman and Saks (2010), Gabaix and Landier (2008)) What's missing in the literature is the implication of the Roberts' law on the CEO pay distribution. The Roberts' law implies that the CEO pay distribution is directly tied to the firm size distribution. That is, if the Zipf's law holds for the firm size so that the firm size follow a Pareto distribution with the parameter 1, then according to the Roberts' law, the CEO pay distribution will follow a Pareto distribution with the parameter $1/\phi$, and more importantly, the CEO pay distribution should be stable over time. This contradicts the empirical evidence that CEO pay distribution has been more unequal since the 1980s. Therefore, it calls for empirical investigation of the time changes of κ in the Roberts' law and theoretical work to explain the changes in the relationship between a CEO pay and the firm size.

Our model is built on the CEO-firm assignment model of Gabaix and Landier (2008) where heterogeneous CEOs and firms are assortatively matched in the equilibrium. While Gabaix and Landier (2008) assume that a CEO pay is determined in the assortative matching equilibrium where firms maximize their profits, we further model CEO pay bargaining between a matched CEO and the firm (the board of directors, to be more precise). In this model, the board's endogenous choice of the bargaining power can be a channel through which pay inequality among top executives can vary over time while the firm size distribution remains stable. If the board's bargaining power weakens, this strengthens the 'superstar' effect so that the pay gap widens more than the managerial talent gap.

3. Changes in CEO Pay Distribution and Firm Size Distribution

(*Incomplete*. Figures and discussions to be filled in)

Using Compustat and Execucomp from 1994-2017, we show the following:

- Fact 1. During our data period, the average market value of the largest 1000 firms (debt plus equity) has increased (in real terms) by a factor of 3, and we also observe a threefold rise of CEO compensation. Although the magnitude is a bit different, the comovement of the firm size and the CEO pay is consistent with Gabaix and Landier (2008).
- Fact 2. However, the dynamics of firm size was not always in line with that of CEO pay.
- Fact 2-1: (The dynamics of firm size distribution) We confirm that Zipf's law is pretty robust if we look at the data annually. This implies that the firm size distribution has been fairly stable.
- Fact 2-2: (The dynamics of CEO pay distribution) We document that CEO pay distribution has become more equal after the Great Recession.
- Fact 2-3: (Roberts' Law) We document that the exponent in the Roberts' law declined after the Great Recession, which is implied by Fact 2-1 and 2-2.

4. A Theoretical Model of CEO-Firm Matching and Pay Bargaining

4.1. The Economic Environment

We consider an economy with the final output sector and intermediate goods sector. We assume that the final output sector is competitive and there is one representative final output firm. The intermediate goods sector is monopolistic and consists of a continuum of measure 1 intermediate goods firms with heterogeneous productivity levels. Each intermediate goods firm hires a CEO and workers. CEOs of measure 1 are heterogeneous in talents and are matched to intermediate goods firms in a competitive assignment setting with CEO pay bargaining.

The production function for the final output good is given by $Y = \left(\int_0^1 Y_i^{\rho} di\right)^{\frac{1}{\rho}}$, where the elasticity of substitution between intermediate goods Y_i is $1/(1-\rho) > 1$. Intermediate goods firms are sorted by their productivity levels so that an intermediate goods firm $i \in (0,1)$ has a productivity level A_i, which is the top $100i$th percentile productivity level. CEOs are also sorted by their talents so that a CEO rank $m \in (0,1)$ indicates that the CEO's talent $T(m)$ is the top $100m$th percentile. Note that highly productive firms have low i, and highly talented CEOs have low m. The production function of the intermediate goods firm i which hired a CEO m is then given by $Y_i(m) = A_i L_i(m)^{\alpha} T(m)$, where $L_i(m)$ is labor employment. The total amount of labor in this economy is L.

An intermediate goods firm i which hires a CEO m will solve the following profit optimization problem:

$$\max_{L_i(m)} p_i(m) Y_i(m) - w_L L_i(m) - w_m(i), [e] \tag{1}$$

where $p_i(m)$ is the price of the intermediate good i when the firm is matched with CEO m, w_L is the price of labor, and $w_m(i)$ is how much the firm i pays to CEO m.

4.2. The Pareto Distribution and Firm Size Distribution

We assume that firm sales revenues follow a Pareto distribution with the tail index ξ_s. Support for this assumption comes from the well-documented Zipf's law for firms, which says that firm size follows a Pareto distribution at the upper tail with the tail index close to 1. The Zipf's law holds true for different measures of firm size such as sales revenue, employment, and market capitalization (Axtell (2001), Luttmer (2007), Gabaix and Landier (2008), Di Giovanni and Levchenko (2013), Fujiwara, Di Guilmi, Aoyama, Gallegati and Souma (2004)).[3]

If X follows a Pareto distribution with index α, its cumulative distribution function $F(X)$ satisfies the following equation for $X(m)$, the top $100m$th percentile value of X:

$$X(m) = Km^{-\frac{1}{\alpha}},$$

for some $K > 0$.

As the firm's optimal choice of employment in (1) gives employment proportional to firm's sales revenue, Pareto tail assumption on firm's sales revenues make firm's employment has Pareto tail with index 1, satisfying the Zipf's law on employment.

4.3. The CEO-Firm Assignment with CEO Pay Bargaining

We now describe the CEO-firm assignment problem. As described in Gabaix and Landier (2008), an assignment equilibrium with pay bargaining consists of (1) a compensation function $W(m)$,[4] the pay of a rank m CEO which will be determined via asymmetric Nash bargaining and (2) an assignment function $\sigma(i)$ where $m = \sigma(i)$ is the rank of the CEO matched with firm i in equilibrium such that the CEO market clears.[5] We assume that a firm and a CEO with asymmetric bargaining power will bargain over the firm profit $\pi(i, m)$ (= sales revenue minus labor costs) given as follows.

$$\pi(i, m) = A_0(A_i T(m))^{\frac{\rho}{1-\alpha\rho}}, \text{ with } \quad A_0 = (1-\alpha\rho)Y^{\frac{1-\rho}{1-\alpha\rho}}\left(\frac{\alpha\rho}{w_L}\right)^{\frac{\alpha\rho}{1-\alpha\rho}}$$

[f]

[3] The Zipf's law is robust across different countries and over time. Di Giovanni and Levchenko (2013) estimated the Pareto tail index for firm sales in many countries to be close to 1. Fujiwara, Di Guilmi, Aoyama, Gallegati and Souma (2004) showed that the Pareto tail indices for total assets, sales, and the number of employees in Italy, Spain, France, and UK were all close to 1 and didn't change much from 1993 to 2001.

[4] Note that $W(m) = w_m(\sigma^{-1}(m))$.

[5] Rigorously speaking, this means that matching function σ is a measure-preserving transformation so that σ satisfies $\mu(\sigma^{-1}(A)) = \mu(A)$ for all μ-measurable subset A in $(0, 1)$.

Our CEO pay bargaining setup is similar to Piketty, Saez and Stantcheva (2014), where asymmetric Nash bargaining involves bargaining cost to attain some level of bargaining power. We assume that the board of director of a firm has a control over the bargaining power, thereby bearing the cost of bargaining to achieve the bargaining power λ $(0 < \lambda < 1)$. While it is the board that exercises the bargaining power over CEO compensation, we will use the term 'firm' and 'board' interchangeably. We then specify the board in the firm i's utility when it is matched with CEO m as

$$U_i(m,\lambda) = \frac{[\lambda\pi(i,m)]^{1-\gamma}}{1-\gamma} + \delta\frac{(1-\lambda)^{-\kappa+1}}{-\kappa+1} \quad \text{for some } \gamma < 1, \kappa > 1, \delta > 0, \tag{2}$$

where the first term is the utility comes from earning a share λ of the profit as a result of asymmetric Nash bargaining, and the second term is the cost to achieve the bargaining power λ where δ is the relative weight placed on the bargaining cost. Note that the board's utility function is convex in λ and marginal disutility of increasing λ increases in κ. We will later see how κ affects inequality among CEO pays. On the other hand, CEOs' utility depends only on their compensation $w_m(i) = (1-\lambda_i(m))\pi(i,m)$.

Given matched with CEO m, each firm sets their bargaining power $\lambda_i^*(m)$ which maximizes its utility (2). The first order condition from this optimization is given as follows.

$$\pi(i,m) = A_0(A_iT(m))^{\frac{\rho}{1-\alpha\rho}} = \left[\frac{\delta(\lambda_i^*(m))^{\gamma}}{(1-\lambda_i^*(m))^{\kappa}}\right]^{\frac{1}{1-\gamma}}. \tag{3}$$

[g][h]

4.4. The CEO Talent Distribution and The Firm Size Distribution

Regarding CEO talents, we follow Gabaix and Landier (2008) and extreme value theory to assume that $T'(m)$, the spacing of the CEO talent distribution $T(m)$, is given by the equation below for the high talent CEOs (m close to 0).

$$T'(m) = -Bm^{\beta-1}, \text{for some constant } B > 0, \beta \in \mathbb{R}.$$

This implies that $T(m)$ can be more explicitly written as

$$T(m) = T_{\max} - \frac{B}{\beta}m^{\beta} \text{ if } \beta > 0, \tag{4}$$

where $T_{\max}$ serves as the maximum talent. Also note that talent diverges to infinity as m goes to 0 if $\beta \leq 0$. We will focus on the case of $\beta > 0$ following the empirical estimate of $\beta = 2/3$ in Gabaix and Landier (2008).

Proposition 1. (The Firm Size Distribution): *Suppose the distribution of the firm productivity A_i has a Pareto tail with the index ξ_a, and the talent $T(m)$ is given by (4). Then, the distribution of the firm size, measured by the sales revenue, has a Pareto tail with the index $\xi_s = \xi_a(1 - \alpha\rho)$.*

Proof. (proof to be written) □

4.5. Positive Assortative Matching and Pareto CEO pay distribution

Firms and CEOs having perfect information on firm's productivity, CEO's talent, and $\lambda_i^*(m)$ will aim to be matched with the partner that maximizes their gain. As a consequence, each firm prefers more talented CEO and each CEO prefers larger firm as presented in following two lemmas.

Lemma 1. *(Firm's preference) Firm prefers more talented CEO, that is, $U_i(m_1, \lambda_i^*(m_1)) > U_i(m_2, \lambda_i^*(m_2))$ for $m_1 < m_2$, for all i.*

Proof. Differentiating (3) with respect to m gives

$$\left(\frac{\rho}{1 - \alpha\rho}\right) \frac{T'(m)}{T(m)} = \frac{1}{1 - \gamma} \left(\frac{\kappa}{1 - \lambda} + \frac{\gamma}{\lambda}\right) \frac{d\lambda_i^*(m)}{dm} \tag{5}$$

and it follows that $d\lambda_i^*(m)/dm < 0$. In other words, each firm choose lower bargaining weight for less talented CEO. Since the marginal gain on share of profit of increasing λ is relatively small for the less talented CEO while the marginal cost of increasing λ is constant with respect to the talent of CEO, firm has lower incentives to achieve higher bargaining weight. By (3), firm's utility function can be written as an increasing function of $\lambda_i^*(m)$.

$$U_i(m, \lambda_i^*(m)) = \frac{\delta\lambda_i^*(m)(1 - \lambda_i^*(m))^{-\kappa}}{1 - \gamma} + \frac{\delta(1 - \lambda_i^*(m))^{1-\kappa}}{1 - \kappa}$$

Putting this together with the relation between $\lambda_i^*(m)$ and m we found in above paragraph, we can conclude that $U_i(m, \lambda_i^*(m))$ is a decreasing function of m. That is, each firm prefers more talented CEO to attain higher utility. □

Lemma 2. *(CEO's preference) CEO prefers larger firm, that is,* $w_m(i_1) > w_m(i_2)$ *for* $i_1 < i_2$*, for all* m.

Proof. By differentiating (3) with respect to i gives

$$\left(\frac{\rho}{1-\alpha\rho}\right)\frac{dA_i/di}{A_i} = \frac{1}{1-\gamma}\left(\frac{\kappa}{1-\lambda}+\frac{\gamma}{\lambda}\right)\frac{d\lambda_i^*(m)}{dm} \tag{6}$$

and it follows that $d\lambda_i^*(m)/di < 0$, i.e., smaller firm gets lower bargain weight. Smaller firm, with less marginal gain on 'sales-labor wage' of increasing λ, has less incentive to increase λ. Using RHS of (3) as a 'sales-labor wage', CEO compensation can be treated as a function of $\lambda_i^*(m)$ and has positive derivative with respect to $\lambda_i^*(m)$.

$$w_m(i) \quad = \quad (1-\lambda_i^*(m))\left[\frac{\delta\lambda_i^*(m)^\gamma}{(1-\lambda_i^*(m))^\kappa}\right]^{\dfrac{1}{1-\gamma}}$$

With these sequential relations between i, $\lambda_i^*(m)$, and $w_m(i)$, one can say that $w_m(i)$is a decreasing function of i. Each CEO prefers larger firm to take higher compensation. □

These firms' and CEOs' preference on their partner result in positive assortative matching, $\sigma(i) = i, \forall i$. Then this positive assortative matching is the only stable matching.

Proposition 2. (Uniqueness and Stability of Positive Assortative Matching): *Positive assortative matching* $\sigma(i) = i, \forall i$ *is the only stable equilibrium assignment.*

Proof. From Lemma 1 and Lemma 2, one can easily conclude that the assortative matching is stable, i.e. no deviation is possible once matching is completed assortatively. A rational individual (firm or CEO) will try to deviate from the assortative matching only if they can be matched with a better partner. Hence no CEO m, currently matched with firm m, will change their partner to $i < m$ so that no firm i will be able to be matched with a better CEO m and exactly the same argument holds for firms.[i]

For the uniqueness, we use proof by contradiction. Suppose that set $A = \{i \in (0,1) \mid \sigma(i) \neq i\}$ is not empty and assume that there exist $j \in A$ with $j > \sigma(j)$. Then, $\sigma((0,j]) \subset$

$(0, \sigma(j))$ since $\forall i \in(0, j)$, $\sigma(i) < \sigma(j)$. (Otherwise, firm i and CEO $\sigma(j)$ prefers each other than their current partner contradicting the assumption that $\sigma(\cdot)$ is stable.) With measure-preserving nature of $\sigma(\cdot)$, it is followed that

$$\begin{aligned} m((0,j]) = m(\sigma((0,j])) &\leq m((0,\sigma(j))) \\ \Rightarrow j &\leq \sigma(j). \end{aligned}$$

The last inequality contradicts to our assumption that $j > \sigma(j)$. Similar argument can be made for the case of $j < \sigma(j)$. Therefore, for any stable matching $\sigma(\cdot)$, set $A = \{ i \in (0,1) \mid \sigma(i) \neq i\} = \emptyset$. In other words, $\sigma(i) = i, \forall i \in (0,1)$ is the only stable matching. We summarise these results in the following proposition. $\square$

Lemma 3. *(Efficient Positive Assortative Matching) The positive assortative matching $\sigma(i) = i, \forall i$ is efficient.*

Proof. The positive assortative matching maximizes aggregate output $Y = \left(\int_0^1 Y_i^\rho di\right)^{\frac{1}{\rho}}$. With $\rho > 0$, for any $\sigma(\cdot)$, $({A_i}^\rho - {A_{\sigma(i)}}^\rho)(T(i)^\rho - T(\sigma(i))^\rho) \geq 0$ holds. This is equivalent to $(A_iT(i))^\rho + (A_{\sigma(i)}T(\sigma(i)))^\rho \geq (A_iT(\sigma(i)))^\rho + (A_{\sigma(i)}T(i))^\rho$ and thus $Y_i(i)^\rho + Y_{\sigma(i)}(\sigma(i))^\rho \geq Y_i(\sigma(i))^\rho + Y_{\sigma(i)}(i)^\rho$. Thus, as described in Gabaix and Landier (2008) and Tervio (2008), the assignment is efficient if and only if $\sigma(i) = i$ almost everywhere for $i \in (0,1)$. Therefore, positive assortative matching is efficient. $\square$

Under assortative matching, we use $S(m)$ to denote the sales of mth firm with mth CEO and $\lambda(m)$ to denote the bargaining weight of the firm m. Under our assumption that $S(m)$ has Pareto tail with index ξ_s, $S(m)$ can be written as $S_0 m^{-\frac{1}{\xi_s}}$ for m close to 0 and for some S_0. Then, firm m and CEO m bargain over $\pi(m,m) = (1-\alpha\rho)S_0 m^{-\frac{1}{\xi_s}}$. The following lemma shows that the CEO m's bargaining weight $(1 - \lambda_m^*(m))$ has Pareto tail with index $-\frac{\xi_s\kappa}{1-\gamma}$.

Lemma 4. *Firms' optimal choices of bargaining power $\lambda_m^*(m)$ implies that $(1-\lambda_m^*(m)) = ((1-\alpha\rho)S_0)^{-\frac{1-\gamma}{\kappa}} \delta^{\frac{1}{\kappa}} m^{\frac{1-\gamma}{\xi_s\kappa}} + o(m^n, n > \frac{1-\gamma}{\xi_s\kappa})$ for m close to 0.*

Proof. Let $\bar{S}$ be $(1-\alpha\rho)S_0$ Then, (3) can be rewritten as

$$\delta\bar{S}^{1-\gamma}m^{-\frac{1-\gamma}{\xi_s}}t(m)^\kappa \quad = (1-t(m))^\gamma \quad \text{where } t(m) \equiv 1 - \lambda_m^*(m) \tag{7}$$

Since $0 < (1 - t(m)) < 1$, $0 < \gamma < 1$, $1 < \kappa$, LHS of (7) is smaller than 1. Then, the following inequalities give upper bound of t.

$$t(m) \quad < \bar{S}^{-\frac{1-\gamma}{\kappa}} m^{\frac{1-\gamma}{\xi_s \kappa}} \delta^{\frac{1}{\kappa}} \overset{let}{=} \bar{t}(m)$$

Lower bound can also be attained. $0 < (1 - t(m)) < 1$ and $0 < \gamma < 1 < \kappa$ implies that LHS of (7) is greater than $(1 - t(m))^\kappa$. Then the following inequalities give lower bound of $t(m)$.

$$t(m) > \frac{\bar{t}(m)}{\bar{t}(m) + 1} \overset{let}{=} \underline{t}(m)$$

Let $R(m) = \bar{t}(m) - t(m)$, then $R(m) = o(\bar{t}(m))$ since,

$$\begin{aligned} & \tfrac{R(m)}{\bar{t}(m)} = \tfrac{\bar{t}(m) - t(m)}{\bar{t}(m)} = 1 - \tfrac{t(m)}{\bar{t}(m)} \le 1 - \tfrac{\underline{t}(m)}{\bar{t}(m)} = 1 - \tfrac{1}{\bar{t}(m)+1} = \tfrac{\bar{t}(m)}{\bar{t}(m)+1} \\ \Rightarrow \quad & \lim_{m\to 0} \tfrac{R(m)}{\bar{t}(m)} \le \lim_{m\to 0} \tfrac{\bar{t}(m)}{\bar{t}(m)+1} = 0 \end{aligned}$$

Thus, $(1 - \lambda(m)) = t(m) = \bar{S}^{-\frac{1-\gamma}{\kappa}} m^{\frac{1-\gamma}{\xi_s \kappa}} \delta^{\frac{1}{\kappa}} + o(m^{\frac{1-\gamma}{\xi_s \kappa}})$ as follows.

$$\begin{aligned} & \tfrac{1-\lambda(m)}{\bar{t}(m)} = \tfrac{t(m)}{\bar{t}(m)} = \tfrac{\bar{t}(m) - R(m)}{\bar{t}(m)} \\ \Rightarrow \quad & \lim_{m\to 0} \tfrac{1-\lambda(m)}{\bar{t}(m)} = \lim_{m\to 0} (1 - \tfrac{R(m)}{\bar{t}(m)}) = 1 \end{aligned}$$

□

So far, we showed that the optimal choice of $\lambda_m^*(m)$ makes $(1 - \lambda_m^*(m))$ has Pareto tail with index $-\frac{\xi_s \kappa}{1-\gamma}$. Then, it is followed that CEO compensation $w(m)$ has Pareto tail with index $\dfrac{\kappa}{\kappa + \gamma - 1} \xi_s$.

Proposition 3. (The Pareto CEO Pay Distribution): *Suppose matching between firms and CEOs is assortative, i.e. firm i hires CEO i for all $i \in (0, 1)$. Then, the CEO pay distribution has a pareto tail with the index $\dfrac{\kappa}{\kappa + \gamma - 1} \xi_s$, where ξ_s is the Pareto tail parameter of the firm size distribution.*

Proof. Using (3), we can rewrite $w(m)$ as

$$w(m) = (1-\lambda_m^*(m)) \left[\delta^{\left(\frac{1}{1-\gamma}\right)} (\lambda_m^*(m))^{\left(\frac{\gamma}{1-\gamma}\right)} (1-\lambda_m^*(m))^{\left(\frac{-\kappa}{1-\gamma}\right)} \right].$$

Then,

$$\begin{aligned} w(m) &= \delta^{\left(\frac{1}{1-\gamma}\right)} (\lambda_m^*(m))^{\left(\frac{\gamma}{1-\gamma}\right)} (1-\lambda_m^*(m))^{\left(\frac{1-\gamma-\kappa}{1-\gamma}\right)} \\ &\propto m^{\left(\frac{1-\gamma-\kappa}{\kappa\xi_s}\right)} \quad \text{for } m \text{ close to } 0. \end{aligned}$$

In other words, CEO compensation $w(m)$ has Pareto tail with index $\left(\frac{\kappa\xi_s}{\kappa+\gamma-1}\right)$. □

Proposition 3 suggests that changes in inequality in the CEO pay distribution can be driven by the changes in κ, which governs the firm's marginal disutility of increasing bargaining power. When κ increases, firms lower their bargaining power as the bargaining costs increase. This will raise CEO pays, and high talent CEOs will benefit from this raise disproportionately more than low talent CEOs, which then increases inequality (equivalently, decreases the Pareto tail index) in the CEO pay distribution.

Increases in κ can be interpreted as changes in a social norm on CEO pays. If a social norm on CEO pays has changed over time so that it has now become more acceptable that CEOs can take home a larger share, it will become more likely for firms to be generous in the bargaining. This corresponds to the increases in the firms' marginal utility of lowering bargaining power. () Specifically, Whoriskey (2011) in an *Washington Post* article documents that "*... that executive salaries have jumped because corporate boards were simply too generous, or more broadly, because greed became more socially acceptable...*".

Our model also predicts the well-documented Roberts' law, which says that the CEO pay is proportional to (firm size)$^\phi$ (Roberts (1956)), with $\phi = \frac{\kappa+\gamma-1}{\kappa}$. ϕ increases both in γ and κ, which implies that superior CEO talents become more valuable as bargaining conditions become more favorable to CEOs.

5. Discussion

(in progress)

6. Conclusion

In this paper, we present a theoretical model which shows that the board of directors' weakened (strengthened) bargaining power can be a channel through which CEO pay distribution departs from the firm size distribution and becomes more unequal (equal). The bargaining power in our model can be broadly interpreted as social circumstances surrounding CEO pays. For example, the weakened bargaining power of boards of directors can be linked to the changes in the fairness norm - high CEO compensation is nowadays regarded to be more acceptable than four decades ago. On the other hand, the recent institutional changes that made it difficult to raise CEO pays can be understood as strengthened bargaining power on the board side.

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