

# The Redistributive Effects of Monetary Policy in an Overlapping Generation Model<sup>1</sup>

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## Abstract

I study the redistributive effects of monetary policy with attention to a wealth effect for savers generated by a cut in interest rates. To this end, I build a full-fledged Overlapping Generations New Keynesian model (OLGNK) with financial frictions. The model, calibrated to match the U.S. demographic structure and with a marginal propensity to consume and inequality among households, yields empirically realistic covariances between marginal propensity to consume (MPC) and household exposures to interest rates and prices. In this environment, I find that a negative wealth effect of an interest rate cut for savers is substantial, dampening the impulse responses of aggregate output relative to the ones in a Representative Agent New Keynesian (RANK) model. This finding is in contrast to a Heterogeneous Agent New Keynesian (HANK) model, where there is amplification of monetary policy relative to RANK.

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**Keywords:** monetary policy, redistribution, overlapping generations, financial friction, retirement, life cycle

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# 1 Introduction

Analyses of the effect of monetary policy through the redistribution channel have attracted increasing attention in recent years. Monetary policy may have redistributive effects because a decrease in interest rates reduces the debt burden of borrowers while causing a loss in interest income for savers. However, it is commonly argued that such monetary policy may ultimately benefit savers as well because of the powerful general equilibrium channel of the monetary policy. According to this view, decreasing interest rates should create strong demand from borrowers that leads to a general equilibrium increase in labor demand. This generates greater labor income for savers, which offsets losses in their interest income. In fact, this view is consistent with the implications of standard models such as the simple or two-agent New Keynesian model (TANK).

I revisit this argument by analyzing the redistributive effects of monetary policy with attention to a wealth effect for savers generated by an interest rate cut. To this end, I develop a full-fledged overlapping generation New Keynesian model, which pays special attention to age heterogeneity. Unlike other models, individuals in an OLG model have different horizons, implying that they also have different MPC. The model is rich in three regards. First, because it adopts an OLG economy, it captures households consumption and income profiles over the full life cycle (Gourinchas and Parker 2002). The model assumes that households live from age 21 to 80 and work from age 25 to 65.<sup>1</sup> Household labor productivity shows systematic age variations over the life cycle, and those households are subject to labor productivity shocks. Second, the model captures the U.S. demographic structure. Household survival probabilities are taken from the Mortality Tables of the Internal Revenue Service, and the resulting distribution of the population by age is similar to that of the U.S.. Third, the marginal propensity to consume (MPC) implied by the model is consistent with empirical findings for MPC (e.g., Johnson, Parker, and Souleles 2006; Parker, Souleles, Johnson, and McClelland 2013; Broda and Parker 2014; Misra and Surico 2014).

In order to ensure the model implies empirically realistic MPC, I incorporate financial frictions into the model. The financial frictions in this paper also address the criticism of RANK that monetary policy operates almost exclusively through the direct intertemporal substitution channel (Kaplan, Moll, and Violante 2018a). In contrast to this implication of RANK models, micro survey data show that there is a significant proportion of hand-to-mouth households whose con-

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<sup>1</sup>A two-period OLG model lacks life-cycle detail because the length of a period featuring constant interest rates and consumption covers about thirty years in real time.

sumption is insensitive to changes in interest rates. Incorporating this aspect into a model also allows us to match negative covariances between MPC and the exposure of household balance sheets to changes in interest rates and prices seen in data (Auclert 2019).

The nature of financial frictions in this paper differs from models in which households face a fixed borrowing limit (e.g., Bewley-Huggett-Aiyagari models). In particular, I suppose that in each cohort, there is some proportion of households that face borrowing constraints. A key assumption here is that the borrowing constraint constrains borrowers' issuance of *new* debt. The constrained households face high borrowing costs and those costs increase with the amount of new borrowing.<sup>2</sup> In my setting, the degree of the financial frictions can be parameterized to match an empirical MPC. Also, the model nests special cases such as hand-to-mouth households and households without any borrowing constraints.

In order to isolate the effects of some model parameters and facilitate model comparisons, I present results of a first experiment comparing a homogeneous OLGNK model and a OLGNK model without any financial frictions. The homogeneous OLGNK model, in which household labor productivity is constant and age-independent, serves as a benchmark because aggregate outcomes from that model are almost identical to those from RANK in which Ricardian neutrality holds. Therefore, this comparison answers the question: If age-dependent labor productivity is set to match expected income profiles over the life cycle in the US economy, how much of a difference does this make (relative to the homogeneous economy) to the model's prediction for the response of the economy to a monetary shock? In the homogeneous OLGNK model households hold no assets, whereas in the OLGNK model households accumulate assets according to life cycle considerations.

I find that the effects of a monetary policy shock are substantially smaller in the OLGNK model compared to the homogeneous OLGNK model. In the homogeneous OLGNK model, output always stays above the steady-state level in response to a transitory decrease in the nominal interest rate. The monetary policy shock induces a initial increase in aggregate demand and labor income. This increase is large enough so that households can consume more today without sacrificing any consumption in the future. In contrast, in the OLGNK model, there is a smaller initial increase in households' demand, which is eventually countered with a decrease in household demand after the first few periods. The weak initial response is driven by a negative

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<sup>2</sup>This setup can be understood as one-side debt adjustment costs. In the real world, a large amount of new borrowing would decrease credit scores. This may increase borrowing rates endogenously, and so may be costly for borrowers. This financial frictions assumption differs from a basic borrowing constraint with a fixed borrowing limit.

wealth effect that a lower interest rate incurs for older generations, whose MPCs are greater due to a shorter planning horizon. This implies that some generations do not gain as much from the monetary policy as other generations.

To understand the differential effects of monetary policy on different generations, I calculate how a monetary policy shock affects the normalized present value of wealth for each generation. My analysis shows that households of age over 47 lose from a negative monetary policy shock and households close to retirement are the biggest losers, whereas the monetary policy shock tends to benefit younger generations. This is not the case for the homogeneous OLGNK model, in which the monetary policy shock increases the present value of wealth for all generations. In this case, the effect of monetary policy via the general equilibrium channel is powerful; a large increase in wages induced by a strong demand with a negative monetary policy shock allows households to consume more today without sacrificing any consumption tomorrow. However, in the OLGNK model, due to the negative wealth effect that savers (older generations) must bear, a negative monetary policy shock induces a rather moderate increase in aggregate borrowing. This leads to a rather subdued general equilibrium effect on wages, and these increases in wages cannot offset the negative wealth effect of a cut in interest rates that savers have to bear.

However, without financial frictions, the OLGNK model neither captures negative covariances between MPC and household balance-sheet exposure to prices nor fully reflects the inequality among households observed in the data. In the second experiment, I calibrate the OLGNK model with financial frictions (OLGNK-FF) to match empirical MPC and the consumption inequality observed in the data. I then address the question of whether the redistributive channel amplifies the effect of monetary policy on aggregate output. Auclert (2019) argues that monetary policy may be amplified through the redistribution channels such as interest-rate exposure, the Fisher channel, and earnings heterogeneity. He presents the redistribution elasticities associated with those channels as sufficient statistics to gauge the redistributive effects of monetary policy. To compare my results with those of Auclert, I also calculate these redistribution elasticities from model simulations.

I show that the redistribution elasticities in the OLGNK-FF model are also negative, as is the case with empirical and model statistics in Auclert's study. However, the computed model impulse responses show that the redistribution channel still does not amplify the effects of monetary policy on aggregate output. Despite the presence of the financial frictions, the aggregate responses are much weaker than what Auclert suggests. These results show that the redistribu-

tive effect of monetary policy may not be captured by the redistribution elasticities. The main reason for the weaker response to monetary policy is that, unlike the case in a Bewley-Huggett-Aiyagari model, an initial increase in borrowing from the constrained household is limited due to the financial frictions. Therefore, an initial increase in demand is subdued, which translates into a smaller general equilibrium effect. Consequently, older generations would still have to bear a larger negative wealth effect. The point here is that negative redistribution elasticities are not sufficient conditions to induce the constrained households to increase their borrowing substantially in response to a monetary policy shock, which is necessary for the amplification of the shock.

In addition, this result suggests that the effects of monetary policy through the redistribution channels may depend heavily on the nature of the financial frictions. In a Bewley-Huggett-Aiyagari model with a fixed borrowing limit, households that are close to their borrowing limit may respond strongly to a negative monetary policy shock by increasing their amount of borrowing. This strong increase in demand generates a powerful general equilibrium effect through wages which alleviates the burdens savers would bear. In my model, households are constrained on new borrowing. Because borrowing by the constrained younger generations is not as sensitive to changes in interest rates, this specification leads to weaker general equilibrium effects. As a result, the type of the financial frictions considered here turns out to mute the equilibrium dynamics of output in response to a monetary shock.

**Related Literature** My paper is related to the literature on TANK such as Bilbiie (2008), Galí, López-Salido, and Vallés (2007), and Debortoli and Galí (2018). The clearest example about an amplification mechanism in TANK is Bilbiie (2008), who shows that amplification is driven by an elasticity of income of hand-to-mouth consumers to aggregate income. But TANK features zero bond trading in equilibrium and abstracts from the evolution of the households' holding of nominal bonds. As my model features bond trading in equilibrium with the evolution of the households' balance sheets, it complements the literature with an analysis of wealth effects incurred by an interest rate cut.

Recently, TANK has been further developed, allowing stochastic switching between types, to capture realistic intertemporal MPC, idiosyncratic risk and precautionary saving (Bilbiie 2018; Bilbiie 2020); and study optimal stabilization policy in the presence of productivity and cost-push shocks (Nistico 2016). Those studies still feature zero bond trading in equilibrium and their focus is not on the redistributive effects of monetary policy between different types through their

balance sheets. The stochastic evolution of the types of household in my paper builds on the finding of Curdia and Woodford (2010, 2016); the key difference is that in my paper the types of household are categorized based on borrowing constraints into constrained and unconstrained households, instead of into borrowers and savers.

Nistico (2016) also nests a perpetual-youth (PY) model (Yaari 1965; Blanchard 1985)) as a special case. A PY model supports a realistic planning horizon. Also, it helps to resolve the forward guidance puzzle (Del Negro, Giannoni, and Patterson 2012). However, a drawback of this type of model is that it still does not capture life-cycle behavior. There is no saving for retirement; the life-cycle details of earnings, consumption and savings are absent. Moreover, due to its the rigid demographic assumption of a constant mortality rate, all individuals currently alive have identical MPCs. For this reason, the model may not be well suited for the analysis of the redistribution channel in which covariances between MPCs and households exposures are important.

There is a growing literature studying the effects of monetary policy in the presence of agent heterogeneity (e.g., McKay, Nakamura, and Steinsson 2016; Kaplan, Moll, and Violante 2018b; Auclert 2019). Setting aside the nature of financial frictions, the OLGNK-FF model in this paper differs from Bewley-Huggett-Aiyagari models in how it analyzes the redistributive effects of monetary policy in two regards. First, it takes account of systematic age-based variation in income over the life cycle. The empirical evidence from Gourinchas and Parker (2002) suggests that younger generations face upward sloping expected income profiles, whereas middle age households, on the contrary, face declining expected income profiles before retirement. This systematic age-based variation in income translates into systematic variation in asset accumulation across generations. Doepke and Schneider (2006) also illustrate that older households have a large positive net nominal position which makes them more exposed to a cut in interest rates. Second, we can analyze the differential effects of monetary policy on the welfare of households that are in different phases of their working life. One common argument is that we may ignore redistributive effects because they average out in the long run. According to this argument, the same type of households that lose from monetary policy impact may also gain later in their life through a general equilibrium effect. This is the case in a perpetual youth model such as the Blanchard-Yaari which does not allow for retirement or variations in labor income over the life cycle.<sup>3</sup> However, an OLG economy, as modelled in this paper, limits the extent to which older

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<sup>3</sup>In a perpetual youth model the horizon is effectively infinite, although future values are discounted at a higher

generations, who face downward sloping expected income profiles, gain from the policy.<sup>4</sup>

This paper also contributes to the growing literature on the redistributive effects of monetary policy, which includes the empirical works of Doepke and Schneider (2006), Adam and Zhu (2015), and Coibion, Gorodnichenko, Kueng, and Silvia (2017) and the theoretical works of Gornemann, Kuester, and Nakajima (2016), Kaplan, Moll, and Violante (2018b), Auclert (2019), and Broer, Harbo Hansen, Krusell, and Öberg (2019), among many others. In particular, Doepke and Schneider (2006) and Adam and Zhu (2015) document the presence of sizable redistribution of nominal wealth across households. The main difference to such studies is that the present paper develops an overlapping generation model in the presence of nominal rigidities and financial frictions, which is tractable and captures heterogeneity among households.

The remainder of this paper proceeds as follows. Section 2 shows the importance of a wealth effect of an interest rate cut for savers and its implication on aggregate output in a simple two-period borrower-saver model. Section 3 builds the OLGNK model with financial frictions and explores how the redistribution channel affects the impact of monetary policy on aggregate output. Section 4 concludes the paper.

## 2 Simple Two-period Model with Financial Frictions

This section presents a simple two-period borrower-saver model in which there is a constraint on *new* borrowing. It shows that a wealth effect of an interest rate cut can be substantial for savers, acting a dampening force on output relative to RANK. I find that the nature of financial frictions plays a role. An OLGNK model in Section 3 builds upon the insights from this section. A reader more interested in the multi-period dynamic analysis may at this time study equation (6) and skip to Section 3.

### 2.1 Environment

The model shares the same approach with Baek (2019), who analyzes the impact of forward guidance in a three-period borrower-saver model. The model includes two periods,  $t = 0, 1$ , and two types of household: ‘savers’, denoted  $S$ , and ‘borrowers’, denoted  $B$ . The proportions of the population represented by the two types are constant at all times, and equal to  $\frac{1}{2}$ . Households

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rate reflecting survival probabilities.

<sup>4</sup>On the other hand, I abstract from the precautionary saving motive under uncertainty. Due to a large state space, I solve models using log linear approximation.

have preferences

$$u(C_0^i - \theta_0^i v(h_0^i)) + \beta^i u(C_1^i - \theta_1^i v(h_1^i)) \quad (1)$$

where either  $u(c) = \ln c$  or  $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$  with  $\sigma > 0, \sigma \neq 1$ ;  $v', v'' > 0, v(0) = 0$ ;  $\beta < 1$ . Households' budget constraints are given by

$$P_0 C_0^i + \frac{B_1^i}{1+i_0} = W_0 \theta_0^i h_0^i + D_0 + B_0^i, \quad (2)$$

$$P_1 C_1^i = W_1 \theta_1^i h_1^i + D_1 + B_1^i \quad (3)$$

where  $i_0$  is a nominal interest rate in period 0;  $B_1^i$  denotes holdings of nominal bonds at the end of period 0;  $B_0^i = \bar{B}^i$  denotes holding of nominal bonds at the beginning of period 0;  $\theta_t^i$  is household  $i$ 's productivity in period  $t$ ;  $W_t$  is the nominal wage;  $D_t$  denotes nominal profits from monopolistically competitive firms.

Households can take negative positions in the nominal bond, but face a (real) borrowing constraint:

$$\frac{B_1^i}{1+i_0} - B_0^i \geq \frac{-P_1 \phi}{1+i_0}. \quad (4)$$

Note that this is the constraint on households' *new* borrowing, not on the total amount of debt they have to repay in the next period.<sup>5</sup> This feature is important in relation to the effect of monetary policy.<sup>6</sup>

Households' utility maximization implies that labor supply (regardless of types) is given by

$$\frac{W_t}{P_t} = v'(h_t). \quad (5)$$

Firms produce output using a linear technology,  $Y_t = H_t$ , where  $H_t = \sum_i \frac{1}{2} \theta_t^i h_t^i$ . Prices in period 0 are assumed to be fixed; firms must hire whatever labor is necessary to meet demand. For generality, I do not assume a particular model of price setting; rather, I suppose that the inflation function  $\pi_1(i_0)$  is known to the monetary authority and the monetary authority chooses the nominal interest rate  $i_0$  at date 0 (and thus the real interest rate  $R = \frac{1+i_0}{1+\pi_1}$ ).

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<sup>5</sup>There may be several ways to microfound this form of the borrowing constraint. One way is to introduce debt adjustment costs. Another way is to assume that only a fraction of borrowers have opportunity to adjust their asset positions, which constrains the aggregate issuance of new debt. Then in a special case in which every borrower has access to a financial market to adjust their positions, the borrowing constraint would reduce to one with a fixed borrowing limit.

<sup>6</sup>Note that without the second term in expression (4), this becomes a standard borrowing constraint,  $\frac{B_1^i}{P_1} \geq -\phi$ .



Finally, markets clear,

$$\frac{1}{2} (C_t^S + C_t^B) = Y_t.$$

An equilibrium is a collection  $\{C_t^i, h_t^i, B_1^i, Y_t, W_t, D_t\}_{t=0,1}^{i=S,B}$  such that, given the policy  $i_0 \geq 0$ , households  $i = S, B$  choose  $\{C_t^i, h_t^i, B_1^i\}_{t=0,1}$  to maximize (1) subject to (2), (3), and (4); firms maximize profits at date 1; and markets clear.

## 2.2 Equilibrium in a Borrower-Saver Economy

In order to derive the main result as simply as possible, I specialize the model to the following case.

**Assumption 1.** *Savers are more patient than borrowers,  $\beta_S > \beta_B$ . Savers have a (weakly) positive endowment of nominal bonds in period 0,  $\bar{B}^S = -\bar{B}^B \geq 0$ . Borrowers and savers have the same productivity in period 0,  $\theta_0^S = \theta_0^B$ , but borrowers have a weakly higher productivity in period 1,  $\theta_1^B \geq \theta_1^S$ . No new borrowing is possible in period 0:  $\phi = 0$ . Prices are flexible in period 1, but they are fixed in period 0, which we normalize to unity.*

‘Savers’ could represent wealthy, older households, that have a large positive net nominal position (Doepke and Schneider 2006), representing nominal bonds and pensions, but low future income. ‘Borrowers’ represent any households that take the other side of these positions in equilibrium. One interpretation is that borrowers represent households that are young today and will be middle-aged in the future: their negative net nominal position could represent both their fixed-rate mortgage debt, and their tax liabilities, which pay for the savers’ pensions (which are fixed in nominal terms).

The assumption of the same productivity in period 0 implies that households have the same income in period 0,  $Y_0^i = W_0 \theta_0 h_0 + D_0$  for  $i = S, B$ . This assumption helps us to abstract from earning heterogeneity and purely focus on a balance sheet effect. The assumption of flexible prices in period 1 implies that output will always be at its efficient level,  $Y_1 = Y_1^*$ , in period 1. Note that efficient labor supply  $h_t^*$  is determined by  $h_t^* := \arg \max_h \theta_t^i h - \theta_t^i v(h)$ . In light of this, I denote efficient output in period 0 by  $Y_0^*$ . The economy is considered to be in recession in period 0 if the equilibrium level of output in period 0 is less than the flexible-price output,  $Y_0 < Y_0^*$ .

We are interested in equilibria in which borrowers are liquidity constrained and the economy is in recession at date 0. The following lemma describes conditions under which this will be the case. The first inequality in the lemma guarantees that borrowers are at their borrowing

limit. The second inequality ensures that the economy is in recession. Roughly speaking, these conditions are satisfied if borrowers' discount factor  $\beta^B$  is low and savers' discount factor  $\beta^S$  is high (e.g., a preference shock).

**Lemma 1.** *Suppose parameters satisfy*

$$y_0^{*-\sigma} > \beta^B R' [y_1^{B*} - R' \bar{B}^S]^{-\sigma}$$

$$y_0^* > (\beta^S R')^{-1/\sigma} (y_1^{S*} + R' \bar{B}^S)$$

where I define

$$y_0^* = \max_h \theta_0 h - \theta_0 v(h),$$

$$y_1^{i*} = \max_h \theta_1^i h - \theta_1^i v(h), i = S, B.$$

Then if  $R = R'$ , the equilibrium has  $Y_0 < Y_0^*$ , where  $Y_0^* = \theta_0 h_0^*$ .

For proof see Appendix A.

To study the Euler equation derived from our model, it is useful to define net consumption,  $c_t^i := C_t^i - \theta_t^i v(h_t^i)$ . Savers' Euler equations imply that

$$u'(c_0^S) = \beta^S R \cdot u'(c_1^S).$$

In standard models with nominal rigidities, a lower interest rate can mitigate a current recession, inducing individuals who are on their Euler equations to spend more today. For instance, if  $c_1^S$  is fixed, a decrease in  $R$  increases  $c_0^S$ . This mechanism holds as long as savers' future consumption does not decrease even as the interest rate decreases. This will not be true if households' *new* borrowing is constrained. In this case, an interest rate has a negative wealth effect for savers in the opposite direction to the substitution effect. If this wealth effect is strong enough, it could outweigh the substitution effect. The following proposition shows that the elasticity of savers' net consumption with respect to an interest rate depends on the form of the borrowing constraint.

**Proposition 1.** 1. *The elasticity of savers' net consumption  $c_0^S$  with respect to  $R$  is*

$$\frac{dc_0^S}{c_0^S} \left( -\frac{R}{dR} \right) = \left( \frac{1}{\sigma} - \frac{R \cdot \bar{B}^S}{c_1^S} \right), \quad (6)$$

where

$$\begin{aligned} c_0^S &:= Y_0^S - \theta_0 v(h_0), \\ c_1^S &= y_1^{S*} + R \cdot \bar{B}^S. \end{aligned} \tag{7}$$

2. Suppose that a borrowing constraint is given by  $\frac{B_1^B}{P_1} \geq 0$ , instead of (4). Then we have

$$\frac{dc_0^S}{c_0^S} \left( -\frac{R}{dR} \right) = \frac{1}{\sigma}.$$

Proof) 1. It follows from the savers' Euler equation:

$$c_0^S = \left( \beta^S (1 + r_0) \right)^{-1/\sigma} [y_1^{S*} + R \cdot \bar{B}^S]. \square$$

2. When the constraint binds for borrowers, savers' period 1 consumption is

$$c_1^S = y_1^{S*} + \phi, \tag{8}$$

which is fixed, unlike the previous case. Thus the proposition follows.  $\square$

With the constraint on new borrowing, the effect of monetary policy depends on the difference between the savers' intertemporal elasticity of substitution (IES),  $\frac{1}{\sigma}$ , and the share of nominal bonds in their date 1 consumption (i.e., their exposure to interest rate shocks),  $\frac{R \cdot \bar{B}^S}{c_1^S}$ .<sup>7</sup> But if we impose a fixed borrowing limit, the elasticity of  $c_0^S$  with respect to  $R$  coincides with IES (Proposition 1-2). Savers do not need to bear a wealth effect of an interest rate cut.

Proposition 1 points out that the redistribution channels can dampen the effect of monetary policy. To formalize this point in the context of our model, let us define the net nominal position (NNP) and the unhedged interest rate exposure (URE) as in Auclert (2019),  $NNP_0^i = \frac{B_0^i}{P_0}$  and  $URE_0^i = \frac{W_0}{P_0} \theta_0 h_0 + \frac{D_0}{P_0} + \frac{B_0^i}{P_0} - C_0$ . The following proposition shows that the elasticity of net *output* with respect to an interest rate depends on the form of the borrowing constraint. It contrasts with Collorary 1 and Theorem 3 in Auclert.

**Proposition 2.** 1. Suppose that a borrowing constraint is given by expression (4) with  $\phi = 0$  and  $B_0^B < 0$ . Then  $cov(MPC_0^i, URE_0^i) < 0$ ,  $cov(MPC_0^i, NNP_0^i) < 0$ , and  $cov(MPC_0^i, Y_0^i) = 0$ , but

<sup>7</sup>In a special case in which the IES and nominal bond shares are both equal to unity, the consumption elasticity is zero. The proposition also highlights that output could potentially even fall in response to a lower interest rate, if the IES is less than 1 and the nominal bond share is large.

$$\frac{dy_0}{y_0} \left( -\frac{r_0}{dr_0} \right) < \frac{1}{\sigma}.$$

2. Suppose that a borrowing constraint is given by  $\frac{B_1}{P_1} \geq -\phi$ . Then  $cov(MPC_0^i, URE_0^i) < 0$ ,  $cov(MPC_0^i, NNP_0^i) < 0$ , and  $cov(MPC_0^i, Y_0^i) = 0$ , and  $\frac{dy_0}{y_0} \left( -\frac{r_0}{dr_0} \right) > \frac{1}{\sigma}$ .

This proposition shows that the nature of financial frictions plays an important role in the evaluation of the redistributive effects of monetary policy. According to Auclert, whenever those covariances (redistribution elasticities) are negative, the redistribution channels amplify the effect of monetary policy, and in fact, that is true when we also have a fixed borrowing limit (Proposition 1-2).

However, when borrowers' borrowing is insensitive to a change in interest rates, the redistribution channel dampens the effect of monetary policy despite the fact that those redistribution elasticities are negative (Proposition 1-1).<sup>8</sup>

On the other hand, if a decrease in interest rates relaxes the borrowers' constraint and borrowing always increases to the amount of increase in borrowing capacity (Proposition 1-2), this stimulates aggregate demand today to induce a large increase in income. Therefore, there is no negative wealth effect that savers bear at the end, as can be seen in equation (8). However, if there are constraints on households' new borrowing, an initial increase in aggregate demand of this kind is not large enough to offset a negative wealth effect that hurts savers (equation (7)). The point here is that those covariances (and redistribution elasticities) do not guarantee large borrowing from borrowers and may not be sufficient to induce the amplification of monetary policy. This result shows that the nature of borrowing constraints is important when evaluating the effects of monetary policy.

In relation to TANK, the amplification mechanism in TANK usually relies on an elasticity of income of constrained agents to aggregate income (Bilbiie 2020). It usually features zero bond trading in equilibrium and emphasizes the role of transfer policies. My model abstracts from income heterogeneity, as both savers and borrowers have the same income at date 0. This paper complements the literature in that it focuses on the monetary transmission mechanism through the household balance sheet.

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<sup>8</sup>In our model, market clearing for nominal assets implies that savers' assets are borrowers' liabilities. Proposition 1-1 would still hold when savers' assets are government's liabilities, but rebates from a government (due to reductions in interest rates) are delayed.

### 3 A New Keynesian Overlapping Generation Model with Financial Frictions

In an overlapping generation New Keynesian (OLGNK) model, in each period new generations are born and live for a finite period of time, but otherwise, if there exist no financial frictions, the model closely follows RANK. Here I describe the OLGNK model extended with financial frictions, but the model can be parameterized to nest a case in which there are no financial frictions.

One criticism of the representative NK model is that monetary policy operates almost exclusively through intertemporal substitution, whereas micro survey data show that there is a significant fraction of households who behave as hand-to-mouth consumers (Kaplan, Moll, and Violante 2018b).

To incorporate financial frictions, I borrow an idea from Curdia and Woodford (2016), where the types of agents follow a Markov process, as explained in the next section.

#### 3.1 Households

Time is discrete,  $t = 0, 1, 2, 3, \dots$  and the economy lives forever. I use the subscript  $t$  to denote the current period. In each period, new households are born, who live for  $T + 1$  periods and then die. The age of households born in year  $s$  at time  $t$  is given by  $t - s$ . I use the superscript  $t - s$  to denote the age of households. I denote by  $\phi^{t-s}$  the probability of surviving until age  $t - s$ , conditional on being alive at age  $t - s - 1$ . Then the unconditional probability of survival from age 0 to age  $t - s$  is  $\bar{\phi}^{t-s} = \prod_{j=0}^{t-s} \phi^j$ . These probabilities define a mass  $\mu^{t-s}$  of household  $t - s$ , where the total mass of households is normalized to one,  $\sum_{t-s=0}^T \mu^{t-s} = 1$ .

Each generation consists of the two types of households, where  $\tau_i^{t-s} \in \{f, n\}$  indicates household  $i$ 's type at time  $t$ . I use the superscript to denote the type of households.

The net accumulation of savings,  $N_t$ , is defined as

$$N_t^{t-s,\tau} = \phi^{t-s+1} B_{t+1}^{t-s,\tau} - A_t^{t-s,\tau},$$

where  $B_{t+1}^{t-s,\tau}$  denotes the nominal assets of the type  $\tau$  household of generation  $t - s$  at the end of period  $t$  and  $A_t^{t-s,\tau}$  the nominal assets of the household at the beginning of the period.<sup>9</sup>

<sup>9</sup>Further details are provided in equation (14).

**Type- $f$  households** Households of this type are not financially constrained. They are not subject to the borrowing constraint, except the no Ponzi constraint. Their utility function is given by

$$U_t^{t-s,f} = \sum_{k=t}^{s+T} \beta^{k-s} \bar{\phi}^{k-s} v(Z^{k-s}) \left[ \frac{(c_k^{k-s,f})^{1-\sigma}}{1-\sigma} - \frac{(h_k^{k-s,f})^{1+\psi}}{1+\psi} \right], \quad (9)$$

where  $v(Z^{t-s}) = 1 + \bar{v}Z^{t-s}$  which is an age-dependent preference shifter that reflects deterministic household characteristics  $Z^{t-s}$  (e.g., family size),  $c_t^{t-s,f}$  is the consumption of the type  $f$  household of generation  $t-s$  at time  $t$ , and  $h_t^{t-s,f}$  is labor supply.

Generation  $t-s$  is endowed with labor productivity  $\theta^{t-s,f}$  that generates labor income  $\frac{W_t}{P_t} \theta^{t-s,f} h_t^{t-s,f}$ , where  $W_t$  is the nominal wage at time  $t$  and  $P_t$  is the aggregate price level. Labor productivity is age-dependent, but its distribution is time invariant.

**Type- $n$  households** Type- $n$  households receive a negative labor productivity shock

$$\theta^{t-s,n} = (1 - \zeta) \theta^{t-s,f}, \quad (10)$$

where  $0 < \zeta < 1$ . This negative income shock positions the type- $n$  households as natural borrowers in the economy.

However, the type- $n$  households are subject to a financial constraint. The type of the financial constraint here is motivated by the insights from Section 2. The financial constraint is imposed into the household's utility in the form of the penalty function (identically, liquidity demand)  $\varkappa(\cdot)$ :

$$U_t^{t-s,n} = \sum_{k=t}^{s+T} \beta^{k-s} \bar{\phi}^{k-s} v(Z^{k-s}) \left[ \frac{(c_k^{k-s,n})^{1-\sigma}}{1-\sigma} - \frac{(h_k^{k-s,n})^{1+\psi}}{1+\psi} + \varkappa(n_k^{k-s,n}) \right], \quad (11)$$

where  $n_t^{t-s,n} = N_t^{t-s,n} / P_t$  and

$$\varkappa(n) = \varkappa [1 - \exp(-\tilde{\varkappa} \cdot n)], \text{ where } \varkappa, \tilde{\varkappa} \geq 0. \quad (12)$$

This form of financial frictions is useful in that the degree of financial frictions can be parameterized by choosing  $\tilde{\varkappa}$ . The model nests two special cases in which financial frictions are irrelevant ( $\tilde{\varkappa} = 0$ ) and households behave as traditional hand-to-mouth consumers ( $\tilde{\varkappa} \rightarrow \infty$ ) where they face the constraint (4).<sup>10</sup> Also,  $\tilde{\varkappa}$  can be chosen so that the MPC of the model is consistent with

<sup>10</sup>The consumption of the type- $n$  households is characterized as that of traditional hand-to-mouth consumers if

MPC values from empirical studies.

$N_t$  is the net issuance of nominal debt, which is the household's total borrowing less existing debt that is carried from a previous period. Thus, expression (12) represents a constraint on households' new borrowing, not on the total amount of debt they have to repay at  $t + 1$ . As  $\kappa''(n) < 0$ , obtaining new debt,  $n < 0$ , is costly to the type- $n$  households. Although there is little cost for saving, there is an increasing adjustment cost of raising the amount of debt. Another way to microfound this type of financial frictions would be to use a setting in which only a certain fraction of the constrained households have access to financial markets for further borrowing in each period.<sup>11</sup>

Our borrowing constraint is different from a traditional one with a fixed borrowing limit in that a lower interest rate does not relax the constraint directly, which shares the same spirit with the constraint (4)<sup>12</sup>; an interest rate does not enter directly into expression (12).<sup>13</sup>

On the other hand, the constrained households are similar to hand-to-mouth consumers in that their current consumption is sensitive to transitory income (Also the model nests them as a special case). But the model is more general in that that the constrained households still display direct intertemporal substitution responses to interest rates; the intertemporal substitution channel still operates. This allows us to describe the evolution of the assets of the constrained households.

Arguably, borrowing opportunities/constraints would differ depending on employment status and new borrowing would be harder for the unemployed. Also, there are empirical surveys such as the SCE Credit Access Survey (by the Federal Reserve Bank of New York) which could be used as inputs into the model, which opens possibilities for future study.

Another advantage of the model from a technical perspective is that it is not necessary to keep track of the wealth distribution in the economy with a set of assumptions, as described below.

**Markov Process for the Evolution of Types and Financial Contracting** In order to keep the model tractable, I follow Curdia and Woodford (2016) who develop a two-agent New Keynesian model without the curse of dimensionality.

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$\bar{z} \rightarrow \infty$  and the size of a negative labor productivity shock  $\zeta$  is sufficiently large. This leads type- $n$  households to borrow as much as they can to increase consumption in the current period, but the marginal cost of issuing new debt becomes infinity.

<sup>11</sup>For instance, in each period, the fraction  $\gamma$  of the constrained households do not have an opportunity to obtain more debt, in which case their positions would be equal to their existing position,  $B_t^{t-s} = B_{t-1}^{t-s}$ .

<sup>12</sup>Plugging (4) into (2),  $\frac{N_0}{P_0} = \frac{W_0}{P_0} \theta_0 h_0 + \frac{\Pi_0}{P_0} - C_0 = -\phi$ . In the case of hand-to-mouth consumers,  $\phi = 0$ .

<sup>13</sup>With a fixed borrowing limit, individuals with wealth close to a borrowing limit responds strongly by increasing borrowing.

Each type  $\tau$  evolves as an independent two-state Markov chain. In particular, at the beginning of each period, a new type for each household of generation  $t - s$  is drawn with probability  $1 - \delta$ . When a new type is drawn, the household becomes type- $f$  with probability  $\kappa^{t-s}$  and type- $n$  with probability  $1 - \kappa^{t-s}$ . When a new type is not drawn, which occurs with probability  $\delta$ , the household remains the same type as in the previous period.<sup>14</sup>

The analysis becomes tractable with an additional form of financial contracting. Thanks to the following form of financial contracting, all households of the same type within the same generation make identical choices, independent of type history. This implies that we can obtain stationary equilibrium without keeping track of the history of the type of individual households.

Households sign state-contingent contracts, which insure against both aggregate and idiosyncratic risks, with one another, but only within the same generation. At the beginning of each period, household  $i$  receives the state-contingent transfer,  $\tilde{T}_t^{t-s}(i)$ , from the insurance agency intermittently. Household  $i$  receives the insurance payment only if a new type is drawn for that household, which occurs with probability  $1 - \delta$ ; otherwise (with probability  $\delta$ ), households do not have access to the insurance agency and thus,  $\tilde{T}_t^{t-s}(i) = 0$ . For those households of generation  $t - s$  who have access to the insurance agency, post-transfer wealth at the beginning of period  $t$  is the same. Those transfers through the insurance agency aggregate to zero each period,

$$\int \tilde{T}_t^{t-s}(i) di = 0 \quad \text{for all } t - s. \quad (13)$$

In Curdia and Woodford (2016), infinitely-lived households expect that they are eventually able to receive transfers from the insurance agency, which equalizes the expected marginal utilities of income across all households of the same type within a generation.<sup>15</sup> In order to obtain a similar result with my model, in which households have finite lifetimes, I suppose that all households have access to the insurance agency with probability 1 at the end of life,  $\delta^T = 0$ . With this final condition, the aggregation of debt becomes linear, and we obtain stationary equilibrium without keeping track of the wealth distribution in the economy. This simplifies the methodology because it enables the use of local log-linearization methods to characterize the model's solutions.

**Budget Constraints** The nominal assets of household  $i$  of generation  $t - s$  at the beginning of

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<sup>14</sup> $\kappa$  may depend on age, but it is time invariant. Thus, the distribution of the agent's type is stationary.

<sup>15</sup>Note that the probability that households do not access to the insurance agency goes to zero,  $\lim_{t \rightarrow \infty} \delta^t = 0$ .



any period  $t$ , after insurance payments (if any), are given by

$$A_t^{t-s}(i) = (1 + i_{t-1}) B_t^{t-s}(i) [1 - \omega + \omega P_t] + \tilde{T}_t^{t-s}(i), \quad (14)$$

where  $B_t^{t-s}(i)$  denotes household holdings of nominal assets at the beginning of period  $t$  and  $B_t^{T+1} = 0$ ;  $i_{t-1}$  is a nominal interest rate between periods  $t-1$  and  $t$ ; and  $\omega$  is a constant share of the portfolio invested in real indexed bonds.<sup>16</sup> It is important to note here that the insurance transfer  $\tilde{T}_t^{t-s}(i)$  depends only on the history of aggregate and idiosyncratic exogenous states, not on the choice of  $B_t^{t-s}(i)$ . This implies that households cannot manipulate the amount of the transfer  $\tilde{T}_t^{t-s}(i)$  by choosing a lower level of  $B_t^{t-s}(i)$ .

The end-of-period asset position of a household is given by

$$\phi^{t-s+1} B_{t+1}^{t-s}(i) = A_t^{t-s}(i) - P_t c_t^{t-s}(i) + W_t \theta^{t-s}(i) h_t^{t-s}(i) + D_t^{t-s}(i) + G_t^{t-s}, \quad (15)$$

where  $D_t^{t-s}(i)$  denotes the lump-sum distribution of nominal profits from monopolistically competitive firms; and  $G_t^{t-s}$  denotes lump-sum transfers from the government. Additionally, I assume that there are annuity markets in which households insure themselves against mortality risk. Specifically, every household continues to keep the assets of the deceased in the same generation. This implies that  $B_{t+1}^{t-s}(i)$  is multiplied by  $\phi^{t-s+1}$  in the left side of equation (15).

**Optimality Conditions for Households** The Euler equation for a generation  $t-s$  household with the current type  $f$  and any type history  $\tau^t$  is given by

$$\lambda_t^{t-s,f} = \beta \phi^{t-s+1} (1 - \delta) E_t \left[ \frac{(1 + i_t)}{\Pi_{t+1}} \tilde{\lambda}_{t+1}^{t-s+1} \right] + \beta \phi^{t-s+1} \delta E_t \left[ \frac{(1 + i_t)}{\Pi_{t+1}} \lambda_{t+1}^{t-s+1,f} \right], \quad (16)$$

$$\lambda_t^{t-s,f} = v(Z^{t-s}) (c_t^{t-s,f})^{-\sigma}, \quad \lambda_t^{t-s,n} = v(Z^{t-s}) (c_t^{t-s,n})^{-\sigma}, \quad (17)$$

where  $\tilde{\lambda}_{t+1}^{t-s+1} = \kappa^{t-s+1} \lambda_{t+1}^{t-s+1,f} + (1 - \kappa^{t-s+1}) \lambda_{t+1}^{t-s+1,n}$ . The Euler equation for a household with the current type  $n$  and any type history  $\tau^t$  is given by

$$\begin{aligned} \lambda_t^{t-s,n} - \varkappa' (n_t^{t-s,n}) &= \beta \phi^{t-s+1} (1 - \delta) E_t \left[ \frac{(1 + i_t)}{\Pi_{t+1}} \tilde{\lambda}_{t+1}^{t-s+1} \right] \\ &+ \beta \phi^{t-s+1} \delta E_t \left[ \frac{(1 + i_t)}{\Pi_{t+1}} (\lambda_{t+1}^{t-s+1,n} - \varkappa' n_{t+1}^{t-s+1,n}) \right] \end{aligned} \quad (18)$$

<sup>16</sup>Ideally, households should be able to choose real and nominal assets independently. However, it is well known that this causes indeterminacy because households are indifferent between holding nominal or real assets where there are no arbitrage conditions.

Both equations (16) and (18) can be solved forward:

$$\begin{aligned} \lambda_t^{t-s,f} = & \sum_{k=0}^{T-1-(t-s)} \beta^{k+1} \bar{\phi}_k (1-\delta) \delta^k E_t \left[ \frac{(1+i_{t+k})}{\Pi_{t+1+k}} \tilde{\lambda}_{t+1+k}^{t-s+1+k} \right] \\ & + \beta^{T+1-t-s} \bar{\phi}_{T-(t-s)} \delta^{T-(t-s)} E_t \left[ \frac{(1+i_{T+s})}{\Pi_{T+s+1}} \tilde{\lambda}_{T+s+1}^{T+1} \right], \end{aligned} \quad (19)$$

$$\begin{aligned} \lambda_t^{t-s,n} - \mathcal{Z}' \left( n_t^{t-s,n} \right) = & \sum_{k=0}^{T-1-(t-s)} \beta^{k+1} \bar{\phi}_k (1-\delta) \delta^k E_t \left[ \frac{(1+i_{t+k})}{\Pi_{t+1+k}} \tilde{\lambda}_{t+1+k}^{t-s+1+k} \right] \\ & + \beta^{T+1-t-s} \bar{\phi}_{T-(t-s)} \delta^{T-(t-s)} E_t \left[ \frac{(1+i_{T+s})}{\Pi_{T+s+1}} \tilde{\lambda}_{T+s+1}^{T+1} \right], \end{aligned} \quad (20)$$

where  $\bar{\phi}_k = \prod_{i=0}^k \phi^{t-s+1+i}$ .<sup>17</sup>

Equations (19) and (20) tell us that because  $\tilde{\lambda}$ , which appears in the right-hand side, is independent of the history of the type,  $\{\lambda_t^{t-s,f}, \lambda_t^{t-s,n}, n_t^{t-s,n}\}$  is history independent; all the households in the same generation with the same type choose the same amount of consumption and labor supply in equilibrium.<sup>18</sup> While the type- $n$  households, which suffer from a negative labor productivity shock, are natural borrowers in the economy, the shock is temporary with  $\delta < 1$  or they expect the access to the insurance agency eventually.

A household, which is a wage-taker, supplies hours  $h_t^{t-s,\tau}$  that satisfies the labor supply equation:

$$\frac{\theta^{t-s,\tau} W_t}{P_t} = \frac{\left( h_t^{t-s,\tau} \right)^\psi}{\lambda_t^{t-s,\tau}}. \quad (21)$$

### 3.2 Standard Part of the Model

**Firms** A monopolistically competitive firm produces a differentiated good of type  $j$  using linear technology

$$Y_t(j) = A_t h_t(j), \quad (22)$$

where  $h_t(j) \equiv \int h_t(j;i) di$  is aggregate labor hired of type  $j$ ; and  $h_t(j;i) = \theta^{t-s,\tau_t(i)} h_t^{t-s,\tau_t(i)}(j)$  is the efficiency units of labor. Households consume a Dixit-Stiglitz aggregate of differentiated

<sup>17</sup>The last terms in equations (19) and (20) reflect the assumption that all households have access to the insurance agency at the end of their life ( $\delta = 0$  at the end of the life cycle).

<sup>18</sup>Equation (15) can be written as  $n_t^{t-s,n} = -c_t^{t-s,n} + \frac{W_t}{P_t} \theta^{t-s,n} h_t^{t-s,n} + \frac{D_t^{t-s,n}}{P_t} + \frac{G_t^{t-s}}{P_t}$ . If households with the same type choose the same  $c_t$  and  $h_t$ , then their choice of  $n_t$  is also the same. That is, this equilibrium is consistent with equation (20).

goods,  $Y_t = \left[ \int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}$ , where  $\varepsilon > 1$  is the constant elasticity of substitution. This implies that each firm faces the demand schedule:

$$Y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\varepsilon} Y_t,$$

and the corresponding price index can be written  $P_t = \left[ \int_0^1 P_t(j)^{1-\varepsilon} dj \right]^{1/(1-\varepsilon)}$ .

Before addressing the firm's pricing decision, consider its cost minimization problem:

$$\min_{H_t(j)} \frac{W_t}{P_t} h_t(j) + \varphi_t (Y_t(j) - A_t h_t(j)),$$

where  $\varphi_t$  is a Lagrangian multiplier associated with the production function (22). The first order condition is given by

$$\frac{W_t}{P_t} = \varphi_t A_t. \quad (23)$$

$\varphi_t$  is thus defined as the firm's real marginal cost.

The firm's pricing setting problem is then to choose  $P_t(j)$  to maximize its profits. An individual firm takes as given the demand curve, the price index  $P_t$ , and the wage  $W_t$ , and chooses its price  $P$  to maximize expected discounted profits. However, each firm has a sticky price, in that it can change the price of its product in a given period with probability  $1 - \alpha$  (Calvo 1983). This part of the model remains the same as a standard RANK model. Solving the firm's price setting decision problem, we obtain the standard Phillips curve (after log-linearization):

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \hat{\varphi}_t, \quad (24)$$

where  $\kappa = \frac{(1-\varepsilon)(1-\beta\varepsilon)}{\varepsilon}$ ,  $\pi_t = \log \frac{P_t}{P_{t-1}}$ ,  $\hat{\varphi}_t = \log \frac{\varphi_t}{\varphi}$ , and  $\varphi = \frac{\varepsilon-1}{\varepsilon}$ .

In a model in which households are heterogeneous, how firms' profits are distributed across households will affect equilibrium dynamics. As in Kaplan, Moll, and Violante (2018b), I assume that profits are distributed in proportion to household productivity each period in a lump-sum manner,

$$D_t^{t-s, \tau_t(i)} = \frac{\theta^{t-s, \tau_t(i)}}{E[\theta]} \int \left[ P_t(j) - \frac{W_t}{A_t} \right] Y_t(j) dj. \quad (25)$$

This implies that households' additional income as the profit-sharing component of worker compensation (e.g., dividends, bonuses) is proportional to their labor income.

**Monetary Policy** Monetary policy is straightforward.

$$1 + i_t = (1 + i^*) \left( \frac{1 + \pi_t}{1 + \pi^*} \right)^{\phi_\pi} e^{\varepsilon_t^m}, \quad (26)$$

where  $i^*$  is a target interest rate and  $\pi^*$  is the central bank's inflation target, which is assumed to be equal to 0.

**Fiscal Policy** The government budget constraint is given by

$$B_{t+1} = G_t + (1 + i_{t-1}) B_t. \quad (27)$$

I suppose that the government runs a balanced budget to maintain a stable level of real debt in each period,  $\frac{B_{t+1}}{P_t} = \bar{b}$  for all  $t$  where  $\bar{b}$  denotes (real) government. Then the government budget constraint becomes

$$\bar{b} = \frac{G_t}{P_t} + (1 + i_{t-1}) \frac{\bar{b}}{1 + \pi_t}. \quad (28)$$

**Market Clearing** The labor market clears:

$$h_t = \int h_t(j) dj. \quad (29)$$

The goods market clears:

$$\sum_{t-s=0}^T \mu^{t-s} c_t^{t-s} \equiv c_t = Y_t = \frac{A_t h_t}{\Delta_t}, \text{ where } c_t^{t-s} = \int c_t^{t-s, \tau_t(i)} di. \quad (30)$$

where  $\Delta_t \equiv \int_0^1 \left( \frac{P_t(j)}{P_t} \right)^{-\varepsilon} dj$  is the measure of price dispersion.

The bond market clears:

$$\sum_{t-s=0}^T \mu^{t-s} \int \frac{B_{t+1}^{t-s}(i)}{P_t} di = \bar{b}. \quad (31)$$

### 3.3 The Dynamics of Savings

The insurance contract guarantees that all households  $t - s$  with access to the insurance agency have identical post-transfer wealth,  $\tilde{A}_t^{t-s}$ , at the beginning of period  $t$ . Integrating equation (14) over all households  $t - s$  with access to the insurance agency in period  $t$ , the post-transfer wealth

(per capita),  $\tilde{A}_t^{t-s}$ , is given by

$$\tilde{A}_t^{t-s} = P_{t-1} (1 + i_{t-1}) \bar{b}_t^{t-s} [1 - \omega + \omega P_t],$$

where  $\bar{b}_t^{t-s} = \int B_t^{t-s-1}(i) di$ . This equation is derived using equation (13) and the fact that all households have a equal chance to access to the insurance agency, so that the pooling of wealth (per capita) of those households is drawn randomly from among all households. The beginning-of-period wealth of households who do not have access to the insurance agency is instead given by equation (14) with  $\tilde{T}_t^{t-s}(i) = 0$ .

For the aggregation of individual savings, it is convenient to classify households  $t - s$  according to the period in which they last had access to the insurance agency. Let us denote the type of a household who last had access to the insurance agency  $j$  periods ago by  $\tau(j)$ . Aggregating across individual generation  $t - s$  households, the aggregate end-of-period assets of the households of type- $\tau$ ,  $B_{t+1}^{t-s,\tau}$ , can be written as

$$B_{t+1}^{t-s,\tau} \equiv \sum_{j=0}^{t-s} \kappa^{t-s-j,\tau} (1 - \delta) \delta^j B_{t+1}^{t-s}(\tau(j)), \quad (32)$$

where  $B_{t+1}^{t-s}(\tau(j))$  is given as follows.  $B_{t+1}^{t-s}(\tau(0))$ , the end-of-period assets of the households that have access to the insurance agency in period  $t$  and then draw type  $\tau$ , will equal

$$\phi^{t-s+1} B_{t+1}^{t-s}(\tau(0)) = \tilde{A}_t^{t-s} - P_t c_t^{t-s,\tau} + W_t \theta^{t-s,\tau} h_t^{t-s,\tau} + D_t^{t-s,\tau} + G_t. \quad (33)$$

The right hand side of equation (33) takes into account the fact that the households of the same type within the same generations make the same choices.  $B_{t+1}^{t-s}(\tau(j+1))$ , the end-of-period assets of the households who have not had access to the insurance agency, is expressed as

$$\phi^{t-s+1} B_{t+1}^{t-s}(\tau(j+1)) = B_t^{t-s-1}(\tau(j)) - P_t c_t^{t-s,\tau} + W_t \theta^{t-s,\tau} h_t^{t-s,\tau} + D_t^{t-s,\tau} + G_t, \quad (34)$$

for any  $j \geq 0$ .

According to equation (32), the law of motion for aggregate savings for each type- $\tau$  can be obtained by summing equation (33), multiplied by  $\kappa^{t-s,\tau} (1 - \delta) P_t^{-1}$ , and equation (34) for each

value  $j \geq 0$ , multiplied by  $\kappa^{t-s-j,\tau} (1 - \delta) \delta^{j+1} P_t^{-1}$  :

$$\begin{aligned} \phi^{t-s+1} b_{t+1}^{t-s,\tau} &= \kappa^{t-s,\tau} (1 - \delta) (1 + i_{t-1}) \bar{b}_t^{t-s} \left( \frac{1 - \omega}{\Pi_t} + \omega \right) + [\delta (1 + i_{t-1}) b_t^{t-s-1,\tau} \left( \frac{1 - \omega}{\Pi_t} + \omega \right) \\ &\quad - \bar{\kappa}^{t-s,\tau} \left( c_t^{t-s,\tau} - \frac{W_t}{P_t} \theta^{t-s,\tau} h_t^{t-s,\tau} - \frac{D_t^{t-s,\tau} + G_t}{P_t} \right)], \end{aligned} \quad (35)$$

where

$$\begin{aligned} \bar{b}_{t+1}^{t-s} &= b_{t+1}^{t-s,f} + b_{t+1}^{t-s,n}, \\ \kappa^{t-s,f} &= \kappa^{t-s}, \quad \kappa^{t-s,n} = (1 - \kappa^{t-s}), \\ \bar{\kappa}^{t-s,\tau} &= \bar{\kappa}^{t-s-1,\tau} \delta + (1 - \delta) \kappa^{t-s,\tau}, \\ \bar{\kappa}^{0,\tau} &= \kappa^{0,\tau}. \end{aligned}$$

Integrating equation (35) over each type of the households of generation  $t - s$ , we obtain the expression for the dynamics of aggregate savings for generation  $t - s$ :

$$\begin{aligned} \phi^{t-s+1} \bar{b}_{t+1}^{t-s} &= (1 + i_{t-1}) \bar{b}_t^{t-s} \left( \frac{1 - \omega}{\Pi_t} + \omega \right) \\ &\quad - \left( \left( \bar{\kappa}^{t-s} c_t^{t-s,f} + (1 - \bar{\kappa}^{t-s}) c_t^{t-s,n} \right) - W_t \left( \bar{\kappa}^{t-s} \theta^{t-s,f} h_t^{t-s,f} + (1 - \bar{\kappa}^{t-s}) \theta^{t-s,n} h_t^{t-s,n} \right) - \frac{D_t^{t-s,\tau} + G_t}{P_t} \right). \end{aligned} \quad (36)$$

Thus the dynamics of aggregate savings for each generation depend on the aggregate savings in the previous period, its own past level, and the generation consumption and the various sources of income such as labor income, dividends, and transfers.<sup>19</sup>

The system of equations that describe the model consist of the households' optimality conditions, (16)-(21), the distribution of firm profits (25), the market clearing conditions, (29)-(31), the aggregate-supply relationship, (23) and (24), and the dynamics of aggregate savings, (35)-(36), together with a monetary policy reaction function (26) to specify the nominal interest rate  $i_t$  and a fiscal rule (28) to specify the government debt  $G_t$ . Those equations suffice to determine the evolution of the variables  $\left\{ c_t^{t-s,\tau}, h_t^{t-s,\tau}, \lambda_t^{t-s,\tau}, b_{t+1}^{t-s,\tau}, \bar{b}_{t+1}^{t-s}, W_t, \Pi_t, \varphi_t, Y_t, i_t, D_t^{t-s,\tau}, G_t \right\}$ .

<sup>19</sup>As the model is non-Ricardian with heterogeneous households, we need to specify the evolution of savings for each generation along with the distribution policy of firm profits and the fiscal policy.

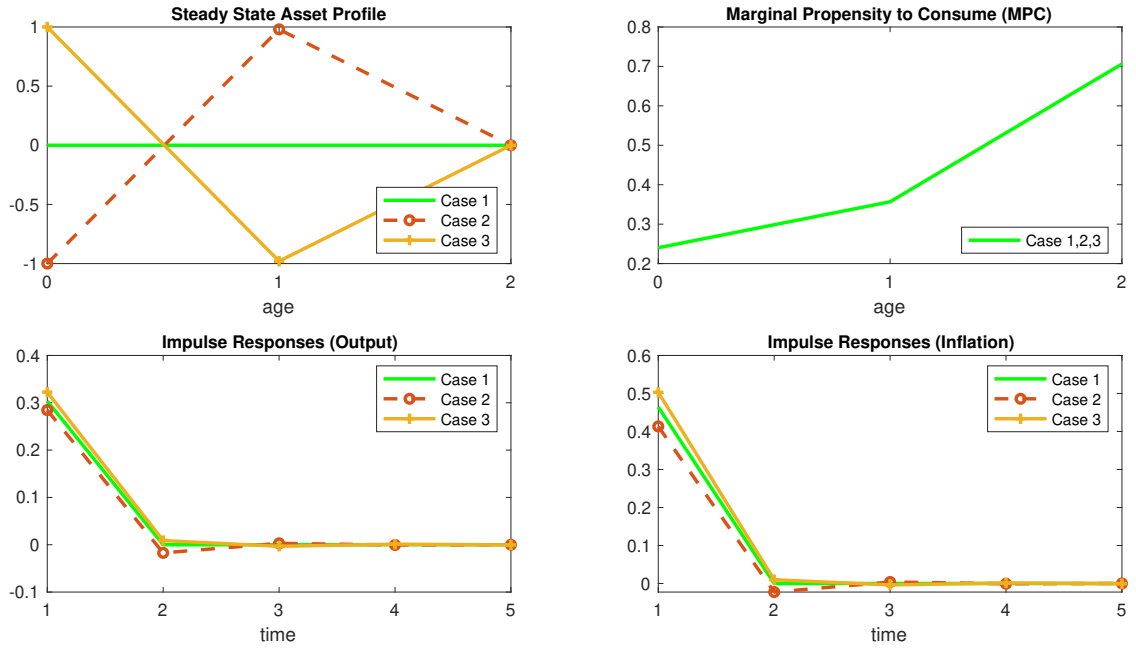


Figure 1: Comparisons of Simple OLG Economies

### 3.4 Quantitative Results

In this section I numerically analyze the effects of a monetary shock. Before I turn to fully calibrated economies, I present a simple three-generation model to illustrate how the household balance-sheet channel operates in an OLG economy.

#### 3.4.1 Simple Three-generation Model

I start by comparing three simple OLG economies to present how age heterogeneity in an OLG setting affects the impact of monetary policy. In this simple setting, households live for only three periods. I use the same values of parameters as presented in Table 2 in the next section, but I abstract from demographics and financial frictions. I also abstract from earning channels by using the same labor productivity profiles across all cases ( $\theta = 1$  for all ages and all cases).

Here I focus on the role of ex-ante heterogeneity in households' asset positions. In order to induce the accumulation of assets over the life cycle, I add a lump-sum transfer,  $e_i$ , to the budget constraint of households of age  $i$ . Each case is different in lump-sum transfers  $\{e_i\}$  among ages, which generates a different steady-state asset profile over the life cycle,  $\{b_i\}$ .

In Case 1, there is no transfer among generations,  $e_i = 0$  for all  $i$ , which implies that there is

Table 1: Effect of the Household Balance Sheet in Simple OLG Models

	Case 1	Case 2	Case 3
Aggregate consumption (model)	0.31%	0.28%	0.33%
Aggregate consumption (predicted)	0.31%	0.28%	0.33%
$M$ (Aggregate Income channel)	0.58	0.58	0.58
$S$ (Substitution channel)	0.42	0.42	0.42
$\Xi_P$ (Redistribution elasticity for $P$ )	0	0.07	-0.07
$\Xi_R$ (Redistribution elasticity of $R$ )	0	0.06	-0.06

no asset accumulation. In this case, dynamic responses to an exogenous shock closely resemble those from RANK. In case 2, I set  $e_0 = -1$ ,  $e_1 = 2$ , and  $e_2 = -1$ . This leads households of age 0 to borrow and age 1 to save (Figure 1, upper-left panel). Young generations are borrowers as household income is greater later in the life cycle. Case 3 is the opposite to Case 2,  $e_0 = 1$ ,  $e_1 = -2$ , and  $e_2 = 1$ ; young generations are savers as household income is greater early in the life cycle.

In contrast to an infinite horizon model, an OLG structure also results in different MPCs across age groups (Figure 1, upper-right panel). Older generations tend to have greater MPCs, simply because they have a shorter planning horizon to smooth their consumption.

In order to decompose the effect of monetary policy on aggregate consumption, I calculate redistribution elasticities for a real interest rate,  $\Xi_R$ , and inflation,  $\Xi_P$ , following Auclert.<sup>20</sup> Those redistribution elasticities may help us to understand how households' heterogeneous exposure to an interest rate and inflation among different age groups can affect aggregate outcomes.

The lower left and right panels show the impulse responses of output and inflation to a temporary cut in the nominal interest rate. In addition to the response of aggregate consumption from direct model simulations, Table 1 also reports the predicted response of aggregate consumption calculated using Auclert's statistics. As can be seen in the table, the predicted responses are almost the same as the actual responses, meaning that those statistics would provide a good approximation of actual responses in an environment in which there is no financial friction.

In comparison to Case 1 which resembles RANK, output and inflation responses are dampened in Case 2, while they are amplified in Case 3. Those different responses can be explained how the revaluation of households' wealth, caused by changes in an interest rate and inflation, interacts with the MPCs of the households of different age groups. For instance, in case 2, age-1

<sup>20</sup>Theorem 3 in Auclert (2019) is used to calculate relevant statistics.  $M = E \left( MPC_i \frac{Y_i}{E[c_i]} \right)$ ,  $\Xi_Y = Cov \left( MPC_i, \frac{Y_i}{E[c_i]} \right)$ ,  $\Xi_P = Cov \left( MPC_i, \frac{NNP_i}{E[c_i]} \right)$ ,  $\Xi_R = Cov \left( MPC_i, \frac{URE_i}{E[c_i]} \right)$ ,  $S = E \left[ (1 - MPC_i) \frac{c_i}{E[c_i]} \right]$ .



Table 2: Parameter Values for the Model

Parameter	Description	Value
$\beta$	time discount factor	0.98
$\sigma$	risk aversion	1
$\frac{1}{\psi}$	elasticity of labor supply	1/2
$\theta$	constant elasticity of substitution	10
$\alpha$	Calvo parameter	0.5
$\phi^\pi$	Taylor coefficient	1.5
$\omega$	constant share of indexed bonds	2/3

households (savers), whose UREs are positive, face a negative wealth effects from a fall in interest rates. But due to a shorter planning horizon, Age-1 households have higher MPCs than age-0 households, implying that MPCs and UREs covary positively. Noting that a change in the consumption of unconstrained households is given by their MPC out of an change in wealth, a decrease in the consumption of age-1 households is larger than an increase in that of age-0 households. Therefore, the redistributive channels dampens aggregate responses. Similarly, in case 3, aggregate responses are amplified as MPCs and UREs covary negatively.

In the next section, I present a fully calibrated model that resembles the U.S. economy to see how the redistribution channel alters the effects of monetary policy.

### 3.4.2 Calibration

Many of the model's parameters are also parameters of the basic New Keynesian model. These include the discount rate  $\beta$ , the coefficient of risk aversion  $\sigma$ , the inverse of the Frisch elasticity of labor supply  $\psi$ , the desired markup of monopolistically competitive firms  $\frac{\theta}{\theta-1}$ , the probability of maintaining a fixed price  $\alpha$ , the coefficient associated with the Taylor rule  $\phi^\pi$ . As Table 2 reports, I choose conventional values for these parameters. I choose the degree of inflation indexation  $\omega = 2/3$  in order to take into account the fact that a considerable portion of households' portfolio is invested in real assets. I also conduct robust checks using the values  $\omega = 1/3$  and  $\omega = 4/5$  and find that they do not affect the conclusion of the paper.

**Demographics** I suppose each generation lives for 60 periods. In order to take into account the effects of demographic structure on consumption and saving, mortality rates are taken from the Static Mortality Tables of the Internal Revenue Service. The resulting distribution of the

population by age, which is reported in Figure 2, is similar to that of the U.S. in 2017.<sup>21,22</sup>

Parameter  $\bar{v}$ , which is associated with the age-dependent preference shifter, is set to 0.2315, following similar strategies used by Fernández-Villaverde and Krueger (2007) and Attanasio, Kitao, and Violante (2007). The preference shifter is necessary to generate a realistic consumption profile over life cycles.<sup>23</sup> The calibration of the age profile of household characteristics  $Z^{t-s}$  is done to match the age profile of consumption, as documented for example in Gourinchas and Parker (2002). They show about a 16 percent rise from the initial value to its peak at age 43, followed by a continuous decline until the end of the life cycle. Consumption at the end of the life cycle is about 60% of the level at its peak. Figure 2 also reports the steady-state consumption across generation in the model.

**Labor Productivity** I allow for heterogeneity among agents, in a manner, by setting generation-specific labor productivity. By choosing the life-cycle profiles of labor productivity  $\{\theta^{t-s}\}$ , we can construct realistic life-cycle profiles of income. In Gourinchas and Parker (2002), households income peaks around age 50. Income at age 50 is higher by about 40% than income at age 25, but income at age 65 is lower by about 15% than income at age 50.<sup>24</sup> I choose the generation-specific labor productivity to match such life-cycle profiles of income using spline methods. I assume that households under age 25 do not participate in the labor market, and that households retire at age 65. As no labor income is available for these households, the labor productivity of households outside working life, i.e., those below age 25 or over 65, is set to 0. The expected changes in income create households' desire to smooth consumption over their life cycle, which leads them to accumulate wealth. Figure 2 reports labor productivity over the life cycle. Generation-specific labor productivity  $\theta$  is normalized by the level of productivity at age 25.

**Financial Frictions** To compare this model to the model without financial frictions, the new parameters that are needed are those relating to the evolution of types of households and the specification of financial frictions. These parameters include the proportion  $\kappa^{age}$  of households that are constrained, the degree of financial frictions,  $\tilde{\alpha}$ , the magnitude of the labor productivity shock (expression (10)),  $\xi$ , and the degree of persistence  $\delta$  of a household's type.

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<sup>21</sup>The age group of 20 to 24 years represents 10% (data) vs 8.8% (model) of the population over 20. For the age group 25-34, the values are 15.8% (data) vs 17.5% (model). For 35-54, 34.2% (data) vs 34.8% (model). For 55-64, 17.1% (data) vs 16.8% (model). For 65+, 21% (data) vs 21.9% (model).

<sup>22</sup>The impact of monetary policy may be different for countries with older populations. In Japan, the age group of 65 years and over accounts for about 34% of the population over age 20.

<sup>23</sup>See also Gourinchas and Parker (2002). Without this preference shifter, consumption is constant over the life-cycle at the steady state.

<sup>24</sup>Auclert (2019) also obtains similar statistics from PSID and SHIW data.



Figure 2: Labor Productivity, Demographic Structure, Steady State Consumption, and Steady State Asset

In the calculation reported here, I assume that  $\kappa^{age} = 0.45 - \frac{0.55}{45} \times age$ . This implies that the proportion of households that are constrained among the youngest generation (the age of 20) is 0.45 and households become less credit constrained over the life cycle. Gourinchas (2000) and Gourinchas and Parker (2002) show that younger generations are more credit constrained and their marginal propensity to consume is higher than older generations. The choice implies that on average, the proportion of households that are constrained is 0.3. This matches the value in Kaplan, Violante, and Weidner (2014), who report that a reasonable estimate for the proportion of hand-to-mouth consumers in the United States is 0.3.

The other three parameters are chosen to target the following three steady state outcomes. First, the average MPC across all households is 0.24, which is within a reasonable range of values reported in the literature (e.g., Johnson, Parker, and Souleles 2006; Parker, Souleles, Johnson, and McClelland 2013; Broda and Parker 2014; Misra and Surico 2014). I also calculate results for an alternative case in which the average MPC equals 0.30, and the results are similar. Second, the variance of log-consumption is equal to 0.23.<sup>25</sup> This number is comparable to 0.25 in Krueger

<sup>25</sup>The equations for the evolution of wealth can be given in terms of the aggregate for each generation. For this reason, I instead adopt the consumption inequality seen in the data as a measure of inequality.

and Perri (2006) and 0.32 in Heathcote, Perri, and Violante (2010). Third, the steady-state level of government debt relative to GDP,  $\frac{\bar{b}}{y}$ , equals 0.93. This is in line with the debt-to-GDP ratio of the U.S., which has been above 0.9 since 2011.<sup>26</sup>

To obtain these steady state values, I set the degree of financial frictions,  $\tilde{\kappa}$ , to 7, I set the magnitude of the labor productivity shock,  $\zeta$ , to 0.83, and I set the degree of persistence of a household's type,  $\delta$ , to 0.9.<sup>27</sup> The choice of  $\delta$  means that the expected time until a household has access to the insurance agency is 10 years. Table 3 reports results for other values of  $\tilde{\kappa}$  for comparison of models.

With those parameter values, the model also replicates some measures of inequality. The Gini coefficient of household consumption at the steady state is 0.21. The 90-10 ratio for the measure of consumption at the steady state is 3. Meyer and Sullivan (2013) and Meyer and Sullivan (2017) show that the 90-10 ratio has fluctuated around 4.0 since 1982 in the U.S.. Therefore, the values of these measures in the model are close to their counterparts in real data. Figure A.3 in Appendix C also reports MPCs across generations and intertemporal MPCs.

**Model Comparisons** I compare the main model to a 'Homogeneous' model, a case in which households are homogeneous; all households are financially unconstrained and the household's labor productivity remains the same throughout the life cycle, but the model is otherwise identical. As a result, nominal asset positions are zero across generations at the steady state, and thus we abstract from the redistributive effects of monetary policy. The behavior of households is similar to a standard New Keynesian model. In the 'Homogeneous' model, I set  $\theta^{t-s} = 0.6125$  for all generations, because this produces the same steady-state output as the 'OLGNK' model.<sup>28</sup>

I also use a model without financial frictions—the 'OLGNK-noFF' model—for the purpose of comparisons, in which financial frictions are excluded because I set  $\tilde{\kappa} = 0$ . The type- $n$  households that receive a negative labor productivity shock can borrow as much as they want in order to smooth their consumption. As the type- $n$  households are not constrained in this model, they face the same kind of the Euler equation (16) as the type- $f$  households face. The model is otherwise identical to the 'OLGNK-FF' model.

<sup>26</sup>According to the Worldbank database, the debt-to-GDP ratio of the U.S. has been rising since 2007 and was 0.99 as of 2016.

<sup>27</sup>Also I set  $\kappa\tilde{\kappa}$  to 0.5, which is the coefficient on  $\kappa'$  ( $n$ ). A lower value of  $\zeta$  would induce constrained households to borrow less at the steady state, which increases households' aggregate savings,  $\bar{b}$ . For  $\zeta = 0.8$ ,  $\frac{\bar{b}}{y}$  equals 0.42. However, the qualitative results are very similar.

<sup>28</sup>As is consistent with a standard New Keynesian model, impulse responses to a monetary shock in the 'Homogeneous' model do not depend on a specific choice of  $\theta$ .

In numerical exercises, I log-linearize all OLGNK models around the same steady state for comparison purposes.<sup>29,30</sup>

### 3.4.3 Numerical Results: Homogeneous and OLGNK-noFF Models

In order to understand how the age-productivity profile  $\theta^{t-s}$  affects the predicted responses to a monetary shock, I begin by comparing the ‘Homogeneous’ and ‘OLGNK-noFF’ models. Comparison of these cases shows the difference made by household asset positions across generations.

**Impulse Response to a Monetary Shock** I consider the equilibrium responses to a temporary monetary policy shock, represented by a unit decrease in  $\varepsilon_t^m$ . Figure 3 shows the impulse responses of output and inflation along with the nominal interest rate. An expansionary monetary shock generates a boom accompanied with increased inflation. However, the impact of the shock on output and inflation is smaller in the ‘OLGNK-noFF’ model than the other model. The immediate impact of the shock on output is lower by about 15% in the ‘OLGNK-noFF’ model, and output is below the steady state level in the next few periods. Also, the initial boom is followed by a slump for longer periods before tracking back to the steady state through endogenous movements in the wealth distribution.<sup>31</sup>

The reason for such different responses is that in the ‘Homogeneous’ model, households immediately increase demand (through the intertemporal substitution channel) to the extent to which the higher wages induced by an increase in demand allow them to raise consumption today without sacrificing any consumption in the future.

Figure 2 indicates that households’ asset positions across generations play a role in producing my numerical results. In the ‘Homogeneous’ model, households are identical and thus their asset position is zero, which implies that Ricardian neutrality holds like in RANK. In the ‘OLGNK’ model, households are borrowers when they are younger. But as consumption-smoothing households accumulate wealth for retirement, older generations hold positive asset positions.

$dc_0/c_0$ (model)	$dc_0/c_0$ (predicted)	S	$\Xi_P$	$\Xi_R$
0.26%	0.16%	0.933	0.084	0.218

<sup>29</sup>Although the ‘Homogeneous’ model has a different steady state to the other models, its first order dynamic responses only depend on the common parameters such as  $\beta$ ,  $\sigma$ , and  $\alpha$ .

<sup>30</sup>To simulate the model for a different value of  $\tilde{\kappa}$  at the same steady state, I add a constant to expression (12). Results are presented in Table 3.

<sup>31</sup>The persistence in the responses of inflation and wage is relatively short because of a low value of Calvo parameter (0.5).

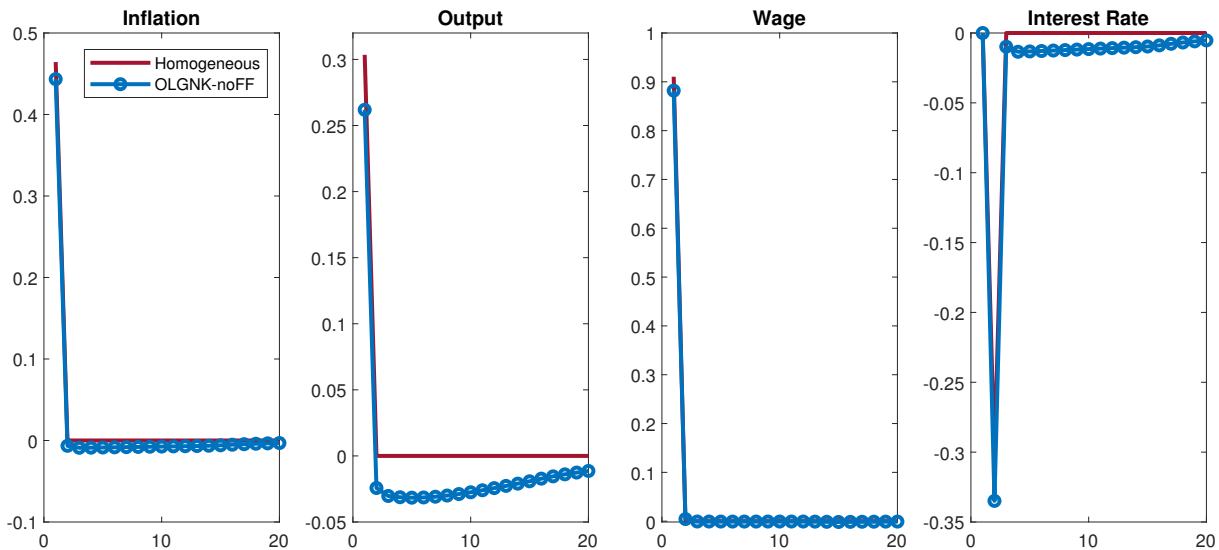


Figure 3: Temporary Nominal Interest Rate Cut: Homogeneous vs OLGNK-NoFF

The positive covariances, reported in the above table, between MPCs and households' exposure to an interest rate and inflation suggest that the effect of monetary policy is dampened through the redistribution channel.<sup>32</sup> The predicted consumption response calculated using those covariances also shows that the aggregate response is smaller in OLGNK relative to RANK.

While the sign of the redistributive effect is correct, there is some quantitative difference between the actual and predicted responses. This is because even a transitory shock leads to endogenous persistence through the evolution of household balance sheets, in which case it is harder to apply Auclert's decomposition.

In order to overcome this issue, I follow Kaplan, Moll, and Violante (2018b) to decompose the direct effect,  $\frac{\partial c_0(i,W,D,G)}{\partial i}$ , into substitution and wealth effects.<sup>33</sup> The wealth effect of the interest rate change captures the effect of household exposure to an interest rate and inflation on consumption. Figure 4 shows that the wealth effect on consumption varies across generations and it is negative for older generations. In aggregate, out of the total direct effect of 0.21 percent, the substitution and wealth effects on consumption are 0.31 and  $-0.1$  percent respectively, implying

<sup>32</sup>I omitted  $\Xi_Y$  (covariance between earnings and MPCs) as it does not affect the aggregate outcome much here.  $\Xi_Y$  is negative but very close to 0, due to the proportional distribution of firm profits.

<sup>33</sup>The effects of the monetary shock on aggregate consumption can be decomposed into direct and indirect components,  $dc_t = \underbrace{\frac{\partial c_t}{\partial i} di_t}_{\text{direct effect}} + \underbrace{\frac{\partial c_t}{\partial W_t} dW_t + \frac{\partial c_t}{\partial D_t} dD_t + \frac{\partial c_t}{\partial G_t} dG_t}_{\text{indirect effects}}$ . I compute the direct effect numerically. The wealth effect is obtained by subtracting the substitution effect  $\sigma(1 - MPC_i)$  from the direct effect for each generation. For more details, see also Appendix B.

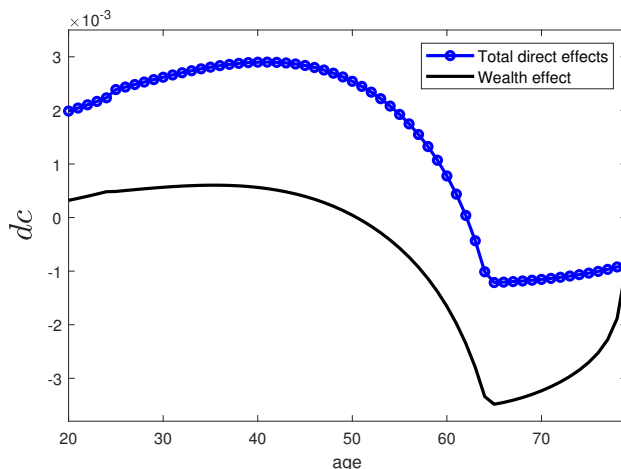


Figure 4: Wealth Effects of Monetary Policy in OLGNK

that the wealth effect is substantial.

**Redistributive Effects of Monetary Policy** Heterogeneity in households asset positions affects equilibrium dynamics. Even though a decrease in the interest rate raises the present value of wealth at time  $t$ , the purchasing power of some generations may decrease. This occurs because a lower interest rate also raises the present value of consumption at time  $t$  (equivalently, the price of consumption). This implies that it is necessary to adjust the present value of wealth using the price of consumption in order to correctly evaluate the impact of an interest rate cut on the value of household wealth.

To this end, I compute gains and losses for generations as a percentage deviation of the normalized present value of wealth from the steady state values throughout the rest of their life. The present value of wealth is normalized by the present value of consumption at the steady state, which is discounted by a new time path of  $\left\{ \frac{1}{1+i_k} \right\}_{k=t}^{\infty}$  after a monetary shock at time  $t$ .

Figure 5 shows the nominal asset holding profile and the redistributive effects of monetary policy by contrasting the two models. In the ‘Homogeneous’ model, households gain universally across generations from a monetary shock. Because the shock is transitory, older generations gain more in terms of the present value of wealth than younger generations. The redistributive effect is prominent in the ‘OLGNK-noFF’ model, in which different generations hold different asset positions. Noticeably, the monetary shock has a negative impact on the present value of wealth of older households. Older households (over the age of 51) tend to lose more, whereas younger households gain more from the monetary shock. In particular, a lower (real) interest

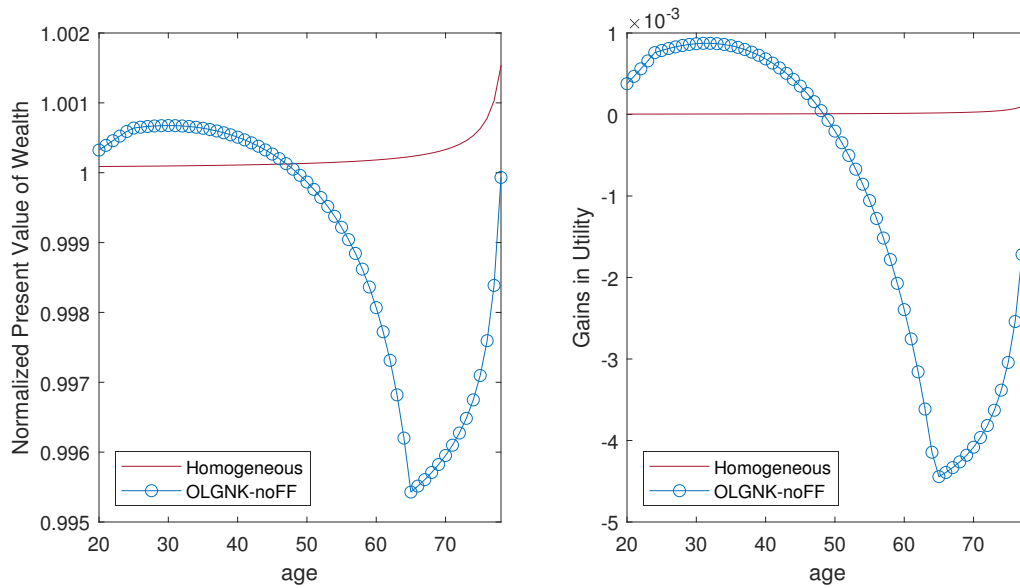


Figure 5: Redistributive Effects Across Generations

rate accompanied by higher inflation hurts middle generations who are closer to retirement. As they carry a large amount of savings for retirement, the wealth of these households is exposed directly to the lower interest rate, but wages do not rise sufficiently to offset this negative wealth effect of the monetary policy due to the subdued increase in aggregate demand. In contrast, households around the age of 35, who are borrowers, benefit most from a lower real interest rate.

In Figure 5, I also compute gains and losses for generations as a (cumulative/average) deviation of utility from the steady state values throughout the rest of their lives. The average gains (or losses) are computed by dividing cumulative gains by remaining years of life for each respective generation. In the ‘Homogeneous’ model, in which the equilibrium responses are expected to be similar to a standard NK model, all generations benefit from the monetary expansion and there is no distinctive redistribution across generations in this model. In terms of the average gains in utility, older generations benefit more than younger generations, because the boom is short-lived. In the ‘OLGNK-noFF’ model, the redistributive effects are noticeable, as older generations lose whereas gains accrue to younger generations. This is because older generations, who are savers, lose from higher inflation and a lower interest rate whereas younger generations, who are borrowers, gain from them.



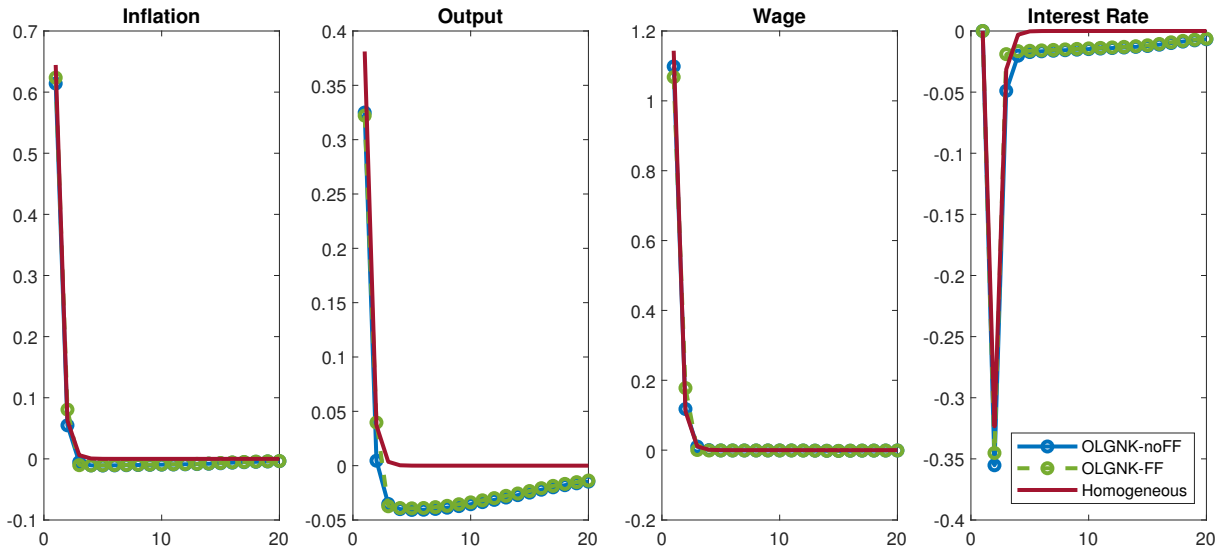


Figure 6: Persistent Nominal Interest Rate Cut ( $\rho = 0.3$ ): OLGNK vs OLGNK-noFF

### 3.4.4 Numerical Results: OLGNK-FF and OLGNK-noFF Models

Like a standard NK model, traditional interest channels are the main transmission mechanism of monetary policy in the OLGNK model without financial frictions. Also, the MPC among households is about  $1/60$ , which is much smaller than the values empirical studies suggest. In this section, I address these issues by assuming that there exist constrained households with  $\tilde{z} > 0$  and study how this assumption affects the previous results.

**Impulse Response to a Monetary Shock** The main result in this section is that the financial frictions I introduce in this study do not amplify equilibrium dynamics in response to a monetary policy shock. Indeed, equilibrium dynamic responses are still muted in the model with financial frictions. To illustrate this, I consider a monetary policy experiment using the ‘OLGNK-FF’ and ‘OLGNK-noFF’ models.

Figure 6 shows the impulse responses of output and inflation to a persistent monetary policy shock ( $\rho = 0.3$ ), which shows that the qualitative results do not change even when a monetary policy shock lasts a few periods.<sup>34</sup> The initial output response is 16 percent smaller in ‘OLGNK-FF’ than in ‘Homogeneous’. As the type- $n$  households cannot take on debt to take advantage of a lower interest rate, aggregate demand does not respond as much, which leads to a dampened response in output. Overall, the effect of a monetary shock in the ‘OLGNK-FF’ model is similar

<sup>34</sup>The impulse responses to a temporary shock is also very similar to Figure 3

to the one in the ‘OLGNK-noFF’ model.

**Relationship to TANK** The similar impulse responses of output from ‘OLGNK-ff’ and ‘OLGNK-noff’ model can be understood by analyzing the relationship between a change in consumption and other parameters. Roughly speaking,  $dC \approx MPC(dY + \omega \cdot dW) + \sigma C(1 - MPC)dR$ , where  $dY$  is a change in income (including both labor income and transfers),  $dW$  a change in wealth due to the interest and price changes, and  $\omega$  the sensitivity of the household demand to  $dW$ . In TANK,  $dW$  is 0 and therefore  $dC/C$  is constant in equilibrium ( $dC = dY$ ) regardless of the value of MPC, unless there is some heterogeneity in  $dY$  across households (Bilbiie 2020).<sup>35</sup> In my model,  $dW$  is positive, but  $\omega$  is small, which leads to similar observational responses in aggregate output. Appendix B further reports the decomposition of direct and indirect effects in OLGNK models.

Quantitatively, our number is much smaller than Kaplan, Moll, and Violante (2018a) in which the elasticity of consumption is larger by 50 percent relative to RANK. In a real world,  $\omega$  may vary depending on other financial conditions, but my point here is that unless monetary policy induces a large value of  $\omega$ , it might be difficult to obtain such a large effect through the redistribution channel.

**The Role of the Distribution of Firm Profits** I now discuss some implications of the distribution of lump-sum transfers in the OLG model. The literature has highlighted the role of the distribution of firm profits in the New Keynesian transmission mechanism (e.g., Broer, Harbo Hansen, Krusell, and Öberg 2019; Bilbiie 2020). Transfers received by households include firms’ profits,  $D$ , and government subsidies/taxes,  $G$ . As firm profits vary inversely with the real wage, a cut in an interest rate reduces the amount of the profit distribution to firms’ shareholders. I continue to assume that  $G$  is evenly distributed among households. Instead, I simulate the model for alternative assumptions about how firm profits are distributed among households.

In our benchmark case (labeled ‘Proportionate’) where firms’ are owned by households proportionally to their labor productivity  $\theta$ , the worker compensation, which is the sum of labor and capital incomes, is proportional to  $\theta$ . By comparison, in the ‘Disproportionate’ model, I suppose that all profits are distributed to the unconstrained households. This implies that the worker compensation received by the constrained households increases disproportionately when

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<sup>35</sup>The aggregate outcome in TANK is the same as in the representative agent NK model because a decreased direct response to the interest rate is offset by an increased indirect general equilibrium effect.

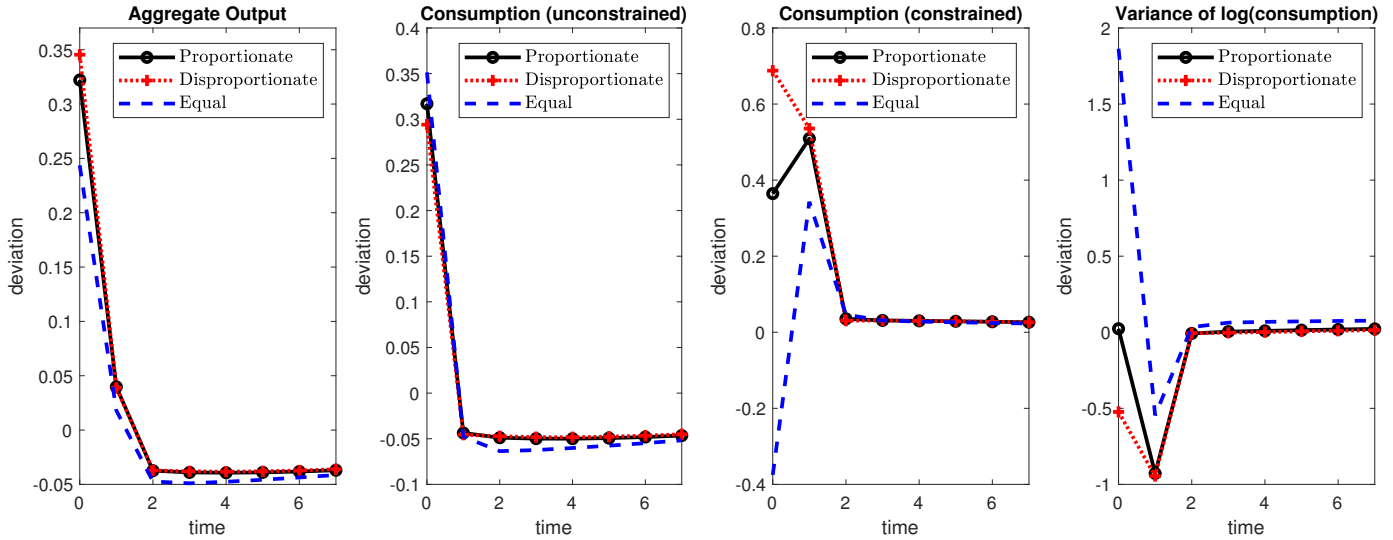


Figure 7: Role of the Distribution of Firm Profits

the real wage increases. The the ‘Equal’ model is contrast to the ‘Disproportionate’ model in that all profits are evenly distributed among all households. There is a disproportionate increase in the worker compensation received by the unconstrained households.

Figure 7 reports the impulse responses of output, consumption decomposition between the types, and inequality to a monetary policy shock. As one can expect, the inequality, which is measured by the variance of log consumption, is countercyclical in ‘Disproportionate’ and procyclical in ‘Equal’ (the last panel).<sup>36</sup> The first panel of the figure shows that across all models, the response of aggregate output is still dampened relative to RANK. The second and third panels compute the impulse responses of the (aggregate) consumption of the unconstrained and constrained households respectively. They show that how firm profits are distributed mainly affects the response of the consumption of the constrained households. Their consumption is highly sensitive to a lump-sum transfer due to their high MPCs.<sup>37</sup> Still its overall impact on the aggregate output is relatively small, because the aggregate consumption of the constrained households is about 13.8% of the total consumption in our benchmark case.

**Relationship to Auclert (2019)** Auclert (2019) categorizes the redistributive effects into an earnings heterogeneity channel, a Fisher channel, and an interest rate exposure channel. To evaluate these effects, he proposes sufficient statistics, such as the redistribution elasticities of

<sup>36</sup>It implies that, in the ‘Disproportionate’ model, inequality is countercyclical and an elasticity of income of the constrained to aggregate income,  $\chi$  in Bilbiie (2020), is larger than 1.

<sup>37</sup>There is also a large increase in the consumption of the constrained households at time 1, because, thanks to the high debt to gdp ratio (1.12 in my calibration), a low interest rate incurs a large tax rebate to households.

Table 3: Consumption and Redistribution Elasticities (Temporary Shock)

	OLG-noFF	Disproportionate	$\tilde{\kappa} = 7$	$\tilde{\kappa} = 15$
Aggregate Consumption (model)	0.26%	0.27%	0.26%	0.25%
Consumption_unconstrained (model)	0.24%	0.20%	0.22%	0.21%
Consumption_constrained (model)	0.02%	0.07%	0.04%	0.04%
Aggregate Consumption (Auclert)	0.16%	0.64%	0.63%	0.86%
Consumption_unconstrained (Auclert)	0.11%	0.13%	0.15%	0.15%
Consumption_constrained (Auclert)	0.05%	0.51%	0.48%	0.71%
$M$ (Income-weighted MPC)	0.0333	0.0381	0.0399	0.0510
$\Xi_Y$ (Redistribution elasticity for $Y$ )	-0.0423	-0.0843	0.0345	0.0620
$\Xi_P$ (Redistribution elasticity for $P$ )	0.0841	-0.3018	-0.2978	-0.5162
$\Xi_R$ (Redistribution elasticity for $R$ )	0.2178	-0.9881	-0.9744	-1.6578
$S$ (Substitution channel)	0.9320	0.8531	0.8506	0.8030
$E [MPC_i]$	0.06	0.24	0.24	0.35

consumption for real interest rates, prices, and income, which may be used to estimate the redistributive effects of monetary policy on aggregate outcomes. Table 3 calculates these redistribution statistics to evaluate whether they help us to predict the redistributive effects of monetary policy in the OLG framework. The table reports two sets of statistics. One set (labeled “model”) is calculated numerically from direct model simulations. The other set (those labeled “Auclert” or redistribution elasticity) consists of the redistribution elasticities proposed by Auclert, calculated using the equilibrium dynamics of output from model simulations. Table 4 reports the same set of statistics for a persistent monetary policy shock, which shows the same qualitative results.

As can be seen in Tables 3 and 4, the signs of redistribution elasticities for income  $\Xi_Y$ , prices  $\Xi_P$ , and real interest rates  $\Xi_R$ , obtained from model simulations are largely negative in the presence of financial frictions  $\tilde{\kappa} > 0$ . This result is consistent with empirical statistics in Auclert.<sup>38</sup> Based upon the signs of those empirical statistics, which are also negative in his study, he argues that the redistributive effects of monetary policy are likely to amplify the effects of monetary policy.

Even though the signs of these statistics are also negative in the OLG models with financial frictions, the redistributive channels of monetary policy still do not amplify its effect on aggregate outcomes. One important difference between my model and a model with a fixed borrowing limit (e.g,  $b_{t+1} \geq -\phi$ ) is related to the way high MPC households initially respond to a monetary policy shock. In a model with a fixed borrowing limit, households that are close to their borrowing

<sup>38</sup>See Table 4 in Auclert (2019).

Table 4: Consumption and Redistribution Elasticities (Persistent Shock,  $\rho = 0.3$ )

	OLG-noFF	Disproportionate	$\tilde{\kappa} = 7$	$\tilde{\kappa} = 15$
Aggregate Consumption (model)	0.33%	0.35%	0.32%	0.32%
Consumption_unconstrained (model)	0.30%	0.25%	0.27%	0.26%
Consumption_constrained (model)	0.03%	0.10%	0.05%	0.06%
Aggregate Consumption (Auclert)	0.19%	0.81%	0.79%	1.09%
Consumption_unconstrained (Auclert)	0.13%	0.16%	0.17%	0.18%
Consumption_constrained (Auclert)	0.06%	0.65%	0.62%	0.91%
$M$ (Income-weighted MPC)	0.0333	0.0367	0.0383	0.0491
$\Xi_Y$ (Redistribution elasticity for $Y$ )	-0.0324	-0.1782	-0.1748	-0.2601
$\Xi_P$ (Redistribution elasticity for $P$ )	0.0841	-0.3018	-0.2978	-0.5162
$\Xi_R$ (Redistribution elasticity for $R$ )	0.2178	-0.9881	-0.9744	-1.6578
$S$ (Substitution channel)	0.9330	0.8531	0.8506	0.8030
$E [MPC_i]$	0.06	0.24	0.24	0.35

limit strongly respond to a monetary policy shock by increasing the amount of borrowing. A sizeable increase in demand induces a large increase in wages, which provides sufficient benefits to savers to offset any negative income effects from a decrease in real interest rates. Thus the effect of monetary policy may be amplified via this general equilibrium channel.

In contrast, in the OLGNK-FF model, the borrowing of the constrained households (high MPC households) is not as responsive to a monetary policy shock because the issuance of new debt is costly for those households. This implies that increases in aggregate demand and wages are smaller and older generations have to bear the negative wealth effect of a monetary policy shock, which decreases the net present value of the consumption of those households. This leads to a further decrease in the responsiveness of aggregate demand to a negative monetary policy shock.<sup>39</sup>

## 4 Conclusion

I build an OLGNK model in which some households are constrained on the issuance of new debt. The model captures demographic structure and inequality among households in the U.S.. The nature of financial frictions in my model is different to that in a Bewley-Huggett-Aiyagari model with a fixed borrowing limit. Although the redistribution elasticities computed from the numerical simulations of the OLGNK-FF model are consistent with empirical statistics, monetary

<sup>39</sup>The redistributive effects across generations remain similar as in Figure 5.

policy has significantly smaller effects on aggregate output than in RANK. This result suggests that the nature of financial frictions is important in evaluating the redistributive effects of monetary policy. One direction for future research is to better understand the differences in how different kinds of financial frictions affect results and evaluate which are more important.

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## Appendix : proofs

### Proof of Lemma 1

Suppose these parameter restrictions are satisfied. We construct an equilibrium with  $i_0 = i'_0$ , as follows. Let  $\pi_1 := \pi_1(i_0)$  denote the resulting rate of inflation given  $i_0$ , and  $r_0$  the corresponding real interest rate. From the households' labor supply,

$$h_t^i = h_t, \text{ for } i = S, B, t = 0, 1.$$

Since prices are flexible at date 1, equilibrium at date 1 is efficient,  $Y_1 = Y_1^*$ , and thus

$$v(h_1^*) = 1, \text{ where } h_1^* := \arg \max_h \theta_1^i h - \theta_1^i v(h), \text{ for } i = S, B. \quad (37)$$

Since  $\theta_0 := \theta_0^S = \theta_0^B$ , profit functions are given by

$$D_0 = P_0 \theta_0 h_0 - W_0 \theta_0 h_0 \text{ and } D_1 = 0,$$

and net income becomes

$$y_t^i := \frac{W_t}{P_t} \theta_t^i h_t + \frac{D_t}{P_t} - \theta_t^i v(h_t) = \theta_t^i h_t - \theta_t^i v(h_t), \quad (38)$$

$$y_0^i = y_0 := \theta_0 h_0 - \theta_0 v(h_0),$$

$$y_1^i = y_1^{i*} := \theta_1^i h_1^* - \theta_1^i v(h_1^*),$$

for  $i = S, B$ .

The asset market clearing and the borrowing constraint imply that

$$B_1^S = -B_1^B, \quad (39)$$

$$B_1^i = (1 + i_0) \bar{B}_1^i, \text{ for } i = S, B,$$

Define the net consumption as  $c_t^i := C_t^i - \theta_t^i v(h_t)$ . Plugging (38) and (39) into the budget constraints,

$$c_0^i = y_0, \quad c_1^i = y_1^{i*} + \frac{(1 + i_0) \bar{B}_1^i}{P_1} \text{ for } i = S, B.$$

Using the savers' Euler equation which is given by

$$y_0^{-\sigma} = \beta^S (1 + r'_0) \left( y_1^{S*} + (1 + r_0^*) \bar{B}^S \right)^{-\sigma}, \quad \text{where } r'_0 = i'_0 - \pi_1(i'_0), \quad (40)$$

we construct  $y_0$  and  $y_1$  as follows:

$$y_1 = y_1^*,$$

$$y_0 = \left( \beta^S (1 + r_0^*) \right)^{-1/\sigma} \left( y_1^{S*} + (1 + r_0^*) \bar{B}^S \right).$$

Note that given  $y_0$ ,  $h_0$  is determined by (38).

From equation (40) and the second inequality in the lemma, output is less than its efficient level at date 0,  $y_0 < y_0^*$ .

It remains to check that the borrowers' Euler equation with a strict inequality. That is:

$$y_0^{-\sigma} > \beta^B (1 + r_0^*) \left( y_1^{B*} - (1 + r_0^*) \bar{B}^S \right)^{-\sigma},$$

where we use the fact that  $\bar{B}^B = -\bar{B}^S$ . This inequality is satisfied given the first inequality in the lemma and by the fact that  $y_0 < y_0^*$ .  $\square$

### Proof of Proposition 2

(i) Note that  $MPC_0^S = \frac{1}{1+\beta}$  and  $MPC_0^B = 1$ . Also, observe that  $NNP_0^S = -NNP_0^B > 0$ ,  $URE_0^S = -URE_0^B > 0$ , and  $E[NNP_0^i] = E[URE_0^i] = 0$ . Then  $cov(MPC_0^i, URE_0^i) = -\frac{\beta}{1+\beta} URE_0^S < 0$  and  $cov(MPC_0^i, NNP_0^i) = -\frac{\beta}{1+\beta} NNP_0^S < 0$ . Using  $Y_0 = Y_0^i$  for  $i = S, B$ , we have  $cov(MPC_0^i, Y_0^i) = E(MPC_0^i Y_0) - E(MPC_0^i) E(Y_0) = 0$ . Also, note that  $c_0^S = c_0^B = y_0 = \theta_0 h_0$ . Then from Proposition 1-1, it follows that  $\frac{dy_0}{y_0} \left( -\frac{r_0}{dr_0} \right) < \frac{1}{\sigma}$ .

(ii) Combining the fixed borrowing limit with the households budget constraints, we have  $c_0^B = y_0 + \frac{\phi}{R}$  and  $c_0^S = y_0 - \frac{\phi}{R}$ . Noting that  $\frac{dc_0^S}{c_0^S} \left( -\frac{r_0}{dr_0} \right) = \frac{1}{\sigma}$  from Proposition 1-2, it must be that  $\frac{dy_0}{y_0} \left( -\frac{r_0}{dr_0} \right) > \frac{1}{\sigma}$ .

### Relationship to Theorem 3 in Auclert (2019).

Borrowers at their borrowing limit always increase their consumption to the amount of increase in borrowing capacity,  $dZ_0^B$ , with the changes in  $dR$  and  $dP$ . If a borrowing constraint is given by  $\frac{B_1}{P_1} \geq -\phi$ , an increase in their consumption,  $dC_0^B$ , happens to be that  $dZ_0^B =$

$-NNP_0^B \frac{dP_0}{P_0} - URE_0^B \left(-\frac{dR_0}{R_0}\right)$ . Then, using  $E_i [NNP_0^i] = 0$  and  $E_i [URE_0^i] = 0$ , we have

$$\begin{aligned} \sum_{i=S,B} dC_0^i &= \left(MPC_0^S NNP_0^S + MPC_0^B NNP_0^B\right) \frac{dP_0}{P_0} + \left(MPC_0^S URE_0^S + MPC_0^B URE_0^B\right) \frac{dR_0}{R_0} \\ &= cov \left(MPC_0^i, NNP_0^i\right) \frac{dP_0}{P_0} + cov \left(MPC_0^i, URE_0^i\right) \frac{dR_0}{R_0}. \end{aligned}$$

Thus, the aggregation result (Theorem 3 in Auclert) holds.

However, if a borrowing constraint is given by  $\frac{B_1}{1+i_0} - B_0 \geq 0$ , then  $dZ_0^B = 0$ . Then, we have

$$\begin{aligned} \left(MPC_0^S NNP_0^S + 0\right) \frac{dP_0}{P_0} &\neq cov \left(MPC_0^i, NNP_0^i\right) \frac{dP_0}{P_0}, \\ \left(MPC_0^S IRE_0^S + MPC_0^B IRE_0^B\right) \frac{dR_0}{R_0} &\neq cov \left(MPC_0^i, IRE_0^i\right) \frac{dR_0}{R_0}. \end{aligned}$$

Therefore, the aggregation result does not hold.  $\square$

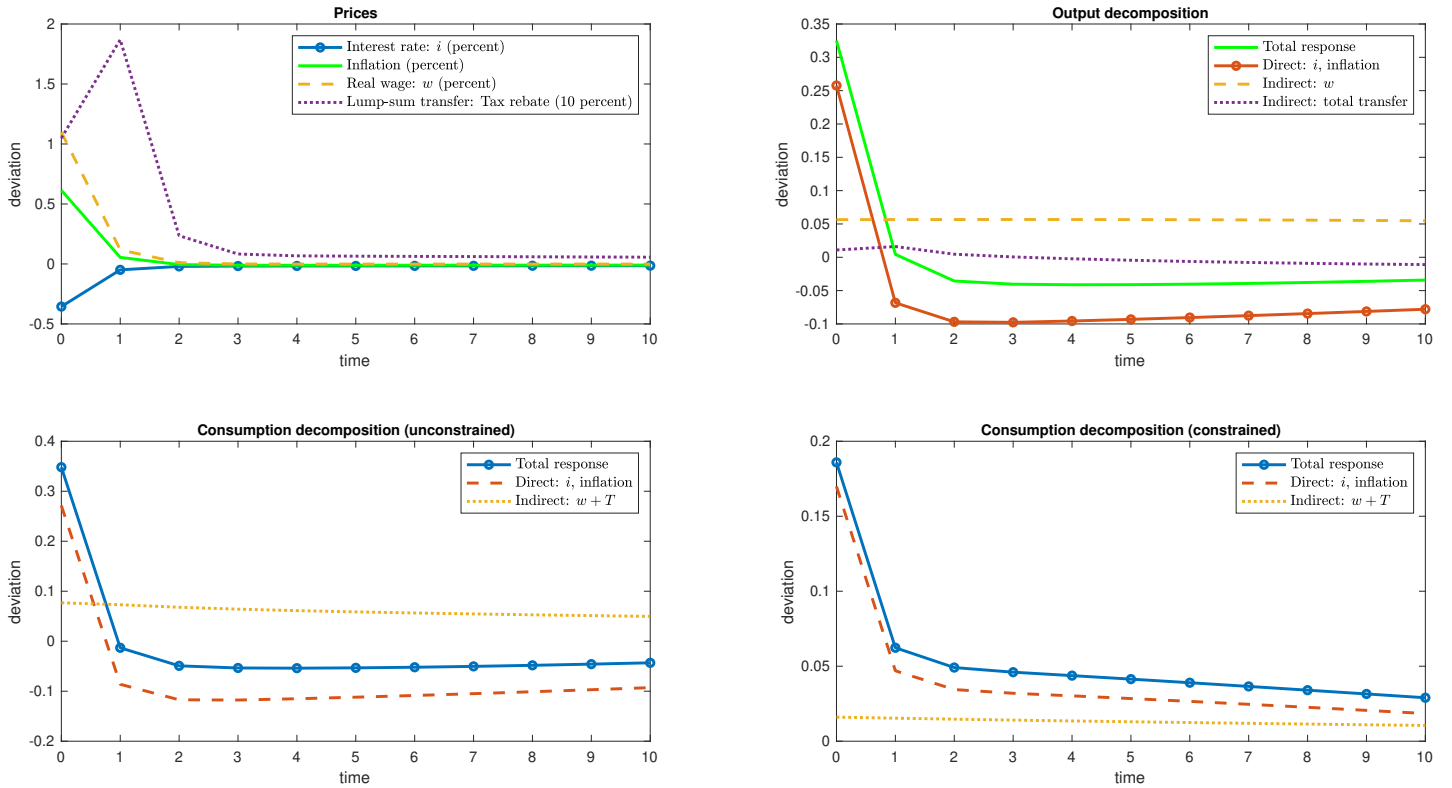


Figure A.1: Direct and Indirect Effects of Monetary Policy in ‘OLGNK-noff’

## Appendix B

The impulse responses of aggregate output from ‘OLGNK-ff’ and ‘OLGNK-noff’ model are observationally similar. To investigate this issue, I decompose output and the consumption of unconstrained and constrained households into the direct and indirect effects following Kaplan, Moll, and Violante (2018b). The upper-left panel in Figures A.1 and A.3 present the equilibrium time paths for interest rates, inflation, real wages, and lump-sum transfers induced by a monetary policy shock. In the right panels, I feed those into the household problem to calculate each element of the direct and indirect responses. Comparing the upper-right panels in the figures, one can see that the direct effects are smaller, but the indirect effects are larger in ‘OLGNK-ff’ than ‘OLGNK-noff’. (The large indirect effects (total transfer) occur at time 1 because, thanks to the high debt to gdp ratio, a low interest rate incurs a large tax rebate to households, and the high MPC constrained households immediately respond to it). This difference is mainly driven by the constrained households. In the lower-right panels of Figures A.1 and A.3, I calculate aggregate

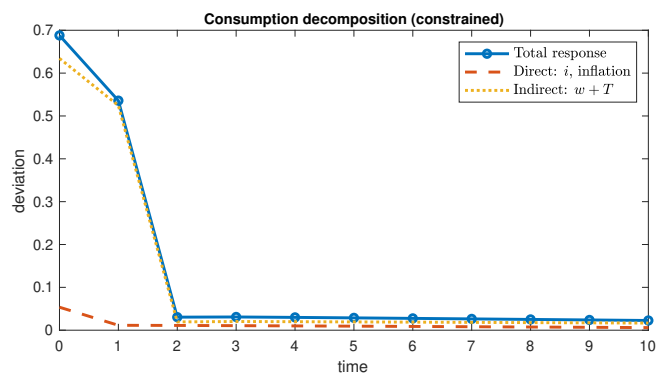
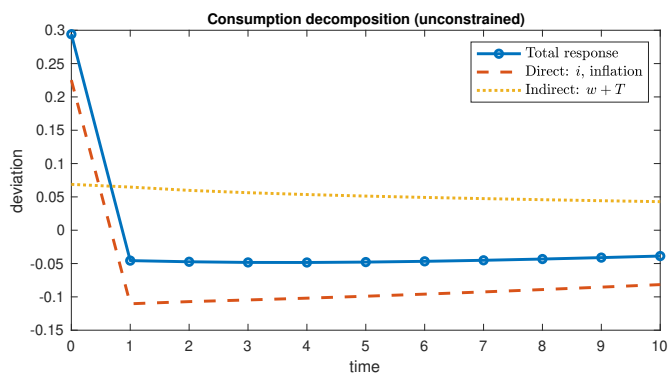
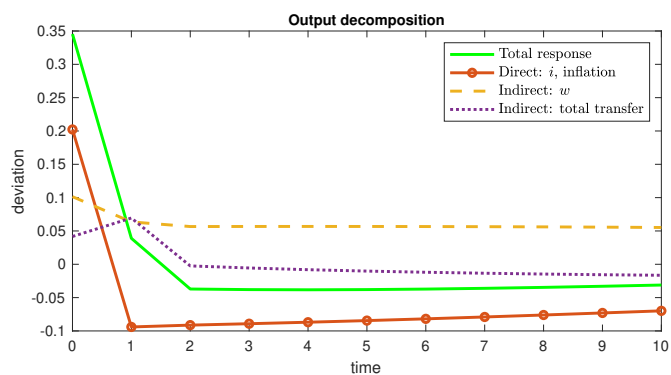
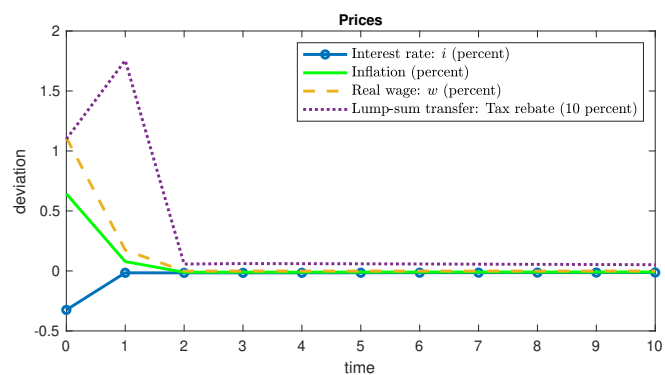


Figure A.2: Direct and Indirect Effects of Monetary Policy in 'OLGNK-ff'

consumption responses among the unconstrained and the constrained respectively. Comparing those figures, we see little difference among the unconstrained, but among the constrained, the indirect effects are much larger while the direct effects are smaller in 'OLGNK-ff'.

## Appendix C: Additional Figures

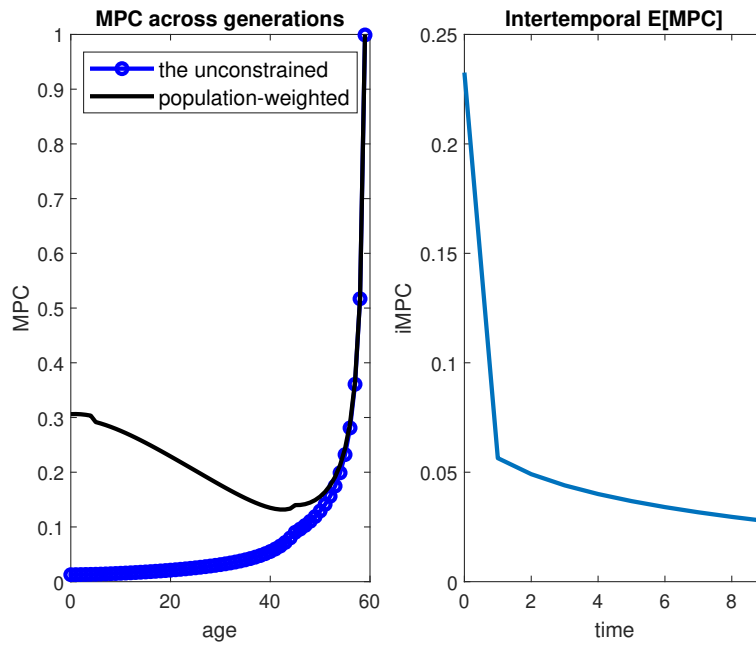


Figure A.3: MPCs in the OLGNK model: the black solid line in the left panel presents the weighted average of the unconstrained and constrained for each generation in the OLG-ff model.