Optimal Monetary Policy under Heterogeneous Consumption Baskets

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Abstract

This paper studies optimal monetary policy in a multi-sector model with heterogeneous consumption baskets and different price indices across households. Based on micro-founded welfare, the first-best outcome is not achievable even in the absence of nominal rigidities: Optimal monetary policy targets non-zero output gaps and benefits borrowing-constrained households. Heterogeneity opens up new redistributive channels for monetary policy that operate through sectoral inflation and relative prices, and leads the central bank to target inflation rates that are weighted toward the goods consumed more intensively by the constrained households and not merely the goods with less flexible prices. Income inequality across households strengthens the results. A policy neglecting heterogeneous baskets benefits the richer households more than optimal at the cost of the poorer.

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Keywords: Optimal monetary policy; Inflation targeting policy; Inflation heterogeneity; Distributional consequences; Inequality; Target output gaps

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1 Introduction

Consumption baskets are heterogeneous across households of different income levels. Cravino et al. (2020) and Vieyra (2018) find that the prices of luxuries, which are consumed more intensively by higher-income households, are stickier and less volatile than those of necessities. Argente and Lee (2020) and Cavallo (2020) document that lower-income groups experienced higher inflation rates during the Great Recession and the recent pandemic, respectively. Since heterogeneous consumption baskets translate into different price indices across households, shocks that have differential effects on sectoral inflation alter relative prices to generate distributional effects through households' budget sets. Monetary policy also has redistributive effects, because it can respond to and influence sectoral inflation differently, affecting relative prices. This phenomenon calls for better understanding of how monetary policy affects different groups in the economy differently and how policy should address the distributional issues that arise from heterogeneous consumption baskets.

We extend the optimal monetary policy work of Aoki (2001), Benigno (2004), and Bilbiie (2008) to analyze consumption basket heterogeneity and its distributional implications for the policy. How do heterogeneous consumption baskets affect equilibrium dynamics? Does heterogeneity generate new inefficiencies and policy trade-offs? How do the new redistributive channels of monetary policy operate? How does optimal monetary policy change? What are the consequences if the central bank neglects heterogeneity? What are the implications of income inequality in this environment? Answers to these questions will fill a gap in the literature.

This paper contributes to the literature in three respects: study the new redistributive channels of monetary policy that are absent under homogeneous consumption baskets; derive micro-founded welfare loss functions and conduct normative analyses by comparing heterogeneous and homogeneous consumption baskets; and draw implications for designing an inflation rate a central bank targets that accounts for the distributional consequences of heterogeneity.

We show that to maximize social welfare, the central bank can and should deal with distributional issues at the cost of overall price instability. Two main conclusions emerge: (1) optimal monetary policy targets non-zero output gaps; (2) optimal policy benefits borrowing-constrained households at the expense of the unconstrained households by targeting inflation rates weighted toward the goods that are consumed more intensively by the constrained households. The existing literature, such as Aoki (2001), Benigno (2004), Mankiw and Reis (2003), and Eusepi et al. (2011), find that a central bank should stabilize a price index that is weighted heavily toward sectors with less flexible prices. In contrast, this paper finds that optimal policy does not necessarily seek to stabilize less flexible prices, and identifies a new rationale for stabilizing inflation in sectors with more flexible prices.

We employ a two-agent—financially constrained and unconstrained—New Keynesian (TANK) framework to model the fact that 25-40% of households live hand-to-mouth based on either net worth or liquid wealth, with limited access to financial markets. They are at a kink in their budget set and insensitive to small changes in interest rates (Kaplan et al., 2014, 2018; Aguiar et al., 2020; Bilbiie, 2008; Debortoli and Galí, 2018). We extend the TANK model to two sectors, which are subject to aggregate and sector-specific productivity shocks. To be consistent with the empirical evidence that consumption baskets are heterogeneous across different income levels and that hand-to-mouth households are relatively poor, we assume that the two types of households consume different shares of goods. They have different CES preferences over the goods, consume different baskets, and face different price indices. This causes households to face different real wages, even in an economy-wide labor market with perfect labor mobility and substitutability, and thus they face idiosyncratic real wage risk. Households also face idiosyncratic non-labor income risk due to the asymmetric distribution of dividend and transfers.

In this economy, monetary policy has redistributive channels through sectoral inflation and relative prices that are absent under homogeneous consumption baskets. Although monetary policy cannot fully stabilize sectoral inflation in both sectors simultaneously under asymmetric disturbances, it can still choose which sectoral inflation to stabilize more, effectively redistributing *across sectors*. When consumption baskets are homogeneous across households, monetary policy has few distributional consequences across households through sectoral inflation, because households face the same price indices and real wages, and hence sectoral inflation and relative prices affect them symmetrically. Thus, optimal policy under homogeneous consumption baskets focuses mostly on price rigidities as demonstrated in existing work. As we introduce heterogeneous baskets, however, we find that monetary policy has significant distributional implications for the welfare of households, because stabilizing inflation in a specific sector more is more beneficial to households that consume goods more intensively from the corresponding sector, and translates into effectively redistributing *across households*. The more stable are a household's consumption-relevant inflation rates and real wages, the lower its consumption volatility (*Real Wage Stabilization Channel*), the less its consumption loss from price dispersion (*Consumption Support Channel*), and the higher its expected welfare. Consequently, optimal policy considers the redistributive effects as well as the distortions from price rigidities.

Under heterogeneous consumption baskets, imperfect risk-sharing gives monetary policy a new role to deal with the distributional inefficiencies. First, the impossibility of achieving the first-best outcome and new trade-offs lead optimal policy to *target non-zero output gaps.* Suppose asymmetric productivity across sectors under flexible prices. The more a household consumes from the higher productivity sector, the lower its price index and the higher its real wage become. Thus, the labor hours of households diverge. They would trade financial instruments to insure against idiosyncratic real wage risk in the frictionless economy, but due to the borrowing constraints, households cannot equalize the marginal disutility of labor and fail to achieve the first-best outcome even in the absence of nominal frictions. Consequently, monetary policy confronts a trade-off whereby sectoral output gaps and labor hour gaps cannot be closed simultaneously. This is the distributional inefficiency from imperfect sharing of idiosyncratic real wage risk. In order to balance the marginal utilities of consumption and marginal disutilities of labor across households, optimal policy targets non-zero output gaps, as we show in the micro-founded welfare-theoretic loss function.

Second, due to the asymmetric responsiveness of consumption across households, optimal inflation targeting policy *benefits the constrained households more and targets an inflation rate weighted toward them.* The constrained households have higher wage elasticity of consumption than the unconstrained households due to the countercyclicality of markups under demand shocks and imperfect risk-sharing. Hence the marginal utility of consumption diverges inefficiently between households. This is the distributional inefficiency from imperfect sharing of idiosyncratic non-labor income risk. Optimal policy benefits the hand-to-mouth households more in order to redistribute toward reducing differences between households' marginal utility. By stabilizing the constrained households' consumption-relevant inflation rates to a greater degree, the variations of their real wage and consumption are subdued (*Real Wage Stabilization Channel*) and consumption loss from price dispersion is also reduced (*Consumption Support Channel*). As such, the central bank effectively redistributes resources from households with lower marginal utility to those with higher marginal utility, which maximizes social welfare. In the end, heterogeneous consumption baskets lead the central bank to target inflation rates that are weighted toward the goods that are consumed more intensively by the constrained households—and not merely the goods with less flexible prices as existing work finds.

Under homogeneous baskets, however, this is not the case. First, in the absence of nominal rigidities, households face no idiosyncratic real wage risk, thus the borrowing constraints are not binding. There is no trade-off between distributional variables and optimal policy targets zero output gaps. Second, despite imperfect sharing of non-labor income risk and the asymmetric responsiveness across households, the central bank cannot redistribute marginal utility across households through sectoral inflation, because the redistributive channels of monetary policy that operate through different price indices degenerate. The inefficient variations of distributional variables are rather at the aggregate level and cannot be addressed by redistribution across sectors.

This study finds that income inequality across households significantly strengthens the main results: As we introduce larger degrees of income inequality, optimal policy assigns even more weight to the stabilization of inflation in the sector of goods that the constrained or the poorer households consume more intensively.[1] Since the hand-to-mouth or the poorer households have higher marginal utility and higher responsiveness of consumption, the utilitarian central bank cares disproportionately more about them and redistributes marginal utilities in their favor to maximize the social welfare.

Through numerical experiments, we also find that if the central bank neglects heterogeneous consumption baskets across different income levels, the policy would worsen inequality. The consequences would then be more beneficial to the richer or unconstrained households than optimal, at the cost of the poorer or constrained households.

Related literature This work contributes to various strands of the literature. First, this study relates to the literature on heterogeneous consumption baskets. Vieyra (2018), Clayton et al. (2019), and Cravino et al. (2020) find the evidence on heterogeneity in consumption baskets across households of different income and education levels and investigate its implication for dynamics in quantitative models. Specifically, Cravino et al. (2020) and Vieyra (2018) find that the prices of luxuries are stickier and less volatile than those of necessities, and Clayton et al. (2019) establish that prices are more rigid in sectors that sell to college-educated households. Argente and Lee (2020) construct income-specific price indices from 2004 to 2010 and investigate the mechanism behind the differences between them. Cavallo (2020) finds a significant difference in inflation rates across income groups after the outbreak of COVID-19. However, these studies do not address the normative questions of optimal monetary policy. We construct a model that allows comparison of heterogeneous and homogeneous consumption baskets, derive a micro-founded welfare-theoretic loss function for each, and conduct normative analysis to draw implications of heterogeneity for the redistributive channels of monetary policy and optimal policy.

This study is also related to the literature that examines heterogeneous agents, particularly in a two-agent framework. Bilbiie (2008) sets up a TANK model and studies the implications of limited asset market participation for dynamics and optimal monetary policy. Debortoli and Galí (2018) also build on a TANK model and study the implications for aggregate dynamics, comparing it with dynamics from RANK and HANK models. These studies

[1] We check that the results are robust to the degrees of heterogeneity in consumption baskets, relative degrees of price stickiness, distortions from monopolistic competition, and whom to tax to finance subsidies.

employ a single-sector framework in which households consume homogeneous baskets. Our multi-sector TANK model nests both heterogeneous and homogeneous consumption baskets, which allows us to extend the existing analyses to heterogeneous consumption baskets in a two-agent two-sector framework. Moreover, we extend our numerical analyses to cases with nonlinear production functions that allow income inequality across households.

This study is also related to the extensive literature on optimal monetary policy. Most research on optimal policy, such as Aoki (2001), Benigno (2004), Woodford (2003), and Bhattarai et al. (2015) has been conducted under a framework in which consumption baskets are homogeneous. There are some studies that consider home bias in the open economy framework. De Paoli (2009) and Faia and Monacelli (2008) study optimal monetary policy in a small open economy characterized by home bias. Auray and Eyquem (2013) examine optimal monetary policy in a monetary union with home bias. However, these works do not fit the study of an economy with hand-to-mouth households and labor mobility. To the best of our knowledge, this paper is the first to derive a micro-founded welfare-analytic loss function and to study the normative implications of heterogeneous consumption baskets for optimal monetary policy in an economy that features heterogeneous-agent with differential access to financial markets and multi-sector with perfect labor mobility.

Lastly, this paper contributes to the literature that studies which price indices central banks should target. Aoki (2001), Benigno (2004), Mankiw and Reis (2003), and Eusepi et al. (2011) find that a central bank should stabilize a price index that is weighted heavily toward sectors with less flexible prices. This implies that a central bank should target core inflation rather than headline inflation. In contrast, we identify a new rationale for stabilizing inflation in sectors with more flexible price and for targeting headline inflation.

The rest of the paper is organized as follows. Section 2 presents the structure of the model and examines equilibrium dynamics for both heterogeneous and homogeneous consumption baskets. Section 3 considers the redistributive channels of monetary policy and the asymmetric responsiveness across households. Section 4 derives the welfare loss functions and optimal monetary policy. Section 5 discusses the consequences of neglecting heterogeneity and studies optimal inflation targeting policy. Section 6 outlines some possible extensions.

2 Model

We build on a two-agent framework to model that some 25-40 percent of households live hand-to-mouth (HtM) based on either net worth or low liquid wealth, facing limited access to financial markets. HtM households are at a kink in their budget set and are insensitive to small changes in interest rates; they have a high marginal propensity to consume out of transitory income changes, which can account for the high correlation between consumption and the transitory component of income growth. (Kaplan et al., 2014, 2018; Aguiar et al., 2020; Bilbiie, 2008; Debortoli and Galí, 2018). We extend a TANK model to a two-sector framework that nests heterogeneous and homogeneous consumption baskets. To be consistent with the empirical evidence that consumption baskets are heterogeneous across different income levels and that hand-to-mouth households are relatively poor, we assume that the two types of households consume different shares of goods.

2.1 Households

Households are either one of the two types, *Constrained* or *Unconstrained*, indexed by $h \in \{C, U\}$. They are populated by measures λ and $1-\lambda$, respectively, so the total population is normalized to 1. Type U households have access to financial markets, while type C households do not.

Both types of households get utility from consumption and disutility from labor supply,

$$\begin{aligned} \mathcal{U}(C_{h,t}, N_{h,t}) &\equiv U(C_{h,t}) - V(N_{h,t}) \\ &\equiv \frac{C_{h,t}{}^{1-\sigma}}{1-\sigma} - \frac{N_{h,t}{}^{1+\varphi}}{1+\varphi} \end{aligned}$$

but their preferences on sectoral good 1 and 2 are different, generating "heterogeneous consumption baskets."[2] Each type of household consumes heterogeneous baskets or different final goods, $C_{U,t}$ and $C_{C,t}$, according to their CES preference parameters, ω_U and ω_C,

$$C_{U,t} \equiv \left[\omega_U^{\frac{1}{\eta}} C_{U,1,t}{}^{\frac{\eta-1}{\eta}} + (1-\omega_U)^{\frac{1}{\eta}} C_{U,2,t}{}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{2.1}$$

$$C_{C,t} \equiv \left[\omega_C^{\frac{1}{\eta}} C_{C,1,t}{}^{\frac{\eta-1}{\eta}} + (1-\omega_C)^{\frac{1}{\eta}} C_{C,2,t}{}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{2.2}$$

where $C_{h,j,t} \equiv \left(\int_{\mathcal{I}_j} (\frac{1}{z_j})^{\frac{1}{\theta}} C_{h,j,t}(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}$, $j \in \{1,2\}$ are indices of household h's consumption of sectoral good j that are CES aggregates of a continuum of differentiated goods, $C_{h,j,t}(i)$, produced in sector 1 if $i \in \mathcal{I}_1 = [0, z_1]$, and in sector 2 if $i \in \mathcal{I}_2 = (z_1, 1]$. The parameters z_1 and $z_2 (= 1-z_1)$ measure the economic size of each sector. σ^{-1} is the elasticity of intertemporal substitution and φ^{-1} is the Frisch elasticity of labor supply, while η and θ denote the elasticity

[2]There are various ways to generate heterogeneous consumption baskets. One of them is to assume non-homothetic preferences where consumption baskets are endogenously different across households of different income levels. Another way is to assume homothetic preference but with exogenously different weight on each good. In this paper, we adopt the latter assumption.

of substitution between sectoral good 1 and 2, and that across differentiated goods produced within each sector, respectively. We assume that (sectoral) good 1 is the numeraire.

Since consumption baskets are different, each type of households face "heterogeneous consumer price indices (CPIs)" of their own final consumption good

$$P_{U,t} = \left[\omega_U P_{1,t}^{1-\eta} + (1-\omega_U) P_{2,t}^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{2.3}$$

$$P_{C,t} = \left[\omega_C P_{1,t}^{1-\eta} + (1-\omega_C) P_{2,t}^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{2.4}$$

where $P_{j,t} = \left(\int_{\mathcal{I}_j} \frac{1}{z_j} P_{j,t}(i)^{1-\theta} di\right)^{\frac{1}{1-\theta}}$, $j \in \{1,2\}$ are price indices of sectoral goods, $C_{h,j,t}$, determined by the supply side as Eq.(2.20) in Section 2.2. The (consumption-relevant) real wages for each type of households are derived as $W_{h,t} = \frac{P_{1,t} W_t}{P_{h,t}}$, and we define the relative price as $Q_t \equiv \frac{P_{2,t}}{P_{1,t}}$.

Labor market is economy-wide with perfect labor mobility across sectors and labor supplies are perfect substitutes.[3] Despite a single equilibrium nominal wage that applies identically to all the households and firms, each household faces "heterogeneous real wages" due to heterogeneous consumer price indices. Thus households face *idiosyncratic real wage risk* under heterogeneous consumption baskets. In addition, they face *idiosyncratic non-labor income risk*, because two types of households have different sources of non-labor income such as dividend, transfer and tax.

2.1.1 The Financially Unconstrained

Type U households, populated with mass $1-\lambda$, have access to the bond market and the stock market, thus earn dividend from the firm's profit as well as labor income. They maximize present value of expected lifetime utility Eq.(2.5) subject to the budget constraint Eq.(2.6),

$$\max_{\{C_{U,t}, N_{U,t}\}_{t=0}^{\infty}} \; E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_{U,t}^{1-\sigma}}{1-\sigma} - \frac{N_{U,t}^{1+\varphi}}{1+\varphi}\right] \tag{2.5}$$

$$\begin{aligned} s.t. \;\; & P_{U,t} C_{U,t} + B_{U,t} + P_{1,t} V_t S_{U,t} \\ & = B_{U,t-1}(1+i_{t-1}) + P_{1,t} W_t N_{U,t} + P_{1,t}(D_t + V_t) S_{U,t-1} + P_{1,t} T_{U,t} \end{aligned} \tag{2.6}$$

[3] We do not make any assumptions on differences in labor productivity nor restrictions on labor mobility to focus on heterogeneous consumption baskets and resulting heterogeneous price indices.

where $B_{U,t}$ and $S_{U,t}$ denote holdings of one-period nominally riskless bond, and of the share in a fund that owns all the firms where the total supply of stock is normalized to 1. In each period t, bonds that mature in period $t+1$ are traded at the nominal interest rate i_t, while shares, a claim to dividend D_t, are traded at price V_t. The dividend D_t is defined as

$$D_t = \sum_{j=1,2} \int_{\mathcal{I}_j} \left(\frac{P_{j,t}(i)}{P_{1,t}} - \frac{(1-\tau)W_t}{A_t A_{j,t}} \right) Y_{j,t}(i) di$$

where τ is subsidy rate on labor cost that will be covered in Section 2.2. $N_{U,t}$ and W_t are labor supply of type U and the wage, and $T_{U,t}$ is the net lump-sum transfers from the government. W_t, D_t, and $T_{U,t}$ are measured in units of the numeraire (good 1). $0<\beta<1$ is the intertemporal discount factor.

The first order conditions with respect to $C_{U,t}$, $N_{U,t}$ and $B_{U,t}$ from Eq.(2.5) and Eq.(2.6) give the Euler equation and optimal condition for labor supply

$$\frac{1}{1+i_t} = E_t\left[\beta \frac{C_{U,t+1}^{-\sigma}}{C_{U,t}^{-\sigma}} \frac{P_{U,t}}{P_{U,t+1}} \right] = E_t\left[\Lambda_{t,t+1}\right] \tag{2.7}$$

$$\frac{N_{U,t}^{\varphi}}{C_{U,t}^{-\sigma}} = \frac{P_{1,t}W_t}{P_{U,t}} \tag{2.8}$$

where $\Lambda_{t,t+1} \equiv \beta \frac{C_{U,t+1}^{-\sigma}}{C_{U,t}^{-\sigma}} \frac{P_{U,t}}{P_{U,t+1}}$ is the stochastic discount factor. Given decisions on $C_{U,t}$, households optimally allocate the expenditure on $C_{U,1,t}$ and $C_{U,2,t}$ by minimizing the total expenditure $P_{U,t}C_{U,t}$ under the constraint given by Eq.(2.1)

$$C_{U,1,t} = \omega_U \left(\frac{P_{1,t}}{P_{U,t}} \right)^{-\eta} C_{U,t} \tag{2.9}$$

$$C_{U,2,t} = (1-\omega_U) \left(\frac{P_{2,t}}{P_{U,t}} \right)^{-\eta} C_{U,t} \tag{2.10}$$

Now given decisions on $C_{U,1,t}$ and $C_{U,2,t}$, households optimally allocate the expenditure on $C_{U,1,t}(i)$ and $C_{U,2,t}(i)$ by minimizing the total expenditure $P_{1,t}C_{U,1,t}$ and $P_{2,t}C_{U,2,t}$ under the constraint given by the definitions of CES aggregates $C_{U,1,t}$ and $C_{U,2,t}$

$$C_{U,1,t}(i) = \frac{1}{z_1} \left(\frac{P_{1,t}(i)}{P_{1,t}} \right)^{-\theta} C_{U,1,t} \tag{2.11}$$

$$C_{U,2,t}(i) = \frac{1}{z_2} \left(\frac{P_{2,t}(i)}{P_{2,t}} \right)^{-\theta} C_{U,2,t} \tag{2.12}$$

2.1.2 The Financially Constrained

Type C households, populated with mass λ, live hand-to-mouth, have no access to the bond market and the stock market, and face borrowing and savings constraints. Wage income is the only source of their income except transfers. They maximize utility Eq.(2.13) each period subject to the budget constraint Eq.(2.14),

$$\max_{\{C_{C,t},N_{C,t}\}} \left[\frac{C_{C,t}^{1-\sigma}}{1-\sigma} - \frac{N_{C,t}^{1+\varphi}}{1+\varphi}\right] \tag{2.13}$$

$$s.t. \quad P_{C,t}C_{C,t} = P_{1,t}W_tN_{C,t} + P_{1,t}T_{C,t} \tag{2.14}$$

where $N_{C,t}$ is labor supply and $T_{C,t}$ is the net lump-sum transfer from the government measured in units of the numeraire (good 1).

The first order conditions with respect to $C_{C,t}$ and $N_{C,t}$ from Eq.(2.13) and Eq.(2.14) give the optimal conditions for labor supply

$$\frac{N_{C,t}^{\varphi}}{C_{C,t}^{-\sigma}} = \frac{P_{1,t}W_t}{P_{C,t}} \tag{2.15}$$

Given decisions on $C_{C,t}$, households optimally allocate the expenditure on $C_{C,1,t}$ and $C_{C,2,t}$ by minimizing the total expenditure $P_{C,t}C_{C,t}$ under the constraint given by Eq.(2.2)

$$C_{C,1,t} = \omega_C \left(\frac{P_{1,t}}{P_{C,t}}\right)^{-\eta} C_{C,t} \tag{2.16}$$

$$C_{C,2,t} = (1-\omega_C)\left(\frac{P_{2,t}}{P_{C,t}}\right)^{-\eta} C_{C,t} \tag{2.17}$$

Now given decisions on $C_{C,1,t}$ and $C_{C,2,t}$, households optimally allocate the expenditure on $C_{C,1,t}(i)$ and $C_{C,2,t}(i)$ by minimizing the total expenditure $P_{1,t}C_{C,1,t}$ and $P_{2,t}C_{C,2,t}$ under the constraint given by the definitions of CES aggregates $C_{C,1,t}$ and $C_{C,2,t}$

$$C_{C,1,t}(i) = \frac{1}{z_1}\left(\frac{P_{1,t}(i)}{P_{1,t}}\right)^{-\theta} C_{C,1,t} \tag{2.18}$$

$$C_{C,2,t}(i) = \frac{1}{z_2}\left(\frac{P_{2,t}(i)}{P_{2,t}}\right)^{-\theta} C_{C,2,t} \tag{2.19}$$

2.1.3 Two Special Cases

For the analytical study, we focus on the comparison of the following two cases for simplicity and tractability:[4]

(1) HetCB completely *hetero*geneous consumption baskets ($\omega_U = 0,\ \omega_C = 1$)

Households specialize their consumption: $\omega_U = 0$ denotes that type U households consume only good 2, thus $C_{U,t} = C_{U,2,t}$, $P_{U,t} = P_{2,t}$ and $W_{U,t} = \frac{W_t}{Q_t}$, while $\omega_C = 1$ denotes that type C households consume only good 1, $C_{C,t} = C_{C,1,t}$, $P_{C,t} = P_{1,t}$ and $W_{C,t} = W_t$. Heterogeneous consumption baskets result in heterogeneous price indices between two household types, which in turn leads to heterogeneous real wages despite one nominal wage under economy-wide labor market.

(2) HomCB completely *homo*geneous consumption baskets ($\omega_U = \omega_C = \frac{1}{2}$)

If $\omega_U = \omega_C = \omega$ holds, both types of households consume the same baskets of goods or final good. Thus, they face identical price indices, $P_{U,t} = P_{C,t}$, and real wages.

2.2 Firms

Firm $i \in [0,1]$ in each sector $j \in \{1,2\}$ is a monopolistically competitive producer that produces differentiated good $Y_{j,t}(i)$ through a constant returns to scale production function

$$Y_{j,t}(i) = A_t A_{j,t} N_{j,t}(i)$$

where $Y_{j,t}(i)$ and $N_{j,t}(i)$ are output and labor employed by firm i.[5] [6] A_t and $A_{j,t}$ are economy-wide and sector-specific productivity, respectively, that follow AR(1) process in log.[7] Each firm faces its own demand function from both types of households' optimization

$$\begin{aligned} Y_{1,t}(i) &= (1-\lambda)\ \frac{\omega_U}{z_1} \left(\frac{P_{1,t}(i)}{P_{1,t}}\right)^{-\theta} \left(\frac{P_{1,t}}{P_{U,t}}\right)^{-\eta} C_{U,t} + \lambda\ \frac{\omega_C}{z_1} \left(\frac{P_{1,t}(i)}{P_{1,t}}\right)^{-\theta} \left(\frac{P_{1,t}}{P_{C,t}}\right)^{-\eta} C_{C,t} \\ Y_{2,t}(i) &= (1-\lambda) \frac{1-\omega_U}{z_2} \left(\frac{P_{2,t}(i)}{P_{2,t}}\right)^{-\theta} \left(\frac{P_{2,t}}{P_{U,t}}\right)^{-\eta} C_{U,t} + \lambda \frac{1-\omega_C}{z_2} \left(\frac{P_{2,t}(i)}{P_{2,t}}\right)^{-\theta} \left(\frac{P_{2,t}}{P_{C,t}}\right)^{-\eta} C_{C,t} \end{aligned}$$

[4] We extend our study to the general cases of heterogeneous consumption baskets in Section 5, and find that the main results are robust.

[5] We extend the model to introduce a decreasing returns to scale production function for numerical study in Section 5, and find that the main results are further strengthened as inequality gets larger.

[6] Firm i is in sector 1 if $i \in \mathcal{I}_1 = [0, z_1]$, and in sector 2 if $i \in \mathcal{I}_2 = (z_1, 1]$.

[7] $a_t = \rho_a a_{t-1} + \sigma_a \varepsilon_t^a,\ \ \varepsilon^a \sim N(0,1)$ where $a_t \equiv log A_t$
$a_{j,t} = \rho_{a_j} a_{j,t-1} + \sigma_{a_j} \varepsilon_t^{a_j},\ \ \varepsilon^{a_j} \sim N(0,1)$ where $a_{j,t} \equiv log A_{j,t}$

Given the outputs and labor employments of a continuum of firms, we define the sectoral output as a CES aggregate of differentiated goods, $Y_{j,t} \equiv \left(\int_{\mathcal{I}_j} (\frac{1}{z_j})^{\frac{1}{\theta}} Y_{j,t}(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}$, and sectoral labor employment as the sum of labor employment in each sector j, $N_{j,t} \equiv \int_{\mathcal{I}_j} N_{j,t}(i)$.[8]

We model nominal friction as in Calvo (1983) and Yun (1996). Firms in each sector re-adjust their prices with probability $1-\alpha_j$ each period. A firm that resets its price $P^*_{j,t}(i)$ at period t maximizes its expected sum of discounted profit

$$\max_{P^*_{j,t}(i)} \; E_t \sum_{s=0}^{\infty} \alpha_j^s \Lambda_{t,t+s} \left[P^*_{j,t}(i) - \frac{(1-\tau) P_{1,t+s} W_{t+s}}{A_{t+s} A_{j,t+s}} \right] \left(\frac{P^*_{j,t}(i)}{P_{j,t+s}} \right)^{-\theta} Y_{j,t+s}$$

where $\Lambda_{t,t+s} = \beta^s \frac{C_{t+s}^{U^{-\sigma}}}{C_t^{U^{-\sigma}}} \frac{P_t^U}{P_{t+s}^U}$ is stochastic discount factor between period t and $t+s$. Since type C households are financially constrained and type U households own all the firms in the economy, the shareholders use their own discount factor in discounting expected future profits of each firm. We eliminate the inefficiency that originates from imperfect competition at the steady state by introducing a proportional subsidy on labor cost at rate τ.[9]

The first-order condition of a price-setting firm's problem is:

$$E_t \sum_{s=0}^{\infty} \alpha_j^s \Lambda_{t,t+s} \left(\frac{P^*_{j,t}(i)}{P_{j,t+s}} \right)^{-\theta} Y_{j,t+s} \left[P^*_{j,t}(i) - \frac{\theta}{\theta-1} \frac{(1-\tau) P_{1,t+s} W_{t+s}}{A_{t+s} A_{j,t+s}} \right] = 0$$

All the price-setting firms at a certain period within each sector choose the same optimal price in equilibrium, $P^*_{j,t}(i) = P^*_{j,t}$. Considering all the firms that adjust prices and do not, the sectoral price level in sector j is determined by:

$$P_{j,t} = \left[(1-\alpha_j) {P^*_{j,t}}^{1-\theta} + \alpha_j {P_{j,t-1}}^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{2.20}$$

Given sectoral price levels, each type of households' price index, $P_{h,t}$, is differently determined by Eq.(2.3) and Eq.(2.4) according to the corresponding consumption baskets.

[8] In equilibrium, sectoral output equals sectoral consumption which is the weighted sum of demand from both types of households: $Y_{j,t} = (1-\lambda) C_{U,j,t} + \lambda C_{C,j,t}$. Thus we have that $Y_{j,t}(i) = \frac{1}{z_j} \left(\frac{P_{j,t}(i)}{P_{j,t}} \right)^{-\theta} Y_{j,t}$.

[9] A positive markup, $\frac{\theta}{\theta-1}$, arising from monopolistic competition, lowers output below its efficient level. Since it is irrelevant to this study, we eliminate this inefficiency by assuming subsidy on a firm's labor employment cost at the rate τ that satisfies $1-\tau = \frac{\theta-1}{\theta}$.

2.3 Fiscal and Monetary Policy

The government budget constraint is given by:

$$B_{G,t-1} \;=\; \frac{B_{G,t}}{1+i_t} + P_{1,t}G_t + (1-\lambda)P_{1,t}T_{U,t} + \lambda P_{1,t}T_{C,t} + \tau P_{1,t}W_t(N_{1,t}+N_{2,t})$$

The government buys goods, G_t, transfers lump-sum (net of tax) to each type of households, $T_{U,t}$ and $T_{C,t}$, and subsidizes firms proportionally for their labor cost at subsidy rate $\tau = \frac{1}{\theta}$ to remove monopolistic distortion in steady state. The government participates in the bond market to borrow ($B_{G,t} < 0$) or lend ($B_{G,t} > 0$), or to implement open market operations.

Fiscal policy is characterized as follows:[10] there is no government expenditure ($G_t = 0$); the government does not transfer lump-sum to and from type C households ($T_{C,t} = 0$), and does not issue nor buy bonds ($B_{G,t} = 0$). The government needs to finance employment subsidy by tax or issuing bonds, and its decision on whom to tax has nontrivial effects on dynamics and income inequality as we discuss in the following section. We assume that the government tax only the unconstrained households.[11] This assumption results in a symmetric steady state with no income inequality between households; since the source of the firms' profit is monopolistic competition under linear production function, subsidy induces no profit and no non-labor income for the unconstrained households at the steady state.[12] But we find in Section 5 that our main results are robust to whom to tax to finance subsidy, and are further strengthened as we introduce income inequality by relaxing assumptions on tax rules and introducing a decreasing returns to scale production function.

[10] Unlike in models with a representative agent, the aggregate and distributional consequences of monetary policy are nontrivially affected by the details of fiscal rules in models with heterogeneous agents, because Ricardian equivalence generically fails to hold. As explained in Kaplan et al. (2018), monetary policy has an indirect effect that operates through fiscal policy; for example, an exogenous shock on interest rate affects the government budget constraint, which in turn affects each households' budget constraints and their decisions through fiscal rules.

[11] Then, bond holdings and transfer (net of tax) terms in the type U households' budget constraint cancel out by the government budget constraint and bond market clearing condition, leaving subsidy term only; this is exactly the same as in models with representative agent. As a result, $B_{G,t}$ plays little role in the bond market mechanism of monetary policy implementation, shutting down the indirect channel of monetary policy through fiscal sides. Hence we can simply assume $B_{G,t} = 0$. Consequently, type U households ends up financing the subsidy, which is ultimately rebated back to them in the form of dividend.

[12] Considering that this study focuses primarily on qualitative aspects rather than on quantitative aspects, we suppose the assumptions are innocuous. Moreover, those assumptions put aside the indirect channel of monetary policy enabling us to shed light more on the implications of heterogeneous consumption baskets itself, and make welfare analysis simpler facilitating comparisons of this study to the findings in the literature such as Benigno (2004).

Lastly, monetary policy characterized by a Taylor rule closes the model.

$$1 + i_t = \frac{1}{\beta}\left(\frac{\Pi_{1,t}}{\overline{\Pi}_1}\right)^{\phi_{\pi_1}}\left(\frac{\Pi_{2,t}}{\overline{\Pi}_2}\right)^{\phi_{\pi_2}}\left(\frac{Y_{1,t}}{Y_{1,t}^E}\right)^{\phi_{y_1}}\left(\frac{Y_{2,t}}{Y_{2,t}^E}\right)^{\phi_{y_2}}\exp(\nu_t)$$

where $Y_{j,t}^E$ is the efficient level of sectoral output j and ν_t is monetary policy shock that follows AR(1) process. We assume zero inflation steady state ($\overline{\Pi}_1 = \overline{\Pi}_2 = 1$).

2.4 Market Clearing

All the markets clear in equilibrium: clearing conditions for the goods markets (sectoral good j and a continuum of differentiated good i), economy-wide labor market, bond market, and stock market are given by

$$\begin{aligned}
Y_{j,t} &= (1-\lambda)C_{U,j,t} + \lambda C_{C,j,t} \\
Y_{j,t}(i) &= (1-\lambda)C_{U,j,t}(i) + \lambda C_{C,j,t}(i) \\
N_{1,t} + N_{2,t} &= (1-\lambda)N_{U,t} + \lambda N_{C,t} \\
0 &= (1-\lambda)B_{U,t} + B_{G,t} \\
1 &= (1-\lambda)S_{U,t}
\end{aligned}$$

2.5 Equilibrium under HetCB

Now we characterize the equilibrium under completely heterogeneous consumption baskets (**HetCB**, $\omega_U = 0,\ \omega_C = 1$). We establish the efficient (first-best) allocation, and characterize the model equilibrium in terms of percentage deviation from the efficient allocation.[13] Imperfect risk-sharing in real wage leads to the impossibility of achieving efficiency and a new trade-off, generating distributional inefficiencies from idiosyncratic real wage risk.

2.5.1 Efficient Allocation

We derive the economy's efficient allocation by solving a social planner's problem that maximizes the weighted sum of utility of both types of households, subject to the resource

[13] We set the parameters z_1 and $z_2 (= 1-z_1)$ as λ and $1-\lambda$ to measure the economic size of each sector.

and technology constraints

$$
\begin{aligned}
\max_{\{C_{h,t},N_{h,t},Y_{j,t}(i)\}} \quad & \left\{\varpi_U(1-\lambda)\left[\frac{C_{U,t}^{1-\sigma}}{1-\sigma}-\frac{N_{U,t}^{1+\varphi}}{1+\varphi}\right]+\varpi_C\lambda\left[\frac{C_{C,t}^{1-\sigma}}{1-\sigma}-\frac{N_{C,t}^{1+\varphi}}{1+\varphi}\right]\right\} \\
s.t. \qquad \lambda C_{C,t} &= \left(\int_{\mathcal{I}_1}\left(\frac{1}{z_1}\right)^{\frac{1}{\theta}} Y_{1,t}(i)^{\frac{\theta-1}{\theta}}di\right)^{\frac{\theta}{\theta-1}} \\
(1-\lambda)C_{U,t} &= \left(\int_{\mathcal{I}_2}\left(\frac{1}{z_2}\right)^{\frac{1}{\theta}} Y_{2,t}(i)^{\frac{\theta-1}{\theta}}di\right)^{\frac{\theta}{\theta-1}} \\
(1-\lambda)N_{U,t}+\lambda N_{C,t} &= \int_{\mathcal{I}_1}\frac{Y_{1,t}(i)}{A_tA_{1,t}}di+\int_{\mathcal{I}_2}\frac{Y_{2,t}(i)}{A_tA_{2,t}}di
\end{aligned}
$$

where $\{\varpi_h\}$ denotes Pareto weights. First order conditions with respect to $C_{h,t}$, $N_{h,t}$, and $Y_{j,t}(i)$ are given by

$$
\begin{aligned}
\varpi_C C_{C,t}^{-\sigma} &= \mu_1 \\
\varpi_U C_{U,t}^{-\sigma} &= \mu_2 \\
\varpi_C N_{C,t}^{\varphi} &= \mu_3 \\
\varpi_U N_{U,t}^{\varphi} &= \mu_3 \\
\mu_1 Y_{1,t}^{\frac{1}{\theta}}\ z_1^{-\frac{1}{\theta}} Y_{1,t}(i)^{-\frac{1}{\theta}} &= \mu_3\frac{1}{A_tA_{1,t}} \\
\mu_2 Y_{2,t}^{\frac{1}{\theta}}\ z_2^{-\frac{1}{\theta}} Y_{2,t}(i)^{-\frac{1}{\theta}} &= \mu_3\frac{1}{A_tA_{2,t}}
\end{aligned}
$$

where μ_1, μ_2 and μ_3 are Lagrange multipliers. According to the last two conditions, $Y_{j,t}(i)$ should have a common value, $Y_{j,t}(i)=\frac{Y_{j,t}}{z_j}$, implying no output dispersion within sector in the efficient allocation. By simplifying the first order conditions and the constraints, the efficient allocation is characterized by

$$
\begin{aligned}
{N_{C,t}^E}^{\varphi} &= {C_{C,t}^E}^{-\sigma} A_tA_{1,t} \\
{N_{U,t}^E}^{\varphi} &= {C_{U,t}^E}^{-\sigma} A_tA_{2,t} \\
\frac{N_{C,t}^E}{N_{U,t}^E} &= \left(\frac{\varpi_C}{\varpi_U}\right)^{-\varphi} \\
\lambda C_{C,t}^E &= Y_{1,t}^E \\
(1-\lambda)C_{U,t}^E &= Y_{2,t}^E \\
(1-\lambda)N_{U,t}^E+\lambda N_{C,t}^E &= \frac{Y_{1,t}^E}{A_tA_{1,t}}+\frac{Y_{2,t}^E}{A_tA_{2,t}}
\end{aligned}
$$

where E stands for "*Efficient.*" The intuition for the first two efficiency conditions is straightforward: marginal utility earned from the goods marginally produced should equal marginal disutility when a household supplies one more unit of labor to the sector of its consumption.

The efficient allocation is affected by relative Pareto weights, $\frac{\varpi_C}{\varpi_U}$; how much a social planner values each household determines its corresponding efficient allocation. In this study, we assume that a social planner is utilitarian ($\varpi_U = \varpi_C$), so that the market outcome without nominal and financial constraints coincides with the efficient allocation, and the steady state of the market outcome regardless of frictions coincides with that of the efficient allocation.

The dynamics of log-linearized variables expressed in terms of exogenous processes are:

$$
\begin{aligned}
n_t^E &= n_{C,t}^E = n_{U,t}^E = \underbrace{\frac{1-\sigma}{\sigma+\varphi}}_{+/-}\left(a_t + n_1 a_{1,t} + n_2 a_{2,t}\right)\\
y_{1,t}^E &= c_{C,t}^E = \underbrace{\left(\frac{1}{\sigma} - \frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}\right)}_{+/+} a_t \underbrace{+\left(\frac{1}{\sigma} - \frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}n_1\right)}_{+/+} a_{1,t} \underbrace{-\frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}n_2}_{-/+} a_{2,t}\\
y_{2,t}^E &= c_{U,t}^E = \underbrace{\left(\frac{1}{\sigma} - \frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}\right)}_{+/+} a_t \underbrace{-\frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}n_1}_{-/+} a_{1,t} \underbrace{+\left(\frac{1}{\sigma} - \frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}n_2\right)}_{+/+} a_{2,t}\\
n_{1,t}^E &= \underbrace{\left(\frac{1-\sigma}{\sigma} - \frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}\right)}_{+/-} a_t \underbrace{+\left(\frac{1-\sigma}{\sigma} - \frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}n_1\right)}_{+/-} a_{1,t} \underbrace{-\frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}n_2}_{-/+} a_{2,t}\\
n_{2,t}^E &= \underbrace{\left(\frac{1-\sigma}{\sigma} - \frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}\right)}_{+/-} a_t \underbrace{-\frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}n_1}_{-/+} a_{1,t} + \underbrace{\left(\frac{1-\sigma}{\sigma} - \frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}n_2\right)}_{+/-} a_{2,t}
\end{aligned}
$$

where the signs are when $\sigma < 1$ and $\sigma > 1$, respectively. The lower-case letters denote percentage deviations from the steady state, and the sectoral output equals the consumption of the corresponding type of households. The implied wage and relative price in the efficient allocation are derived as $w_t^E = a_t + a_{1,t}$ and $q_t^E = a_{1,t} - a_{2,t}$, so we identify heterogeneous real wages, $w_{C,t}^E (= w_t^E) = a_t + a_{1,t}$ and $w_{U,t}^E (= w_t^E - q_t^E) = a_t + a_{2,t}$.

In the efficient allocation, labor hours are equalized between households: perfect substitutability of labor hours with identical productivity and the convexity of disutility of labor lead the social planner to equalize marginal disutility of labor to minimize the social disutility cost in production of any sets of outputs. However, consumption would not equalize generically due to heterogeneity: because marginal utility gain is higher in the sector with higher productivity given one additional unit of labor hour, the social planner finds it efficient to produce more goods in that sector. Thus it is efficient that households who consume goods from the sector with higher productivity more intensively consume more.

Note that the value of σ matters for the scale and direction of each sectoral and distributional variable in their dynamics, because σ measures the relative size of the income effect compared to the substitution effect in labor supply decision and the extent to which households care about the variation of consumption.[14] Throughout the paper, we make a baseline assumption that $\sigma < 1$, because it is more intuitive that labor supply schedule on wage is upward sloping, and it is shown by some studies on labor supply that the income effect is not big enough to dominate the substitution effect. However, the main results of this paper do not change qualitatively with the assumptions on σ.

Let us check how the efficient allocation can be achieved by the frictionless market outcome under a positive shock on sector-specific productivity $a_{1,t}$.[15] Higher productivity in sector 1 affects the real wages differently: it increases the real wage and consumption of type C households who consume good 1 intensively, and if $\sigma < 1$, a higher wage leads to an increase in labor supply of type C households, which is reconciled with a large increase in demand for good 1 following the shock. However, there is no direct effect on the real wage of type U households who consume good 2 intensively. As the labor hours of both types of household diverge, *an incentive to trade financial instruments to insure against idiosyncratic real wage risk* is created: due to the convexity of disutility of labor, both types benefit from it and achieve Pareto improvement by equalizing marginal disutility of labor; the real wage risk is perfectly shared, achieving efficiency conditions. As a result, labor supply of type U households increases while that of type C households decreases. Since more labor supply translates to a higher disutility for type U households, their consumption decreases.[16] [17] We will discuss more in Section 2.5.3, that if we introduce borrowing and savings constraints into the frictionless economy, the market outcome cannot obtain the first-best allocation.

[14] If $\sigma < 1$, the substitution effect dominates the income effect, and an increase in wage leads to more labor hours. In addition, the elasticity of intertemporal substitution is higher, because households care less about consumption smoothing. If $\sigma > 1$, the opposites hold true.

[15] The symmetric mechanism applies for the other sector-specific shock, $a_{2,t}$.

[16] As the elasticity of intertemporal substitution is high ($\sigma < 1$), households care less about consumption smoothing and the responses of their consumption to shocks are large. Thus, the labor employment in sector 1 increases despite a positive sector-specific productivity shock due to a larger increase in demand for good 1, while labor employment in sector 2 decreases as the demand falls by higher disutility of labor supply of type U households.

[17] If $\sigma > 1$, however, a higher wage lowers labor supply of type C households, and this is reconciled with a small increase in demand for good 1 following the shock. As the real wage risk is perfectly shared, labor hours of type C households decreases, from which they would have lower disutility leading to an increase in their consumption. $\sigma > 1$ implies that households care more about consumption smoothing, and their responses are relatively smaller. Thus, labor employment in sector 1 rather decreases due to a higher sector-specific productivity, while labor employment in sector 2 increases as the demand rises by lower disutility of labor supply of type U households.

2.5.2 Approximate Allocation

We approximate the decentralized model by log-linearizing the equilibrium conditions around the deterministic efficient zero-inflation steady state. The market outcomes with no frictions coincide with the first-best allocation. However, as we introduce nominal friction and financial constraints, the market outcome would deviate from the first-best; we find that the first-best outcome is not implementable even in the absence of nominal rigidities.[18] We provide the system of equations expressed in welfare-relevant gaps: variables with *tilde* denote percentage deviations from the efficient allocation.[19] Note that output is aggregated at the sector-level and we have $\widetilde{c}_{C,t} = \widetilde{y}_{1,t}$ and $\widetilde{c}_{U,t} = \widetilde{y}_{2,t}$, because each type of households consume goods of different sectors under **HetCB**.

The first set of equations are from the household side:

$$\widetilde{y}_{2,t} - E_t[\widetilde{y}_{2,t+1}] = -\frac{1}{\sigma}\big(\widetilde{i}_t - E_t[\pi_{2,t+1}] - r_t^E\big) \tag{2.21}$$

$$\varphi\widetilde{n}_{U,t} + \sigma\widetilde{y}_{2,t} = \widetilde{w}_t - \widetilde{q}_t \tag{2.22}$$

$$\varphi\widetilde{n}_{C,t} + \sigma\widetilde{y}_{1,t} = \widetilde{w}_t \tag{2.23}$$

$$\widetilde{w}_t + \widetilde{n}_{C,t} = \widetilde{y}_{1,t} + \frac{1-\sigma}{\sigma} z_2 q_t^E \tag{2.24}$$

where the real interest rate in the efficient allocation is $r_t^E \equiv \sigma(E_t[y_{2,t+1}^E] - y_{2,t}^E)$.

Eq.(2.21) is the Euler equation of type U households: the output gap in sector 2 is a function of the sum of the current and the expected future real interest rate gaps. Since type U households consume good 2 intensively, the Euler equation is expressed in variables from sector 2. There is no Euler equation for type C households who make purely static decisions due to the financial constraints. Eq.(2.22) and Eq.(2.23) are the labor supply schedules of each type of households who face different real wages and idiosyncratic real wage risk: $w_{C,t}(\equiv w_t) \neq w_{U,t}(\equiv w_t - q_t)$.

Financial constraints are shown in Eq.(2.24), which is the budget constraint of the constrained households.[20] Note the adjustment term in q_t^E, the relative productivity; this term is created due to the impossibility of achieving efficiency under asymmetric disturbances, and implies the amount of bond that type C households would desire to trade to share real wage risk if efficiency were to achieve. We discuss the impossibility of achieving efficiency in

[18] This is discussed in Section 2.5.3

[19] We provide the full system of equations and their derivations in the Appendix Section B.

[20] Type U households' budget constraint, $y_{2,t} = w_t - q_t + n_{U,t} + \frac{1}{z_2\theta}(d_t - t_{U,t})$ is excluded from the system of equations here to focus more on the implications of the financially constrained households, but it plays a nontrivial role in the analysis of optimal monetary policy. For later use, note that $d_t - t_{U,t} = -\theta\{z_1(w_t - a_t - a_{1,t}) + z_2(w_t - q_t - a_t - a_{2,t})\}$

Section 2.5.3, and identify a novel trade-off between output gaps and labor supply gaps in Section 2.5.4, which further leads to shifts in target output gaps in Section 4.2.1.

The second set of equations are from the firm side, the sectoral Phillips curves:

$$\pi_{1,t} = \beta E_t[\pi_{1,t+1}] + \kappa_1 \widetilde{w}_t \tag{2.25}$$

$$\pi_{2,t} = \beta E_t[\pi_{2,t+1}] + \kappa_2 \big(\widetilde{w}_t - \widetilde{q}_t\big) \tag{2.26}$$

where $\kappa_j \equiv \frac{(1-\alpha_j\beta)(1-\alpha_j)}{\alpha_j}$. In the presence of nominal friction in each sector ($\alpha_j \neq 0$), sectoral inflation is the weighted sum of the current and the expected future real marginal costs. In the absence of nominal friction, $\alpha_j = 0$, the real marginal cost is constant and the sectoral Phillips curve in the corresponding sector would degenerate, with inflation causing no inefficiency as standard.

Since the wage is applied economy-wide and measured in units of numeraire (good 1), the real marginal cost in sector 1, $w_t - a_t - a_{1,t}$, equals the real wage gap of the constrained households, and that in sector 2, $w_t - q_t - a_t - a_{2,t}$, equals the real wage gap of the unconstrained households. Thus both real marginal cost terms can be expressed in terms of output gaps using the equilibrium conditions from the demand side. Each sectoral output gap and adjustment terms have asymmetric effects on sectoral inflation as we discuss in Section 2.5.5.

$$\begin{aligned}
\widetilde{w}_t &= \frac{\sigma+\varphi}{1+\varphi}\widetilde{y}_{1,t} + z_2\frac{\varphi}{1+\varphi}\frac{1-\sigma}{\sigma}q_t^E \\
\widetilde{w}_t - \widetilde{q}_t &= \frac{z_1}{z_2}\varphi\frac{\sigma+\varphi}{1+\varphi}\widetilde{y}_{1,t} + (\sigma+\varphi)\widetilde{y}_{2,t} - z_1\frac{\varphi}{1+\varphi}\frac{1-\sigma}{\sigma}q_t^E
\end{aligned}$$

Lastly, the (economy-wide) labor market clearing condition is given by

$$z_1\widetilde{y}_{1,t} + z_2\widetilde{y}_{2,t} = z_1\widetilde{n}_{C,t} + z_2\widetilde{n}_{U,t} \tag{2.27}$$

Eq.(2.27) shows that the weighted sum of output gaps equals that of labor supply gaps.

2.5.3 Impossibility of achieving efficiency

Heterogeneous consumption baskets make market outcomes impossible to achieve the efficient allocation under asymmetric disturbances. For clarity, we check this in a flexible-price variant of the model in which the wage and the relative price trace the efficient levels. N stands for *natural* or flexible-price economy.[21]

Recall that idiosyncratic real wage risk is perfectly shared through bond market in the

[21] We provide the full system of equations of flexible-price allocation in the Appendix Section B.

frictionless economy as seen in Section 2.5.1: a positive shock on sector-specific productivity $a_{1,t}$ affects the real wages differently: it raises the real wage of type C, $w^E_{C,t} = w^E_t = a_t + a_{1,t}$, but has no effect on that of type U, $w^E_{U,t} = w^E_t - q^E_t = a_t + a_{2,t}$. To insure against the idiosyncratic real wage risk and equalize marginal disutility of labor, type C households borrow with their consumption increasing, and type U households save with their consumption decreasing.

Table 1: The effects of $a_{1,t}$ shock with and without risk-sharing

x_t	$w_{C,t}$	$w_{U,t}$	$n_{C,t}$	$y_{1,t}$	$n_{U,t}$	$y_{2,t}$	Risk-sharing
x^N_t	$\uparrow$	$-$	$\Uparrow$	$\uparrow$	$-$	$-$	No
x^E_t	$\uparrow$	$-$	$\uparrow$	$\Uparrow$	$\uparrow$	$\downarrow$	Perfect
$\widetilde{x}^N_t$	$-$	$-$	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	No

Now we introduce financial constraint – no risk-sharing between two types – into the frictionless economy. Then, with binding borrowing and savings constraints, a positive shock on $a_{1,t}$ only affects type C household with their consumption and labor supply increasing, while type U households are unaffected. Due to imperfect risk-sharing, households cannot equalize marginal disutility of labor. This results in failure to achieve efficient distribution of labor hours across households, and hence in failure to achieve the first-best outcome even in the absence of nominal frictions.

The following proposition summarizes the above analysis.

Proposition 1 (Impossibility of achieving efficiency)**.** *Under heterogeneous consumption baskets and financial constraints that prevent perfect sharing of idiosyncratic real wage risk, market outcomes cannot obtain the first-best outcome unless $\sigma = 1$, even in the absence of nominal frictions.*[22]

Proof. Please refer to the Appendix Section A. □

The impossibility is attributable to both heterogeneous consumption basket and the existence of HtM households together. On the one hand, if the consumption basket is homogeneous, both types of households face the same CPI and real wage; even under asymmetric disturbances, they make the same decisions with no idiosyncratic real wage risk. Thus, with the flexible prices, financial constraints are not binding anymore in achieving efficiency, and market outcomes can support the first-best outcome. On the other hand, if there is no borrowing and savings constraint, households can trade bonds to share risk. The bond holdings

[22] If $\sigma = 1$, labor supply schedule degenerates to a constant term because the income effect and the substitution effect exactly cancel out. Thus labor hour is always the same, making borrowing and savings constraint not binding in the absence of nominal rigidity.

terms fix the constrained households' budget constraint so market outcomes can support the efficient allocations.[23]

An immediate result of the impossibility of achieving efficiency is the adjustment term that shows up in fitting the efficient allocation into the constrained households' budget constraint, which cannot support the first-best outcome. We need to add an adjustment term as in Eq.(2.29) to take into account *the amount of bond type C households would sell if they were under perfect risk-sharing.* By definition, Eq.(2.24) is derived by subtracting Eq.(2.29) from Eq.(2.28):

$$w_t + n_{C_t} = y_{1,t} \tag{2.28}$$

$$w_t^E + n_{C_t}^E = y_{1,t}^E - \frac{1-\sigma}{\sigma} z_2 q_t^E \tag{2.29}$$

$$\widetilde{w}_t + \widetilde{n}_{C_t} = \widetilde{y}_{1,t} + \frac{1-\sigma}{\sigma} z_2 q_t^E \tag{2.24}$$

An intuitive interpretation is that: under perfect risk-sharing, households would borrow to equate marginal disutility of labor achieving efficiency (Eq.(2.29)). Due to the financial constraints, however, they cannot borrow anymore (Eq.(2.28)), and cannot consume goods or leisure as much by the amount $\frac{1-\sigma}{\sigma} z_2 q_t^E$, failing to achieve efficiency (Eq.(2.24)). Thus under market outcomes in the absence of risk-sharing, consumption is smaller than wage income by $\frac{1-\sigma}{\sigma} z_2 q_t^E$ than under perfect risk-sharing.

2.5.4 A Trade-off between Output Gaps and Labor Supply gaps

In this section, we discuss the distribution of labor demand – how labor hours from each household are determined in equilibrium – and identify a novel trade-off between output gaps and labor supply gaps that is generated by the impossibility.

Assuming no transfers to them, the constrained households' decisions on labor hours and consumption are affected only by their wage, because they are hand-to-mouth depending entirely on their labor income: given wage, their consumption and labor are optimally chosen by $C_{C,t} = W_t^{\frac{1+\varphi}{\sigma+\varphi}}$, and $N_{C,t} = W_t^{\frac{1-\sigma}{\sigma+\varphi}}$. Defining $LE(X)$ as labor-equivalent of variable X to denote the amount of (market) labor to produce X under technology constraint, we have

$$LE(C_{C,t}) - LE(N_{C,t}) = \frac{\lambda C_{C,t}}{A_t A_{1,t}} - \lambda N_{C,t} = \lambda W_t^{\frac{1-\sigma}{\sigma+\varphi}} \left(\frac{W_t}{A_t A_{1,t}} - 1 \right) \begin{cases} > 0 & \text{if } W_t > A_t A_{1,t} \\ < 0 & \text{if } W_t < A_t A_{1,t}. \end{cases}$$

[23] If we remove borrowing and savings constraint, we can derive type C households' budget constraint as $w_t + n_{C,t} = y_{1,t} + \lambda b_{C,t} + \frac{\lambda}{\beta} b_{C,t-1}$, where $b_{C,t}$ is defined as $b_{C,t} \equiv \frac{B_{C,t}}{P_1 Y_1}$. Since $b_{C,t-1}$ is predetermined, $b_{C,t}$ would trace its corresponding efficient level to support the first-best outcome.

This implies that their labor hours are smaller (larger) than the labor-equivalent of their consumption when their real wage gap is positive (negative), with the rest of the labor demand is, in effect, filled by the unconstrained households' labor hours through the labor market clearing condition. For instance, if an expansionary monetary policy shock raises real wages through sticky prices, hand-to-mouth households consume more labor-equivalent than their labor supply, and type U households backs this up implying that the latter consume less labor-equivalent than their labor supply in equilibrium. This is reconciled with the countercyclicality of non-labor income for the unconstrained, which is the difference in the income sources between households abstracting from heterogeneous CPIs. As standard in New Keynesian models, markups and dividend are countercyclical in response to demand shocks. Due to this negative income effect of non-labor income, type U decides to work more hours.[24] In this way, labor demand is redistributed from type C to type U by the amount $\frac{\sigma+\varphi}{1+\varphi} y_{1,t}$.[25] The relations between labor gaps and output gaps are summarized by:

$$\widetilde{n}_{C,t} = \frac{1-\sigma}{1+\varphi}\widetilde{y}_{1,t} + \frac{1-\sigma}{\sigma}\frac{1}{1+\varphi} z_2 q_t^E \tag{2.30}$$

$$\widetilde{n}_{U,t} = \widetilde{y}_{2,t} + \frac{z_1}{z_2}\frac{\sigma+\varphi}{1+\varphi}\widetilde{y}_{1,t} - \frac{1-\sigma}{\sigma}\frac{1}{1+\varphi} z_1 q_t^E \tag{2.31}$$

Note the adjustment terms: since the budget constraint of HtM households which cannot support the first-best outcome is crucial in deriving them, the relations cannot support efficiency either; hence the adjustment terms should be added to the relations to reflect the lack of risk-sharing.[26]

Imperfect sharing in real wage risk and the impossibility lead to a novel *trade-off* between output gaps and labor supply gaps under asymmetric disturbances that generates *distributional inefficiency from idiosyncratic real wage risk*: we cannot close output gaps and labor supply gaps simultaneously, $\widetilde{y}_{1,t} = \widetilde{y}_{2,t} = \widetilde{n}_{C,t} = \widetilde{n}_{U,t} = 0$. Even though we can close both output gaps, labor gaps cannot be closed due to the lack of risk-sharing, and vice versa. What is

[24] The representative agent in the basic New-Keynesian model is in the same situation, but it receives negative dividend that induces negative income effect. Thus the labor-equivalent of consumption and labor supply are equalized as $\frac{C_t}{A_t} = \frac{Y_t}{A_t} = N_t$.

[25] This has a nontrivial implication for dynamics of sectoral inflation which is discussed in Section 2.5.5.

[26] By definition, Eq.(2.30) is derived as the gap between two equations: $\begin{cases} n_{C,t} = \frac{1-\sigma}{1+\varphi} y_{1,t} \\ n_{C,t}^E = \frac{1-\sigma}{1+\varphi} y_{1,t}^E - \frac{1-\sigma}{\sigma}\frac{1}{1+\varphi} z_2 q_t^E. \end{cases}$ Recalling that $n_{C,t}^N(\Uparrow) > n_{C,t}^E(\uparrow)$, and $y_{1,t}^N(\uparrow) < y_{1,t}^E(\Uparrow)$ under a positive shock on $a_{1,t}$, we can find an adjustment term that captures type C households' borrowing under perfect risk-sharing for this labor supply–output relation to support the efficient outcome. Eq.(2.31) is analogous to this. Note that the adjustment terms are in the opposite directions to each other and of the size by the lack of risk-sharing, so that the population-weighted sum of adjustment terms in each relation is zero.

more, we cannot even close both output gaps simultaneously, regardless of nominal frictions.

Proposition 2 (Trade-off between output gaps and labor supply gaps)**.** *In a model with heterogeneous consumption baskets and borrowing and savings constraints under asymmetric disturbances,*
1) It is impossible to close all the sectoral output gaps and labor supply gaps simultaneously.
2) It is impossible to close both sectoral output gaps simultaneously.

Proof. Please refer to the Appendix Section A. □

The trade-off gives monetary policy a new role to deal with the distributional inefficiency in addition to traditional objectives. We will discuss more in detail in Section 4.2.1, where we find that the trade-off leads the central bank to target non-zero output gaps.

2.5.5 Asymmetric redistribution of inflationary pressure across sectors

The effects of sectoral output gaps and adjustment terms on dynamics of sectoral inflation are asymmetric as shown in the Phillips curves rewritten in terms of sectoral output gaps:[27] (1) inflation in sector 1 is affected only by output gap 1, while (2) inflation in sector 2 is affected by both output gaps; (3) a relative productivity shock q_t^E has the opposite consequences in each sector. (1) and (2) imply the redistribution of inflationary pressure across sectors as the labor demand is redistributed across households, and (3) is due to the lack of risk-sharing. We discuss more in detail in the Appendix Section B.4.

$$\pi_{1,t} = \beta E_t[\pi_{1,t+1}] + \kappa_1 \left(\frac{\sigma+\varphi}{1+\varphi} \widetilde{y}_{1,t} + z_2 \frac{\varphi}{1+\varphi} \frac{1-\sigma}{\sigma} q_t^E \right)$$
$$\pi_{2,t} = \beta E_t[\pi_{2,t+1}] + \kappa_2 \left(\frac{z_1}{z_2} \varphi \frac{\sigma+\varphi}{1+\varphi} \widetilde{y}_{1,t} + (\sigma+\varphi) \widetilde{y}_{2,t} - z_1 \frac{\varphi}{1+\varphi} \frac{1-\sigma}{\sigma} q_t^E \right)$$

Note the inefficient distribution of inflation, which is represented by the adjustment terms in the Phillips curves: they are similar to cost-push shocks in that they add stochasticity to inflation dynamics even under zero output gaps, but different in that the former always disappears as we aggregate sectoral inflation with the economic size of each sector. Suppose a positive shock on sector-specific productivity $a_{1,t}$. Due to financial constraints, type C households work more, and type U households work less than under efficient allocation. Since marginal disutility of labor supply gap is higher (lower) for type C (type U) households, their real wage gap that equals to real marginal cost, $\widetilde{w}_t$ ($\widetilde{w}_t - \widetilde{q}_t$), and inflation in the sector of

[27] In case of **HomCB**, sectoral output gap has symmetric effects on both sectoral inflations aside from asymmetric price stickiness, as shown in Section 2.6.2

goods they consume more intensively, $\pi_{1,t}$ ($\pi_{2,t}$), are higher (lower) in equilibrium due to the lack of risk-sharing, implying that inefficient distribution of labor supply translates to inefficient distribution of inflationary pressure across sectors. As a result, inflation dynamics in both sectors are amplified if $\sigma < 1$, or subdued otherwise, considering that the shock leads to a negative output gap in sector 1 and a positive output gap in sector 2 due to nominal rigidities.[28]

2.6 Equilibrium under HomCB

Now we characterize the equilibrium under completely homogeneous consumption baskets (**HomCB**, $\omega_U = \omega_C = \frac{1}{2}$). The main purpose of studying the case of **HomCB** is to better understand the implications of heterogeneous consumption baskets by comparing **HetCB** and **HomCB**. We first establish the efficient allocation, and then characterize the model equilibrium in percentage deviation from the efficient allocation.[29] Unlike **HetCB**, households face the same CPI and real wages, so there is no distributional inefficiency from idiosyncratic real wage risk with no trade-off between distributional variables.

2.6.1 Efficient Allocation

We derive the economy's efficient allocation by solving a social planner's problem that maximizes the weighted sum of utility of both types of households, subject to the resource and technology constraints

$$\max_{\{C_{h,t}, N_{h,t}, Y_{j,t}(i)\}} \left\{ \varpi_U (1-\lambda) \left[\frac{C_{U,t}^{1-\sigma}}{1-\sigma} - \frac{N_{U,t}^{1+\varphi}}{1+\varphi} \right] + \varpi_C \lambda \left[\frac{C_{C,t}^{1-\sigma}}{1-\sigma} - \frac{N_{C,t}^{1+\varphi}}{1+\varphi} \right] \right\}$$

$$\begin{aligned} s.t. \qquad (1-\lambda) C_{U,1,t} + \lambda C_{C,1,t} &= \left(\int_{\mathcal{I}_1} \left(\frac{1}{z_1} \right)^{\frac{1}{\theta}} Y_{1,t}(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} \\ (1-\lambda) C_{U,2,t} + \lambda C_{C,2,t} &= \left(\int_{\mathcal{I}_2} \left(\frac{1}{z_2} \right)^{\frac{1}{\theta}} Y_{2,t}(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} \\ (1-\lambda) N_{U,t} + \lambda N_{C,t} &= \int_{\mathcal{I}_1} \frac{Y_{1,t}(i)}{A_t A_{1,t}} di + \int_{\mathcal{I}_2} \frac{Y_{2,t}(i)}{A_t A_{2,t}} di \end{aligned}$$

where $\{\varpi_h\}$ denotes Pareto weights, and $C_{h,t}$ are defined as Eq.(2.1) and Eq.(2.2). As we did for the case of **hetCB** in Section 2.5.1, we assume a utilitarian social planner ($\varpi_U = \varpi_C$).

Since both types of households are identical with the same preference consuming homo-

[28] Please refer to the Appendix Section B.4 for more detail.

[29] We set the parameters z_1 and $z_2 (= 1 - z_1)$ as ω and $1 - \omega$ to measure the economic size of each sector.

geneous consumption baskets, both consumption and labor hours are equalized across all the households in the first-best allocation, as if there is a representative household:[30]

$$
\begin{aligned}
y_t^E = c_t^E \equiv c_{C,t}^E = c_{U,t}^E &= \frac{1+\varphi}{\sigma+\varphi}a_t + \frac{1+\varphi}{\sigma+\varphi}z_1 a_{1,t} + \frac{1+\varphi}{\sigma+\varphi}z_2 a_{2,t} \\
y_{1,t}^E = c_{1,t}^E \equiv c_{C,1,t}^E = c_{U,1,t}^E &= \frac{1+\varphi}{\sigma+\varphi}a_t + \left(\frac{1+\varphi}{\sigma+\varphi}z_1 + z_2\eta\right)a_{1,t} + \left(\frac{1+\varphi}{\sigma+\varphi}z_2 - z_2\eta\right)a_{2,t} \\
y_{2,t}^E = c_{2,t}^E \equiv c_{C,2,t}^E = c_{U,2,t}^E &= \frac{1+\varphi}{\sigma+\varphi}a_t + \left(\frac{1+\varphi}{\sigma+\varphi}z_1 - z_1\eta\right)a_{1,t} + \left(\frac{1+\varphi}{\sigma+\varphi}z_2 + z_1\eta\right)a_{2,t} \\
n_t^E \equiv n_{C,t}^E = n_{U,t}^E &= \frac{1-\sigma}{\sigma+\varphi}a_t + \frac{1-\sigma}{\sigma+\varphi}z_1 a_{1,t} + \frac{1-\sigma}{\sigma+\varphi}z_2 a_{2,t}
\end{aligned}
$$

Note that sectoral outputs in the first-best outcomes are different between **HomCB** and **HetCB**, depending on the relative size of the elasticity of substitution between sectors, η, and the elasticity of intertemporal substitution, $\frac{1}{\sigma}$. Suppose a positive shock on sector-specific productivity $a_{1,t}$: output in sector 1 would directly increase in both cases, but under **HomCB**, the increase is larger as households substitute goods from the higher-productivity sector for goods from the lower-productivity sector; however, the intertemporal substitution effect on good 1 would be weaker under **HomCB**, because the positive income effect of the shock is distributed to both sectors. If we assume that the elasticity of substitution between sectors dominates the elasticity of intertemporal substitution, $\eta > \frac{1}{\sigma}$, the former effect outweighs the latter, so output in sector 1 would be larger while output in sector 2 would be smaller under **HomCB** than under **HetCB**.

$$
\begin{aligned}
&\frac{\partial}{\partial a_{1,t}}\left[y_{1,t}^{E,\mathbf{HomCB}} - y_{1,t}^{E,\mathbf{HetCB}}\right] = \left(\eta - \frac{1}{\sigma}\right)z_2; \quad \frac{\partial}{\partial a_{2,t}}\left[y_{1,t}^{E,\mathbf{HomCB}} - y_{1,t}^{E,\mathbf{HetCB}}\right] = -\left(\eta - \frac{1}{\sigma}\right)z_2 \\
&\frac{\partial}{\partial a_{1,t}}\left[y_{2,t}^{E,\mathbf{HomCB}} - y_{2,t}^{E,\mathbf{HetCB}}\right] = -\left(\eta - \frac{1}{\sigma}\right)z_1; \quad \frac{\partial}{\partial a_{2,t}}\left[y_{2,t}^{E,\mathbf{HomCB}} - y_{2,t}^{E,\mathbf{HetCB}}\right] = \left(\eta - \frac{1}{\sigma}\right)z_1
\end{aligned}
$$

2.6.2 Approximate Allocation

We approximate the decentralized model by log-linearizing the equilibrium conditions around the deterministic efficient zero-inflation steady state. We focus on the different features of **HomCB** from **HetCB**.[31]

[30] We provide more details including the log-linearized system of equations in the Appendix Section C.

[31] We provide the full system of equations and some derivations in the Appendix Section C.

The first set of equations are from the household side:

$$\widetilde{c}_{U,t} - E_t[\widetilde{c}_{U,t+1}] = -\frac{1}{\sigma}\left(\widetilde{i}_t - (\omega E_t[\pi_{1,t+1}] + (1-\omega)E_t[\pi_{2,t+1}]) - r_t^E\right) \tag{2.32}$$

$$\varphi\widetilde{n}_{U,t} + \sigma\widetilde{c}_{U,t} = \widetilde{w}_t - (1-\omega)\widetilde{q}_t \tag{2.33}$$

$$\varphi\widetilde{n}_{C,t} + \sigma\widetilde{y}_{1,t} = \widetilde{w}_t - (1-\omega)\widetilde{q}_t \tag{2.34}$$

$$\widetilde{w}_t - (1-\omega)\widetilde{q}_t + \widetilde{n}_{C,t} = \widetilde{c}_{C,t} \tag{2.35}$$

where the real interest rate in the efficient allocation is $r_t^E \equiv \sigma(E_t[c_{U,t+1}^E] - c_{U,t}^E)$.

Homogeneous consumption baskets make non-trivial differences: first, both households face the same real wage, $w_{C,t} = w_{U,t} = w_t - (1-\omega)q_t$ even under asymmetric disturbances.[32] Households do not have idiosyncratic real wage risk to insure against anymore, making borrowing and savings constraint not binding in achieving the first-best outcome in the absence of nominal rigidity. Hence market outcomes can support the efficient allocation, creating no adjustment term in Eq.(2.35), the budget constraint of HtM households, and no trade-off shown in Section 2.5.4.[33] And there is no distributional inefficiency from idiosyncratic real wage risk. We define the aggregate output gap as $\widetilde{y}_t \equiv \omega\widetilde{y}_{1,t} + (1-\omega)\widetilde{y}_{2,t}$. Then the distributional variables are perfectly correlated (in log) with the aggregate output gap, implying that the inefficient variations of distributional variables are rather at an aggregate level under **HomCB**.

$$\widetilde{c}_{C,t} = (1+\varphi)\widetilde{y}_t; \quad \widetilde{c}_{U,t} = \frac{1-\lambda(1+\varphi)}{1-\lambda}\widetilde{y}_t; \quad \widetilde{n}_{C,t} = (1-\sigma)\widetilde{y}_t; \quad \widetilde{n}_{U,t} = \frac{1-\lambda(1-\sigma)}{1-\lambda}\widetilde{y}_t$$

The second set of equations are from the firm side:

$$\pi_{1,t} = \beta E_t[\pi_{1,t+1}] + \frac{(1-\alpha_1\beta)(1-\alpha_1)}{\alpha_1}\widetilde{w}_t$$
$$\pi_{2,t} = \beta E_t[\pi_{2,t+1}] + \frac{(1-\alpha_2\beta)(1-\alpha_2)}{\alpha_2}\left(\widetilde{w}_t - \widetilde{q}_t\right)$$

where the real marginal cost terms in the sectoral Phillips curves are different from those

[32] We can simplify the expression for real wage by defining wage to be expressed in units of the final good, $\ddot{w}_t \equiv w_t - (1-\omega)q_t$. But for consistency with the **HetCB** case, we maintain the previous definition.

[33] Unlike the **HetCB** case, the flexible-price allocation under **HomCB** achieve efficiency closing both output gaps and labor supply gaps simultaneously despite constraints on risk-sharing.

under **HetCB** and given by

$$\begin{aligned}\widetilde{w}_t &= (\sigma+\varphi)\widetilde{y}_t + (1-\omega)\widetilde{q}_t \\ \widetilde{w}_t - \widetilde{q}_t &= (\sigma+\varphi)\widetilde{y}_t - \omega\widetilde{q}_t\end{aligned}$$

The dynamics of sectoral inflation are affected by the current and expected future aggregate output gap and the relative price gap. Hence unlike the **HetCB** case, each sectoral output gap has symmetric effects on both sectoral inflations aside from asymmetric price stickiness.[34]

Lastly, the economy-wide labor market clearing condition is given by

$$\omega\widetilde{y}_{1,t} + (1-\omega)\widetilde{y}_{2,t} = \lambda\widetilde{n}_{C,t} + (1-\lambda)\widetilde{n}_{U,t}$$

3 Model dynamics

This section studies monetary policy transmission mechanism and the redistributive effects that operates through sectoral inflation and relative prices under heterogeneity. Then we examine the features that induce asymmetric responsiveness across households.

3.1 Monetary Policy Transmission Mechanism

Table 2 shows the baseline parameter values assumed in the numerical analysis.[35] We assume $\sigma = 0.67$, because it is more intuitive that labor supply schedule on wage is upward sloping, and it is shown by some studies that the income effect on labor supply is not big enough to dominate the substitution effect. However, the main results of this paper do not depend on the assumptions on σ. The mass of HtM households is 40% to be consistent with empirical evidence ($\lambda = z_1 = 0.4$).[36] The inverse of the Frisch elasticity of labor supply is assumed to be unity as standard in the literature.[37]

[34] We define the aggregate inflation as $\pi_t \equiv \omega\pi_{1,t} + (1-\omega)\pi_{2,t}$. If the price stickiness in both sectors are the same, $\alpha_1 = \alpha_2$, the aggregate Phillips curve that explains the dynamics of the aggregate inflation can easily be established as a weighted sum of sectoral Phillips curves.

[35] We conduct robustness check for a variety of combinations of parameterizations.

[36] To facilitate the comparison of our numerical results under **HetCB** and **HomCB** to those of Benigno (2004), we assume $\lambda = z_1 = 0.5$ in the numerical analysis of optimal monetary policy.

[37] A large share of financially constrained households can lead to "Inverted Aggregate Demand Logic" as shown by Bilbiie (2008) by which an increase in real interest rate is rather expansionary. In this case, we need inverted Taylor principle for determinacy: only passive policy is consistent with a unique rational expectations equilibrium. The IADL occur when the share of non-asset holders is high enough (high λ) and/or the Frisch elasticity of labor supply is low enough (high φ). But we do not face this under baseline specification.

Table 2: Baseline parameter values in the numerical analysis

β	0.99	λ	0.4	AD	4	ρ_a	0.9	σ_a	0.01	ϕ_{π_1}	0.75
φ	1	$1-\lambda$	0.6	RD	0.5	ρ_{a_1}	0.9	σ_{a_1}	0.01	ϕ_{π_2}	0.75
θ	6	z_1	0.4	α_1	0.65	ρ_{a_2}	0.9	σ_{a_1}	0.01	$\phi_{\widetilde{y}_1}$	0
σ	0.67	z_2	0.6	α_2	0.82	ρ_v	0.0	σ_v	0.01	$\phi_{\widetilde{y}_2}$	0

We introduce the concepts of average duration – $AD \equiv (1-\alpha_1)^{-z_1}(1-\alpha_2)^{-z_2}$ – and relative duration – $RD \equiv (1-\alpha_2)(1-\alpha_1)^{-1}$ – as in Benigno (2004), where the duration of price contract in each sector is $(1-\alpha_j)^{-1}$.[38] For the study of transmission mechanism, we follow the empirical evidence (Vieyra, 2018; Cravino et al., 2020; Argente and Lee, 2020; Clayton et al., 2019) that the prices in luxury good sector adjust more frequently than those in necessity good sector, and assume $AD=4$ and $RD=0.5$, or $\alpha_1=0.65$ and $\alpha_2=0.82$, which implies average duration of both sectors is 4 quarters while duration in sector 2 is double that in sector 1. We do not confine this study to this parameterization, but consider a variety of combinations of Calvo parameters, α_1 and α_2, in the normative analysis. Monetary policy is characterized by a simple Taylor rule responding only to sectoral inflation ($\phi_{\pi_1}=\phi_{\pi_1}=0.75$).

Now we examine the monetary policy transmission mechanism of an expansionary shock. In the model, type U households have Euler equation and respond to changes in interest rates, while type C households do not. Thus monetary policy shock is injected in sector 2 whose goods are consumed intensively by type U, then propagated to sector 1 through the labor market: an interest rate cut is followed by an increase in demand for good 2, leading to higher labor demand and wage; a higher marginal cost induces inflation, but the price in sector 1 rises faster than those in sector 2, because the price in sector 1 is stickier, leading to a decrease in relative price.[39] Consequently, the real wage of type U, $w_t - q_t$, is higher than that of type C, w_t, having different effects on consumption and labor supply across households. This is the *redistributive channel* of monetary policy that operates through heterogeneous real wages.

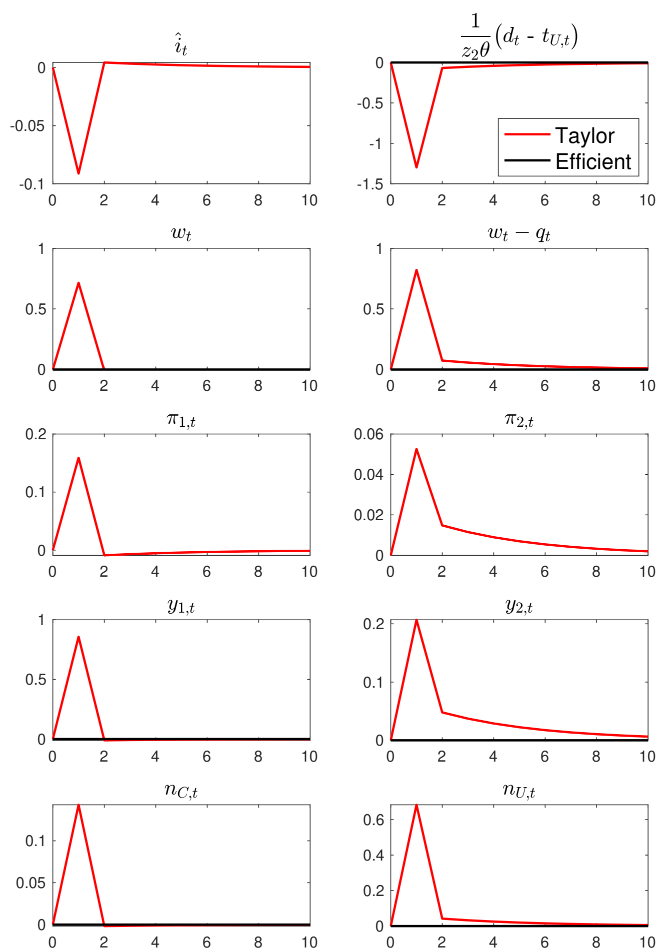
Consumption of type C increases more than that of type U in equilibrium, despite a higher real wage of type U. This is due to the counter-cyclicality of dividend under demand shocks and to the assumption on fiscal policy that finances subsidy on employment cost by lump-sum tax on type U.[40] As the real wage increases, type C raises their labor hours but

[38] From the definitions, we derive that $\alpha_1 \equiv 1-AD^{-1}RD^{-z_2}$ and $\alpha_2 \equiv 1-AD^{-1}RD^{z_1}$

[39] Real marginal cost is higher in sector 2 considering that real wage is higher for type U, although inflation is higher in sector 1. This is explained by the asymmetry in nominal rigidity.

[40] The sum of dividend and lump-sum transfer (net of tax) terms in type U households' budget constraint are linearized as $\frac{1}{z_2\theta}(d_t - t_{U,t}) = -\frac{1}{z_2}\{z_1(w_t - a_t - a_{1,t}) + z_2(w_t - q_t - a_t - a_{2,t})\}$.

Figure 3.1: Transmission Mechanism of an Expansionary Monetary Policy Shock



not enough to cover all their consumption. The rest is backed up by type U in equilibrium who are under negative income effects of dividend. Above illustration is shown in Figure 3.1.

3.2 Redistributive Effects of Monetary Policy

Heterogeneity creates nontrivial redistributive channels of monetary policy, which operate through relative prices and sectoral inflation. Monetary policy can have different effects on the real wages across households through relative prices, affecting their consumption and labor hours differently. In addition, although it cannot stabilize sectoral inflation in both sectors simultaneously under asymmetric disturbances, monetary policy can choose which one to stabilize more than the other, which affects the variations of consumption-relevant inflation rates and real wages differently. This has important distributional implications for the welfare of households: The more stable a households' consumption-relevant inflation rates, the more stable its real wages, the lower volatility of its consumption and labor hours, with its welfare increasing. This is the *Real Wage Stabilization Channel*.[41] Moreover, as inflation in its consumption sector stabilizes more, a household benefits more by lower price dispersion and smaller output loss. This is the *Consumption Support Channel*, that operates in second-order. Under homogeneous baskets, however, those channels do not work, because relative prices and sectoral inflation have only symmetric effects on households through the same real wages;[42] relative price affects the distribution of demands across sectors, but has no distributional consequences across households, because they consume the same composition of baskets.

Heterogeneity also confronts monetary policy with a nontrivial distributional issue on balancing welfare-relevant output gaps. Relative productivity shocks directly affect relative price, but it shows a sluggish adjustment due to nominal rigidity, leading to a negative output gap in the sector with higher productivity and a positive output gap in the sector with lower productivity. Which output gap to close more does not have distributional implications across households under **HomCB**, because its effects are symmetric, but does have under **HetCB**. Monetary policy faces a trade-off regarding whom to care about more: A more expansionary policy would benefit households who consume goods from the sector with higher productivity intensively by reducing the variation of its output gap, while having the opposite effects on households who consume goods from the sector with lower productivity intensively by raising variation of its output gap.

In a similar context, heterogeneity causes monetary policy to balance different efficient rates of interest across households. Suppose a positive shock on the sector-specific productivity $a_{1,t}$. Under **HomCB**, the efficient levels of consumption for both types increase with the efficient rate of real interest ($r_t^E = -\sigma E_t[c_t^E - c_{t+1}^E]$) decreasing; nominal rigidity would lead to a negative aggregate output gap. Although HtM households' Euler equation does not work, the central bank would largely trace the unique efficient rate and implement expansionary policy to benefit both types. Under **HetCB**, however, the efficient rates of real interest diverge: It decreases for type C ($r_{C,t}^E = -\sigma E_t[c_{C,t}^E - c_{C,t+1}^E]$) but increases for type U ($r_{U,t}^E = -\sigma E_t[c_{U,t}^E - c_{U,t+1}^E]$). The population-weighted average of the efficient rates of real interest coincides with that under homogeneous baskets, but since HtM households' Euler equation does not work, monetary policy needs another real interest rate to target. This

[41] We show this in Section 5.

[42] Monetary policy still has a distributional effect under **HomCB** through dividend, that is inversely correlated with price dispersion in second-order. Monetary policy can benefit the unconstrained (constrained) households more by assigning more weight to overall inflation stabilization (output stabilization).

would have distributional consequences for welfare-relevant output gaps; the central bank's objective function will characterize the policy.

Monetary policy has direct effects through intertemporal substitution and indirect effects through labor demand and real wages in general equilibrium. The unconstrained households have the Euler equation (Eq.(2.21), Eq.(2.32)) and respond to changes in interest rates, while HtM households do not. Thus monetary policy affects the former through both direct and indirect channels, but the latter is affected only by indirect effects, with the policy having disproportionate effects on the unconstrained households.

Moreover, monetary policy can have an indirect distributional effect through dividend that is inversely correlated with price dispersion, which transforms into inflation terms in the welfare loss function.

3.3 Asymmetric Responsiveness across Households

In the model, HtM households show larger responsiveness to shocks, as is standard in TANK models. This is attributable to the imperfect sharing of idiosyncratic non-labor income risk and idiosyncratic real wage risk, which leads the marginal utility of consumption and marginal disutility of labor to diverge inefficiently across households. Three main factors determine asymmetric responsiveness of consumption across households: differences in (1) wage elasticities of consumption, (2) real wages, and (3) responses to interest rate changes.

3.3.1 Wage Elasticity of Consumption

To understand the responses of consumption to changes in wages, we use a simple example of a household that makes a static decision on consumption and labor supply given the wage with utility function and budget constraint below:

$$\begin{aligned} \max\ U(C,N) &= \frac{C^{1-\sigma}}{1-\sigma} - \frac{N^{1+\varphi}}{1+\varphi} \\ s.t.\ \ WN+M &= C \end{aligned}$$

where M indicates sources of income other than wage income. From this analysis, we find that the responsiveness or the wage elasticity of consumption of a household depends non-trivially on the dynamics of other sources of income M,

$$\varepsilon_{C,W} \equiv \frac{\partial C}{\partial W}\frac{W}{C} = \frac{1+\varphi(1+\frac{\frac{\partial M}{\partial W}W}{WN})}{\sigma+\varphi(1+\frac{M}{WN})} = \frac{1+\varphi(1+\frac{\varepsilon_{M,W}M}{WN})}{\sigma+\varphi(1+\frac{M}{WN})}$$

where $\varepsilon_{M,W} \equiv \frac{\partial M}{\partial W}\frac{W}{M}$ is the wage elasticity of non-labor income.

Let us consider the cases of both types of households in the model. Since HtM households depend entirely on wage income ($M = 0,\ \varepsilon_{M,W} = 0$), their wage elasticity of consumption would be $\frac{1+\varphi}{\sigma+\varphi}$. However, the unconstrained households have other sources of income, dividend, which is countercyclical ($M > 0,\ \varepsilon_{M,W} < 0$) in response to demand shocks, as in standard New Keynesian models.[43] Thus their wage elasticity of consumption is smaller than that of HtM ($\varepsilon_{C,W,\text{type C}} = \frac{1+\varphi}{\sigma+\varphi} > \varepsilon_{C,W,\text{type U}}$), with the marginal utility of consumption diverging.[44] This is the *distributional inefficiency from idiosyncratic non-labor income risk* that arises from the financial constraints.

We derive similar results for the wage elasticity of labor hours. Refer to Section B.5 for more detail.

3.3.2 Real Wage

Due to heterogeneity in consumption baskets, households face different price indicess and real wages. Thus shocks that affect sectoral inflation differently alter relative prices and have differential effects on households' real wages and their variations, and thereby on their marginal utilities of consumption and marginal disutilities of labor. It is through this mechanism that monetary policy can have redistributive effects; it can respond to and influence sectoral inflation differently affecting relative prices. Thus the policy can redistribute marginal utilities between households to maximize social welfare, as we discuss in the following sections.

3.3.3 Responses to Interest Rate Changes

Unlike type U, HtM households are insensitive to changes in interest rates. Thus monetary policy has a stronger effect on the unconstrained households through the direct channels.

[43] As is standard in New Keynesian models, markup and dividends are countercyclical, leading to a stabilized consumption and labor hours for the unconstrained households. Cyclicality of markups is still controversial, but a recent study such as Hong (2019) shows that markups are countercyclical with an average elasticity of -1.1 with respect to real GDP. In reality, the richer or unconstrained households would be able to smooth their consumption making use of financial instruments, while the poorer or HtM households cannot. In the TANK model, there is effectively no instrument for savings. So we can consider countercyclicality of markups as an important model feature that generates a smoother consumption for the richer or unconstrained households than the poorer or HtM households, even in a simple model with no features such as assets and wage rigidities.

[44] Fiscal rules are important because they affect the dynamics and cyclicality of macroeconomic variables; if we introduce transfers (or tax), they also play nontrivial roles along with other sources of income in the determination of the responsiveness of consumption. We conduct various robustness check for the main results in Section 5.

4 Optimal Monetary Policy

We study optimal monetary policy under commitment by using a linear-quadratic approach following Woodford (2003). First, we take a second-order approximation to the equally-weighted sum of present valued utilities of both types of households around the deterministic efficient zero-inflation steady state, to derive a quadratic welfare-theoretic loss function of the utilitarian central bank. Then, we analyze optimal monetary policy by solving a Ramsey problem of the central bank that minimizes the welfare loss under the constraints that consist of first-order approximations to the equilibrium conditions.

We draw implications of the existence of HtM households and heterogeneous consumption baskets separately. Under **HomCB**, the distributional inefficiencies from idiosyncratic non-labor income risk are rather at the aggregate level, creating no additional trade-off. We find that financial constraint itself makes little difference to the results provided by Benigno (2004). Under **HetCB**, however, optimal policy changes significantly from Benigno (2004). The distributional inefficiencies are non-trivial from both idiosyncratic real wage risk and idiosyncratic non-labor income risk. Since monetary policy has redistributive effects, it should deal with the distributional inefficiencies at the cost of some price instability.

4.1 OMP under HomCB

4.1.1 Welfare-theoretic Loss Function

The welfare-theoretic loss function of the utilitarian central bank is derived as follows:

Proposition 3. *Under homogeneous consumption baskets, a second-order approximation to the equally-weighted present valued sum of both types of households' utilities is given by*

$$
\begin{aligned}
&E_0 \sum_{t=0}^{\infty} \beta^t \Big[\lambda \big\{ U(C_{C,t}) - V(N_{C,t}) \big\} + (1-\lambda) \big\{ U(C_{U,t}) - V(N_{U,t}) \big\} \Big] \\
&\quad = -\frac{U_c \overline{Y}}{2} E_0 \sum_{t=0}^{\infty} \beta^t \mathbb{L}_t + t.i.p. + o(||\xi||^3)
\end{aligned}
$$

where t.i.p. denotes "the terms independent of monetary policy" and $o(||\xi||^3)$ *includes all* *the terms of third order or above. The loss function is defined as*[45]

$$\mathbb{L}_t = \Phi_{\pi_1}\pi_{1,t}^2 + \Phi_{\pi_2}\pi_{2,t}^2 + \Phi_y \widetilde{y}_t^2 + \Phi_q \widetilde{q}_t^2$$

$$\Phi_{\pi_1} \equiv \omega\frac{\theta}{\kappa_1}; \quad \Phi_{\pi_2} \equiv (1-\omega)\frac{\theta}{\kappa_2}; \quad \Phi_y \equiv (\sigma+\varphi)\frac{1-\lambda(1-\sigma\varphi)}{1-\lambda}; \quad \Phi_q \equiv \eta\omega(1-\omega)$$

Proof. Please refer to the Appendix Section A. □

Nominal rigidity is a source of inefficiencies: it causes price dispersion within each sector that leads to output losses in second-order and transforms to sectoral inflation, $\pi_{1,t}$ and $\pi_{2,t}$; it induces inefficient variations in demand for goods, shown by the aggregate output gap, $\widetilde{y}_t$; also, it creates cross-sectoral distortion, $(\widetilde{y}_{1,t}-\widetilde{y}_{2,t})^2$, affected by inefficient variations of relative price, $\widetilde{q}_t$.

Financial constraints generate distributional inefficiencies from idiosyncratic non-labor income risk and are reflected in the coefficient of the aggregate output gap, Φ_y. Since the distributional variables are perfectly correlated with the aggregate output gap, distributional inefficiencies such as differences in consumption or labor hours between two types can be explained by the aggregate output gap. Thus distributional inefficiency is rather at an aggregate level, and the central bank's problem of balancing welfare loss from sectoral inflation, aggregate output gap and relative price gap is essentially unaffected.[46] Note that Φ_y is increasing in λ: as the share of HtM households increases, output stabilization becomes relatively more important than price stabilization. This is because the dividend is inversely correlated with price dispersion in second-order:

$$\int D_t(i)di = \sum_{j=1,2} Y_{j,t}\left[\frac{P_{j,t}}{P_{1,t}} - \frac{W_t}{A_t A_{j,t}}\int_{\mathcal{I}_j}\frac{1}{z_j}\left(\frac{P_{j,t}(i)}{P_{j,t}}\right)^{-\theta} di\right]$$

where $d_{j,t} \equiv \log\frac{1}{z_j}\int_{\mathcal{I}_j}\left(\frac{P_{j,t}(i)}{P_{j,t}}\right)^{-\theta} = \frac{\theta^2}{2}Var_i^j\{p_{j,t}(i)\} + o(||\xi||^3)$, which is associated with sectoral inflation.[47] As λ increases, the share of households receiving dividend decreases, and the central bank care relatively less about price dispersion, putting a relatively higher weight

[45] For a comparison to the loss function under **HetCB**, we can rewrite the loss function in terms of sectoral output gaps:

$$\mathbb{L}_t^{(\text{HomCB})} = \Phi_{\pi_1}\pi_{1,t}^2 + \Phi_{\pi_2}\pi_{2,t}^2 + \Phi_{y_{11}}\widetilde{y}_{1,t}^2 + \Phi_{y_{12}}\widetilde{y}_{1,t}\widetilde{y}_{2,t} + \Phi_{y_{22}}\widetilde{y}_{2,t}^2$$

$$\text{where} \quad \Phi_{y_{11}} \equiv \Phi_y\omega^2 + \frac{\Phi_q}{\eta^2}; \quad \Phi_{y_{22}} \equiv \Phi_y(1-\omega)^2 + \frac{\Phi_q}{\eta^2}; \quad \Phi_{y_{12}} \equiv 2\left(\Phi_y - \frac{1}{\eta}\right)\omega(1-\omega);$$

[46] We will see in Section 4.2.2 that distributional inefficiencies lead to a shift in target output gaps under **HetCB**.

[47] Refer to the proof of Proposition 3 provided in Appendix Section A for the derivation.

on output gap stabilization. This finding is in line with Bilbiie (2008), which studies in a single-sector framework with cost-push shocks. In our multi-sector model, we have the policy trade-off even in the absence of the inefficient cost-push shock due to the asymmetric disturbances.

4.1.2 Optimal Monetary Policy under Commitment

Now we investigate optimal monetary policy of the central bank under commitment that chooses target variables and nominal interest rate to maximize the objective function under equilibrium constraints.[48] Under **HomCB**, the distributional inefficiencies from idiosyncratic non-labor income risk are at the aggregate level, in that inefficient variations of distributional variables are perfectly correlated with aggregate output gap, creating no additional trade-off. Thus, monetary policy focus on dealing with nominal distortions. We find that financial constraint itself makes little difference to the results provided by Benigno (2004) where the market is complete, only raising the relative importance of aggregate output gap in the loss function. We briefly discuss them in the following propositions.

We study in three different cases of price stickiness: (i) flexible price in one sector and sticky price in the other sector ($\alpha_1 = 0$ or $\alpha_2 = 0$); (ii) sticky price in both sectors to the same degree ($0 < \alpha_1 = \alpha_2$); and (iii) sticky price in each sector but to different degrees ($0 < \alpha_1 < \alpha_2$).

(i) Flexible Price in One Sector

Proposition 4. *If the price of either one of the two sectors is fully flexible, it is optimal to fully stabilize inflation of the sticky sector. Under the optimal monetary policy, the market outcome can achieve efficiency.*

Proof. Please refer to the Appendix Section A. □

In this case, the only distortion is from nominal rigidity in the sticky sector. Since the central bank has one instrument and effectively one distortion, it can perfectly fix the distortion achieving efficiency; inflation in the flexible sector is innocuous because there is no price or output dispersion; the price in the sector with no nominal friction adjusts flexibly so that relative price traces its efficient level; if inflation in the sticky sector is fully stabilized, there would be no inefficiency from nominal friction. Since real marginal costs in both sectors are closed to zero inducing no non-labor income source for type U households with dividend and tax summing up to zero, financial constraints are not binding, and the first-best is obtained.

[48] Studying optimal monetary policy is deriving one more condition, a “targeting rule”, to minimize the welfare loss among all the possible candidate rules including simple Taylor rules that can close the model.

(ii) Equal Degrees of Nominal Rigidity across Sectors

Proposition 5. *If the prices of both sectors are sticky to the same degree, it is optimal to fully stabilize the aggregate inflation weighted by sector size. However, the optimal monetary policy cannot achieve efficiency.*

Proof. Please refer to the Appendix Section A. □

In this case, the central bank deals with two distortions – nominal rigidity in each sector – with one instrument. Moreover, monetary policy loses control over relative price which is affected only by the exogenous asymmetric shocks, and it cannot fix the inefficiencies induced by sluggish adjustment of relative price.[49] Although distributional variables, $c_{h,t}$ and $n_{h,t}$, aggregate output and real wage, $w_t-(1-\omega)q_t$, are on their efficient paths, relative price, wage, and sectoral output fail to achieve efficiency.

(iii) General Case

Proposition 6. *If the prices of both sectors are sticky to different degrees, efficiency cannot be obtained. A targeting rule is derived as follows:*

$$\frac{1}{\kappa_2-\kappa_1}\left[\begin{array}{c}\kappa_2\{\theta\pi_t+\frac{1-\lambda(1-\sigma\varphi)}{1-\lambda}A(L)\widetilde{y}_t\}+A(L)\{\theta\pi_t+\frac{1-\lambda(1-\sigma\varphi)}{1-\lambda}A(L)\widetilde{y}_t\}\\ -\beta A(L)\{\theta E_t[\pi_{t+1}]+\frac{1-\lambda(1-\sigma\varphi)}{1-\lambda}A(L)E_t[\widetilde{y}_{t+1}]\}\end{array}\right]$$
$$=(1-\omega)\theta\pi_{2,t}-\eta\omega(1-\omega)A(L)\widetilde{q}_t+(1-\omega)\frac{1-\lambda(1-\sigma\varphi)}{1-\lambda}A(L)\widetilde{y}_t$$

where $A(L)\equiv 1-L$. If the central bank commits to the class of "inflation targeting policy", it is optimal to give higher weight to the sector with higher degrees of nominal rigidity.

Proof. Please refer to the Appendix Section A for proof, and Section 5 for numerical results. □

With one instrument and two distortions to deal with, monetary policy fails to achieve efficiency. Since the targeting rule that we derive is complicated to get an intuition from, we draw implications from the perspective of "optimal inflation targeting policy": what is optimal weight δ that minimize welfare loss among the class of policy rules that fully stabilizes a weighted average inflation? Through numerical experiments in Section 5, we

[49] Note that if $\alpha_1=\alpha_2$, the dynamics of relative price is derived only by sectoral Phillips curves and the definition of relative price.

find that optimal inflation targeting policy give higher weight to the sector whose price is stickier, which is consistent with the findings of Benigno (2004).

$$\begin{cases} \delta^{hom} > z_1, & \text{if } \alpha_1 > \alpha_2 \\ \delta^{hom} < z_1, & \text{if } \alpha_1 < \alpha_2 \end{cases}$$

4.2 OMP under HetCB

4.2.1 Welfare-theoretic Loss Function

We find that the trade-off generated by the impossibility under **HetCB** leads the central bank to target non-zero output gaps, as shown in the welfare-theoretic loss function.

Proposition 7. *Under heterogeneous consumption baskets, a second-order approximation to the equally-weighted present valued sum of both types of households' utilities is given by*

$$\begin{aligned} & E_0 \sum_{t=0}^{\infty} \beta^t \Big[\lambda \big\{ U(C_{C,t}) - V(N_{C,t}) \big\} + (1-\lambda) \big\{ U(C_{U,t}) - V(N_{U,t}) \big\} \Big] \\ & \quad = -\frac{U_c \overline{Y}}{2} E_0 \sum_{t=0}^{\infty} \beta^t \mathbb{L}_t + t.i.p. + o(||\xi||^3) \end{aligned}$$

where t.i.p. denotes "the terms independent of monetary policy", $o(||\xi||^3)$ includes all the terms of third order or above, and the loss function is defined as[50]

$$\begin{aligned} \mathbb{L}_t &= \frac{z_1 \theta}{\kappa_1} \pi_{1,t}^2 + \frac{z_2 \theta}{\kappa_2} \pi_{2,t}^2 + z_1 \sigma \widetilde{y}_{1,t}^2 + z_2 \sigma \widetilde{y}_{2,t}^2 + z_1 \varphi \widetilde{n}_{C,t}^2 + z_2 \varphi \widetilde{n}_{U,t}^2 \\ &= \Gamma_{\pi_1} \pi_{1,t}^2 + \Gamma_{\pi_2} \pi_{2,t}^2 + \Gamma_{y_{11}} (\widetilde{y}_{1,t} - x_{1,t}^*)^2 + \Gamma_{y_{12}} (\widetilde{y}_{1,t} - x_{1,t}^*)(\widetilde{y}_{2,t} - x_{2,t}^*) + \Gamma_{y_{22}} (\widetilde{y}_{2,t} - x_{2,t}^*)^2 \end{aligned}$$

$$x_{1,t}^* \equiv \frac{1-\sigma}{\sigma} \frac{\varphi}{\sigma+\varphi} \frac{(\sigma - z_2) z_2}{\sigma\varphi + z_2} q_t^E; \quad x_{2,t}^* \equiv \frac{1-\sigma}{\sigma} \frac{\varphi}{\sigma+\varphi} \frac{z_1 z_2}{\sigma\varphi + z_2} q_t^E;$$

$$\Gamma_{\pi_1} \equiv \frac{z_1 \theta}{\kappa_1}; \quad \Gamma_{\pi_2} \equiv \frac{z_2 \theta}{\kappa_2};$$

$$\Gamma_{y_{11}} \equiv z_1 \left[\sigma + \left(\frac{1-\sigma}{1+\varphi} \right)^2 \varphi + \frac{z_1}{z_2} \left(\frac{\sigma+\varphi}{1+\varphi} \right)^2 \varphi \right]; \quad \Gamma_{y_{12}} \equiv 2 z_1 \varphi \frac{\sigma+\varphi}{1+\varphi}; \quad \Gamma_{y_{22}} \equiv z_2 (\sigma+\varphi)$$

Proof. Please refer to the Appendix Section A. □

Nominal rigidity is a source of inefficiencies: it causes price dispersion within each sector that leads to output losses in second-order and transforms to sectoral inflation, $\pi_{1,t}$ and $\pi_{2,t}$;

[50] We express the loss function in terms of distributional variables, because **HetCB** creates new trade-offs between distributional variables and cannot be explained by the aggregate variables.

it induces inefficient variations of households' real wages and hence of their demand for goods, sectoral outputs and labor hours, shown by output gaps and labor hour gaps, $\widetilde{y}_{1,t}, \widetilde{y}_{2,t}, \widetilde{n}_{C,t}$ and $\widetilde{n}_{U,t}$.[51] Taking into account the distributional inefficiencies from idiosyncratic real wage risk shown by the relations between labor supply gaps and output gaps (Eqs.(2.30)-(2.31)), we find that the output gaps that the central bank should target, $x^*_{1,t}$ and $x^*_{2,t}$, move away from zero following asymmetric disturbances. This is the consequences of the central bank's optimal balancing of marginal utility of consumption and marginal disutility of labor between households under imperfect risk-sharing.[52]

Suppose a positive shock on sector-specific productivity $a_{1,t}$ when $\sigma < 1$.[53] If we suppose the central bank can close output gaps, marginal disutility of labor of type C is larger and that of type U is smaller than efficient levels. Hence the central bank would try to lower marginal disutility of type C at the cost of their consumption (negative output gap 1), and raise that of type U by boosting consumption of both types (positive output gaps in both sectors). Thus, target output gap 2 should obviously be raised above zero, $x^*_{2,t} > 0$, but the direction of target output gap 1 depends on the value of σ that measures the extent households care about consumption smoothing, the relative size of income effect in labor supply, and the size of redistribution of labor demand.[54]

If σ is small enough ($\sigma < z_2$), the target output gap 1 is lowered below zero, $x^*_{1,t} < 0$. On the one hand, households care less about consumption smoothing and their responses of consumption to shocks are stronger; the shock affects labor hour gaps by a larger amount generating larger inefficiency; the benefit from balancing also increases, because households care relatively more about variations in labor hour gap. On the other hand, since the redistribution of labor demand is smaller when the income effect is smaller, output gap 1 is more effective in adjusting labor hour gap C than labor hour gap U. Consequently, optimal balancing is to lower target output gap 1. If σ is not small enough ($z_2 < \sigma < 1$), the opposite holds, and output gap 1 should be targeted above zero. We summarize the direction of shifts in target output gaps under an increase in relative productivity $q^E_t (\equiv a_{1,t} - a_{2,t})$ in Table 3 with varying values of σ.

The covariance term shows up in the loss function as a result of the redistribution of

[51] $\widetilde{q}_t$ does not appear in the loss function for two reasons. First, since it captures differences in real wages, it is reflected in the distributional variables. Second, since we are assuming completely heterogeneous consumption baskets with no substitution between sectoral goods, the cross-sectoral distortion, $(\widetilde{y}_{1,t} - \widetilde{y}_{2,t})^2$, is not penalized, nor is correlated with inefficient variations of relative price, $\widetilde{q}_t$.

[52] Note that distributional inefficiency from idiosyncratic real wage risk is reflected in the target output gap terms, while distributional inefficiency from idiosyncratic non-labor income risk is reflected in the weight of output gap terms and the covariance term.

[53] If we set $\sigma > 1$, both target output gaps unambiguously decreases below zero. If we set $\sigma = 1$, labor hours degenerate to a constant, so labor hours are equalized always.

[54] Note that target output gap in sector 2 shifts by a larger amount than in sector 1 under $\sigma < 1$.

Table 3: Directions of shifts in target output gaps under an increase in q_t^E

	$\sigma < z_2$	$\sigma = z_2$	$z_2 < \sigma < 1$	$\sigma = 1$	$\sigma > 1$
$x_{1,t}^*$	$\downarrow$	$-$	$\uparrow$	$-$	$\downarrow$
$x_{2,t}^*$	$\uparrow$	$\uparrow$	$\uparrow$	$-$	$\downarrow$

labor demand in equilibrium from type C to type U households, whose labor hour gap is positively correlated with both output gaps; the weight $\Gamma_{y_{12}} \equiv 2z_1\varphi\frac{\sigma+\varphi}{1+\varphi}$ reflects the amount of the redistribution, $\frac{\sigma+\varphi}{1+\varphi}y_{1,t}$. Redistribution is also shown in the weight of output gap 1, $\Gamma_{y_{11}} \equiv z_1[\sigma + (\frac{1-\sigma}{1+\varphi})^2\varphi + \frac{z_1}{z_2}(\frac{\sigma+\varphi}{1+\varphi})^2\varphi]$: the second and the third term indicate labor demanded by sector 1 that is distributed to type C and type U, respectively.

4.2.2 Optimal Monetary Policy under Commitment

Heterogeneous consumption baskets make significant differences to the results under **HomCB** or provided by Benigno (2004) where the market is complete. This is because distributional inefficiencies are non-trivial from both idiosyncratic real wage risk, and idiosyncratic non-labor income risk: the impossibility creates trade-offs at the distributional level, leading the central bank to target non-zero output gaps in order to balance marginal utilities and marginal disutilities between households; optimal policy benefits more HtM households, whose wage elasticity of consumption is higher, to redistribute towards reducing differences between households' marginal utility. Since monetary policy has redistributive channels in operation, it should deal with the distributional inefficiencies as well as nominal rigidity, but at the cost of some price instability.

We study in four different cases of price stickiness: (i) flexible price in sector 1 and sticky price in sector 2 ($\alpha_1 = 0 < \alpha_2$); (ii) flexible price in sector 2 and sticky price in sector 1 ($\alpha_2 = 0 < \alpha_1$); (iii) sticky price in both sectors to the same degree ($0 < \alpha_1 = \alpha_2$); and (iv) sticky price in each sector but to different degrees ($0 < \alpha_1 < \alpha_2$).[55]

(i) Flexible Prices of Goods Consumed Intensively by the Constrained

Proposition 8. *If the price of the goods consumed more intensively by the constrained households is fully flexible, it is optimal to stabilize inflation of the sticky sector. Under the optimal monetary policy, the market outcome fails to obtain efficiency, but achieves flexible-price allocation.*

Proof. Please refer to the Appendix Section A. □

[55] Note that case (i) and (ii) are effectively the same under homogeneous consumption baskets.

In this case, the central bank with one instrument should deal with two distortions – nominal rigidity in sector 2 and distributional inefficiencies. Moreover, due to its flexible price and the insensitivity of its consumers to interest rate, sector 1 is insulated from monetary policy and monetary policy cannot deal with the distributional inefficiency. Thus, it is optimal to eliminate distortion from nominal rigidity in sector 2, achieving flexible-price allocation, which is generically not efficient due to imperfect risk-sharing.[56]

(ii) Flexible Prices of Goods Consumed Intensively by the Unconstrained

Proposition 9. *If the price of the goods consumed more intensively by the unconstrained households is fully flexible, flexible price allocation is feasible by fully stabilizing inflation of the sector with nominal friction, but sub-optimal. Under optimal policy, the deviations of output gaps from their target levels are optimally distributed as functions of the current and the past shocks:*

$$\widetilde{y}_{1,t}^{OMP} = \underbrace{z_2 \frac{1-\sigma}{\sigma} \frac{\varphi}{\sigma+\varphi} \frac{\sigma - z_2}{\sigma\varphi + z_2} q_t^E}_{=x_{1,t}^*} - z_2 \frac{1-\sigma}{\sigma} \frac{\varphi}{\sigma+\varphi} \frac{\sigma\varphi+\sigma}{\sigma\varphi+z_2} \frac{(\lambda_1-1)(1-\lambda_2)}{\lambda_1-\rho} \sum_{k=0}^{\infty} \lambda_2^k q_{t-k}^E$$

$$\widetilde{y}_{2,t}^{OMP} = \underbrace{z_1 \frac{1-\sigma}{\sigma} \frac{\varphi}{\sigma+\varphi} \frac{z_2}{\sigma\varphi + z_2} q_t^E}_{=x_{2,t}^*} + z_1 \frac{1-\sigma}{\sigma} \frac{\varphi}{\sigma+\varphi} \frac{\sigma\varphi}{\sigma\varphi+z_2} \frac{(\lambda_1-1)(1-\lambda_2)}{\lambda_1-\rho} \sum_{k=0}^{\infty} \lambda_2^k q_{t-k}^E$$

Under optimal policy, a weighted average of the deviation of output gaps from the target level is fully stabilized, giving higher weight to the sector with flexible price:

$$\varphi z_1(\widetilde{y}_{1,t} - x_{1,t}^*) + (1+\varphi) z_2(\widetilde{y}_{2,t} - x_{2,t}^*) = 0$$

Proof. Please refer to the Appendix Section A. ∎

We find a policy trade-off in which the central bank has an incentive to deal with the distributional inefficiency from financial constraints at the cost of some price instability: it tolerates inflation or deflation to some degrees. As in case (i), the central bank should deal with two distortions – nominal rigidity in sector 1 and distributional inefficiencies – and can perfectly eliminate nominal distortion by fully stabilizing inflation of the sticky sector. Unlike that however, monetary policy can and should deal with the distributional inefficiency as

[56] Other policy rules may be able to affect sector 2 and type U households, but they are sub-optimal because any effects on them are at the cost of inflation as in the representative-agent New-Keynesian model.

well as distortions from nominal rigidities.[57] Note from the case (i) and (ii) that monetary policy should deal with the distributional inefficiencies when it has redistributive effects. Optimal policy balances between two distortions, although it fails to achieve efficiency.

In the following cases (iii) and (iv), optimal policy balances between three distortions – nominal rigidity in each sector and distributional inefficiencies.

(iii) Equal Degrees of Nominal Rigidity across Sectors

Proposition 10. *If the prices of both sectors are sticky to the same degree, it is no longer optimal to stabilize the aggregate inflation weighted by sector size. A targeting rule is derived as a function of the current and past variables under commitment:*

$$z_1\theta\pi_{1,t} + z_2\theta\pi_{2,t} + \left(z_1 + \frac{z_1}{z_2}\frac{\sigma\varphi}{1+\varphi}\right)A(L)(\widetilde{y}_{1,t} - x^*_{1,t}) + z_2A(L)(\widetilde{y}_{2,t} - x^*_{2,t}) = 0$$

Proof. Please refer to the Appendix Section A. □

Note from the aggregate Phillips curve, $\pi_t \equiv \beta E_t[\pi_{t+1}] + \kappa(\sigma+\varphi)(z_1\widetilde{y}_{1,t} + z_2\widetilde{y}_{2,t})$, and the targeting rule that full stabilization of the aggregate inflation is no longer optimal: optimal plan is a mix of price stabilization and output stabilization. In this case, the central bank loses control over relative price which is affected only by the exogenous asymmetric shocks. Thus it cannot fix the inefficiencies induced by sluggish adjustment of relative price failing to achieve efficiency.

(iv) General Case

Proposition 11. *If the prices of both sectors are sticky to different degrees, a targeting rule is derived as a function of the current and past variables under commitment:*

$$\begin{aligned}
&\frac{\kappa_2}{\kappa_2 - \kappa_1}\left[z_1\theta\pi_{1,t} + z_2\theta\pi_{2,t} + \left(z_1 + \frac{z_1}{z_2}\frac{\sigma\varphi}{1+\varphi}\right)A(L)(\widetilde{y}_{1,t} - x^*_{1,t}) + z_2A(L)(\widetilde{y}_{2,t} - x^*_{2,t})\right] \\
&+ \frac{1}{\kappa_2 - \kappa_1}A(L)\left[z_1\theta\pi_{1,t} + z_2\theta\pi_{2,t} + \left(z_1 + \frac{z_1}{z_2}\frac{\sigma\varphi}{1+\varphi}\right)A(L)(\widetilde{y}_{1,t} - x^*_{1,t}) + z_2A(L)(\widetilde{y}_{2,t} - x^*_{2,t})\right] \\
&- \frac{\beta}{\kappa_2 - \kappa_1}A(L)E_t\left[z_1\theta\pi_{1,t+1} + z_2\theta\pi_{2,t+1} + \left(z_1 + \frac{z_1}{z_2}\frac{\sigma\varphi}{1+\varphi}\right)A(L)(\widetilde{y}_{1,t+1} - x^*_{1,t+1}) + z_2A(L)(\widetilde{y}_{2,t+1} - x^*_{2,t+1})\right] \\
&= z_2\theta\pi_{2,t} + \frac{z_1\varphi}{1+\varphi}A(L)(\widetilde{y}_{1,t} - x^*_{1,t}) + z_2A(L)(\widetilde{y}_{2,t} - x^*_{2,t})
\end{aligned}$$

[57] Since monetary policy that is injected into the flexible sector propagates to the sticky sector through the labor market, sector 1 and type C households are under the effects of monetary policy. Suppose an interest rate cut that changes demand for goods and labor, leading to changes in wage. Real marginal cost gap in sector 2 or real wage gap of type U households, $\widetilde{w}_t - \widetilde{q}_t$ is closed due to flexible price, but adjustments of real marginal cost in sector 1 or real wage of type C households, $\widetilde{w}_t$ are sluggish due to sticky prices in sector 1.

where $A(L) \equiv 1–L$. If the central bank commits to the class of "inflation targeting policy", heterogeneous consumption baskets give higher weight to the sector consumed by the constrained households than the optimal weight implied by homogeneous consumption baskets.

Proof. Please refer to the Appendix Section A for proof, and Section 5 for numerical results. □

To overcome the complexity of the targeting rules in case (iii) and (iv), we get intuition from the "optimal inflation targeting policy": heterogeneous consumption baskets put higher weight to the sector of goods consumed intensively by the constrained households than under homogeneous baskets, $\delta^{het} > \delta^{hom}$, regardless of how nominal rigidities are distributed between sectors. We discuss further in Section 5.

5 Some Numerical Analysis

This section conducts numerical experiments on the consequences of neglecting heterogeneity and the implications of heterogeneity for the optimal inflation targeting policy. We also rationalize the redistributive effects by welfare analysis and discuss the robustness.

5.1 Consequences of Neglecting Heterogeneity

What would the consequences be if the central bank neglects heterogeneous consumption baskets? We posit a scenario in which the central bank minimize welfare loss under **HomCB** instead of the true one under **HetCB**.

The experiment shows significant implications. Neglect of heterogeneity would lead to: (1) *under*stabilization of consumption-relevant inflation and real wages of the constrained households and of the output gap in the sector of goods type C consumes more intensively; (2) *over*stabilization of inflation and real wages of the unconstrained households and of the output gap in the sector of goods type U consumes more intensively.[58]

Let us discuss why under a positive shock on sector-specific productivity $a_{1,t}$.

- Distributional inefficiencies from idiosyncratic real wage risk are not considered, neglecting shifts in target output gap above zero by the shock; monetary policy would be less expansionary than optimal.

- Distributional inefficiencies from idiosyncratic non-labor income risk under **HetCB** require stabilizing more the real wage of HtM households who are more responsive, to

[58]This result is qualitatively robust under plausible values of σ.

Figure 5.1: Consequences of neglecting heterogeneity under $a_{1,t}$ shock

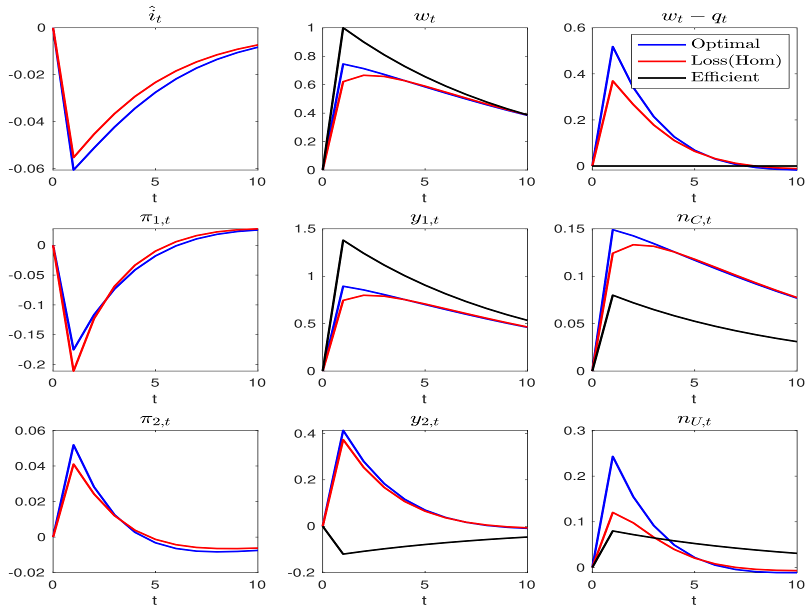
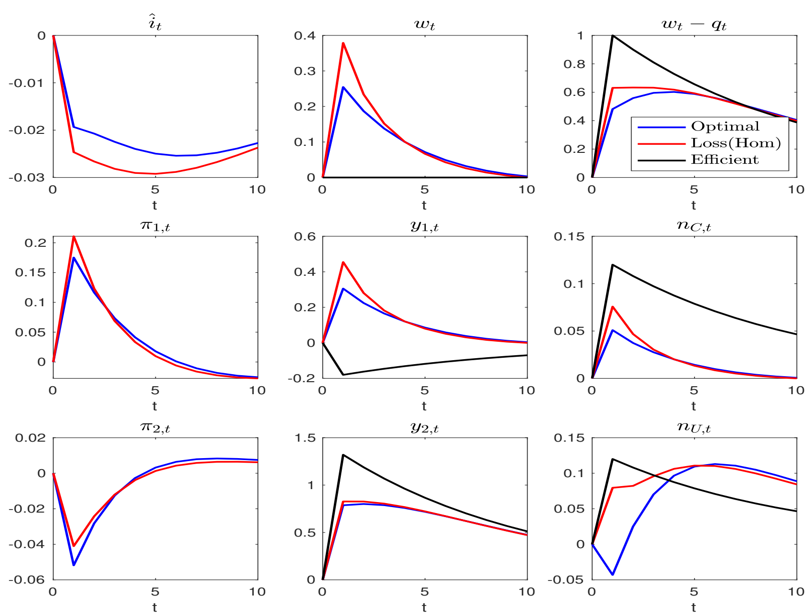


Figure 5.2: Consequences of neglecting heterogeneity under $a_{2,t}$ shock



reduce the difference in marginal utility of consumption between households. Since the real wage gap of the constrained decreases due to nominal rigidity, neglect would lead to less expansionary policy than optimal.

- The loss function under **HomCB** penalizes the relative price gap, $\widetilde{q}_t$, for cross-sectoral distortion, which doesn't need to be cared for under heterogeneity because the substitution between sectors is absent or weak. Since the shock leads to a decrease in $\widetilde{q}_t$, under plausible assumptions on nominal rigidities ($RD < 1$), the loss function under **HomCB** would falsely require a contractionary policy to reduce the gap.

- The loss function under **HomCB** penalizes the covariance more strongly, because the inter-sector connection is tighter than under **HetCB**.[59] Since the shock leads to negative output gap in sector 1 and positive output gap in sector 2, a larger size of the product, $|\widetilde{y}_{1,t}\widetilde{y}_{2,t}|$, would reduce welfare loss more. Thus the central bank would let $\widetilde{y}_{1,t}$, which is more volatile, to deviate more by a contractionary policy than optimal.[60]

- A misperception may arise due to the difference between each efficient allocation, $\frac{\partial}{\partial a_{1,t}}\left[y_{1,t}^{E,\mathbf{HomCB}} - y_{1,t}^{E,\mathbf{HetCB}}\right] = \left(\eta - \frac{1}{\sigma}\right) z_2$, $\frac{\partial}{\partial a_{1,t}}\left[y_{2,t}^{E,\mathbf{HomCB}} - y_{2,t}^{E,\mathbf{HetCB}}\right] = -\left(\eta - \frac{1}{\sigma}\right) z_1$. The loss function under **HomCB** can misperceive with an upward bias on $y_{1,t}$ and a downward bias on $y_{2,t}$, leading to a less expansionary policy.

The results are compatible with those under optimal inflation targeting policy in Section 5.2 that puts more weight on the sector of goods consumed more intensively by HtM households, allowing more variation of inflation in the other sector.

5.2 Optimal Inflation Targeting Policy

We derive the implications of heterogeneous consumption baskets under the inflation targeting policy by solving for the optimal weight δ^* that minimizes welfare loss in the class of policy rules that fully stabilize a weighted average inflation $\pi_t^{\delta} \equiv \delta\pi_{1,t} + (1-\delta)\pi_{2,t}$.[61]

5.2.1 Distributional Consequences and the Expected Welfare

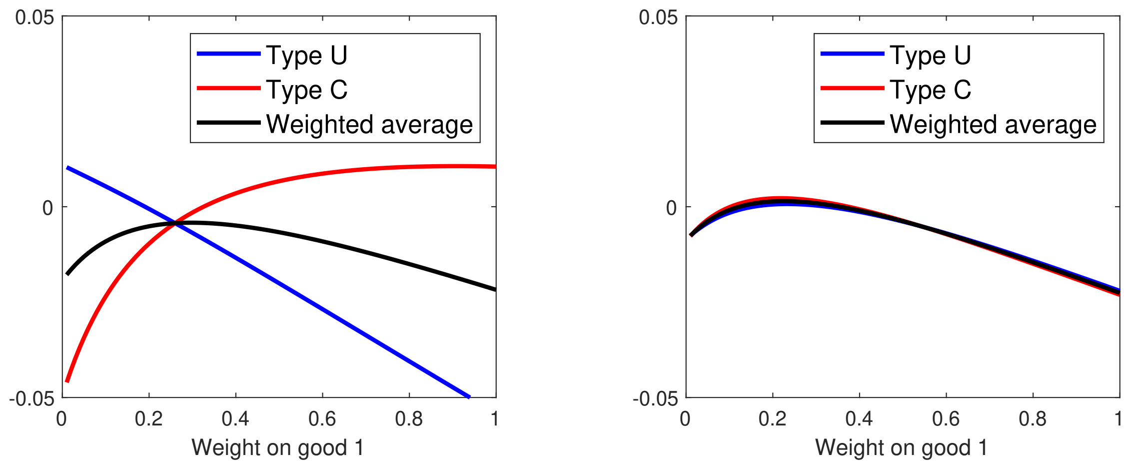
First, we shed light on the distributional consequences of inflation targeting policy. Figure 5.3 shows how the expected welfare, defined as the sum of present-valued utilities, changes with the weight, δ, given to sector 1 on the horizontal axis. We find clear redistributive effects

[59] Under **HetCB**, two sectors are connected as long as labor demand from HtM households' consumption sector is distributed from them to the unconstrained households; the connection through consumption is absent or weak.

[60] $\widetilde{y}_{1,t}$ is more volatile than $\widetilde{y}_{2,t}$, because HtM households, who consume goods from sector 1 more intensively, are more responsive to shocks. Moreover, they do not respond to the interest rates and affected by monetary policy only through the indirect channels.

[61] For the numerical study, we solve the model using a second-order approximation method to the policy functions. For easier comparison with the literature, we set the sector sizes equal, $z_1 = z_2 = 0.5$. We vary them as needed for the robustness checks. The results do not change qualitatively and are robust.

under **HetCB** in Figure 5.3a, whereby the expected welfare of each type of household, $\mathbb{W}_C$ and $\mathbb{W}_U$, is monotonically increasing in the weight the inflation targeting policy assigns to each type's consumption sector: $\frac{\Delta \mathbb{W}_C}{\Delta \delta} > 0$ and $\frac{\Delta \mathbb{W}_U}{\Delta(1-\delta)} > 0$.



(a) **HetCB** (b) **HomCB**

Figure 5.3: Expected welfare and redistributive effects of an inflation targeting policy

The intuition is that the more consumption-relevant inflation is stabilized, the more real wages stabilize, the more consumption and labor hours stabilize. Households dislike volatility due to the concavity of utility from consumption and the convexity of disutility from labor, as shown in the loss functions. This is *the real wage stabilization channel*. Moreover, households benefit from the stabilization of inflation in the sector of goods they consume more intensively, because output loss or consumption loss from price dispersion are also reduced in second-order. This is *the consumption support channel*. Through these channels, the expected welfare of a household increases as its price indices are more stabilized.[62] Thus under **HetCB**, monetary policy can effectively redistribute welfare and marginal utilities across households by changing the weight δ, and deals with the distributional inefficiencies as well as distortions from nominal rigidities.[63]

Let us discuss the implication of a policy change that gives higher weight δ to sector 1. In the welfare loss function below, the red terms are related with type C households, and the blue terms are related with type U households. Note that a household's consumption

[62] This result is robust to the general cases of heterogeneous consumption baskets in which households consume some common share of goods. Please refer to Figure D.1 in the Appendix.

[63] We also find that the curvature of the welfare curves is affected by nominal rigidity in each sector. In Figure 5.3a, the curvature of the welfare curve is larger for HtM households. This is because the sector of goods they consume intensively has a lower degree of nominal rigidity and hence the benefit of reducing the variation of their real wages and output loss by stabilizing their CPIs more gets smaller as δ increases.

and labor hours are functions of its real wages. As the price in sector 1 stabilizes more, type C households' consumption and labor hours stabilize more, and they experience less consumption loss in second-order. Thus their expected welfare increases. However, as the price in sector 2 is less stabilized, type U households' expected welfare decreases. Table 4 summarizes this.

$$\mathbb{L}_t^{(\text{HetCB})} = \frac{z_1\theta}{\kappa_1}\pi_{1,t}^2 + \frac{z_2\theta}{\kappa_2}\pi_{2,t}^2 + z_1\sigma\widetilde{y}_{1,t}^2 + z_2\sigma\widetilde{y}_{2,t}^2 + z_1\varphi\widetilde{n}_{C,t}^2 + z_2\varphi\widetilde{n}_{U,t}^2$$

Channel	Type C	Type U
Real Wage Stabilization	$\underbrace{\widetilde{y}_{1,t},\quad \widetilde{n}_{C,t}}_{\text{more stabilized}}$	$\underbrace{\widetilde{y}_{2,t},\quad \widetilde{n}_{U,t}}_{\text{less stabilized}}$
Consumption Support	$\underbrace{\pi_{1,t}}_{\text{less consumption loss}}$	$\underbrace{\pi_{2,t}}_{\text{more consumption loss}}$
Expected Welfare	$\Uparrow$	$\Downarrow$

Table 4: The distributional effect of higher δ on welfare

Under **HomCB**, however, we confirm that monetary policy has little redistributive effect, because sectoral inflation, relative prices, and sectoral output gaps have only symmetric effects on both types with distributional variables being correlated only with the aggregate output gap. Thus the central bank cannot deal with the distributional inefficiencies, but only addresses distortions from nominal rigidity. We discuss more in detail in the next subsection.

5.2.2 Optimal Weight

Table 5 compares optimal δ under **HomCB** and **HetCB** with 4-quarter average duration and varying relative duration; for example, $RD = 0.5$ is equivalent to $(\alpha_1,\ \alpha_2) = (0.65,\ 0.82)$. Note that we assumed symmetric sectoral size for both **HomCB** and **HetCB**, $z_1 = z_2 = 0.5$.

Under **HomCB**, financial constraint itself induces no significantly different implications from those of Benigno (2004), as the analytical results did: More weight is assigned to the sector with higher nominal rigidity.[64] This result is in line with the previous finding that under **HomCB**, distributional inefficiencies are at the aggregate level; with no redistributive effects through sectoral inflation and relative prices, an inflation targeting policy deals only with distortions from nominal rigidities.

[64] The values of optimal δ^{hom} are very close to those of Benigno (2004).

However, **HetCB** makes significant differences, and gives consistently higher weight to the goods consumed more intensively by HtM households than under **HomCB** regardless of relative degrees of nominal rigidities. This is because heterogeneity gives monetary policy a new role to deal with distributional inefficiencies from imperfect sharing of idiosyncratic real wage risk and idiosyncratic non-labor income risk. The policy redistributes in favor of HtM households who has higher responsiveness of consumption. In order to benefit them more through the redistributive channels, the central bank targets inflation rates that are weighted toward the goods that are consumed more intensively by the constrained households and not merely the goods with less flexible prices. We find that income inequality further strengthens this result in the next Section 5.2.3.

Table 5: Optimal inflation targeting policy under **HomCB** and **HetCB**

AD	$RD \equiv \frac{1-\alpha_2}{1-\alpha_1}$	δ^{hom}	δ^{het}	$\delta^{het} - \delta^{hom}$
4 quarters	2	0.77	0.82	+0.05
	1.5	0.67	0.73	+0.06
	1.2	0.58	0.65	+0.07
	1	0.50	0.58	+0.08
	0.83	0.42	0.50	+0.08
	0.67	0.33	0.40	+0.07
	0.5	0.23	0.30	+0.07

Given RD, the additional weight put on sector 1 by **HetCB** decreases as σ increases. A smaller elasticity of intertemporal substitution and the larger income effect of labor supply lead to a more stabilized variation of consumption for HtM households, and the policy has less incentive to stabilize inflation in their consumption baskets. Table 6 shows the optimal weight under $RD < 1$ that is compatible with empirical findings.

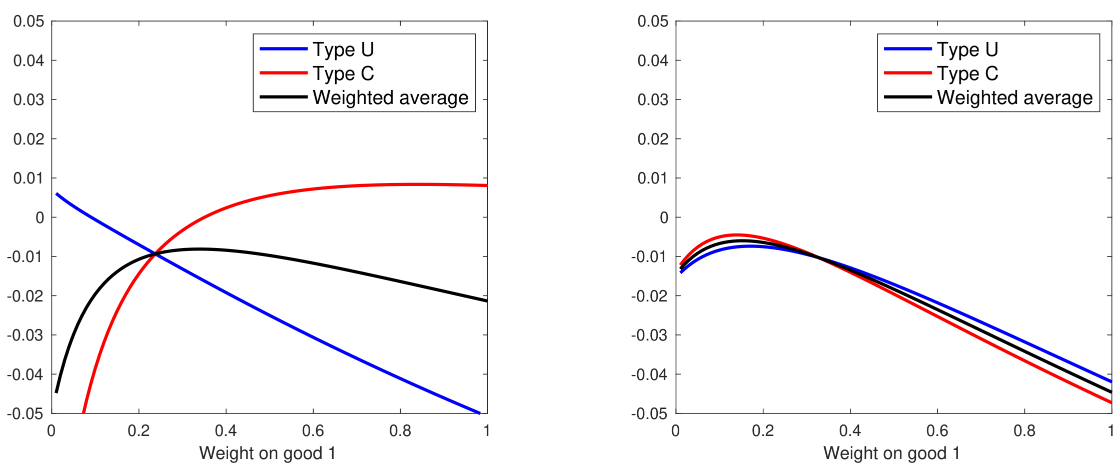
Table 6: Optimal inflation targeting policy with varying σ

$AD = 4$	δ^{hom}	$\delta^{het}\vert_{\sigma}$	$\sigma = \frac{1}{3}$	$\frac{2}{3}$	1	2	3
$RD = 1$	0.50		0.61	0.58	0.55	0.51	0.49
0.83	0.42		0.53	0.50	0.48	0.44	0.43
0.67	0.33		0.43	0.40	0.39	0.36	0.35
0.50	0.23		0.32	0.30	0.29	0.27	0.26

5.2.3 Policy under Inequality

We find that income inequality between households significantly strengthens the main results. To introduce income inequality, we extend the model: (1) A nonlinear production function, $Y_{j,t}(i) = A_t A_{j,t} N_{j,t}(i)^{\alpha}$, that induces an additional source of profits through a convex cost function aside from monopolistic competition; and (2) fiscal rules that finance the share $\bar{s}$ of the subsidy by taxing HtM households: Tax only type U if $\bar{s}=0$, tax both types equally if $\bar{s}=\lambda$, and tax only type C if $\bar{s}=1$. As α decreases from unity and $\bar{s}$ increases from zero, inequality would get wider.

To examine the implications of income inequality for the redistributive effect of monetary policy and optimal inflation targeting policy, we vary α from unity to $\frac{2}{3}$. In this case, a moderate degree of income inequality is generated where the richer households income is about 50% higher than the poorer households, where the size of sector 1 is 0.38. When the sector size is controlled for under **HomCB**, optimal weight δ is 0.15. With no redistributive effects, the policy deals only with the distortions from nominal rigidities, giving much higher weight to the goods with less flexible prices compared to the sector size. Under **HetCB**, however, optimal weight δ is 0.34, which is much higher than under **HomCB**. This is because the policy deals with the distributional inefficiencies as well as the distortions from nominal rigidities.



(a) **HetCB** ($\delta^{het} = 0.34$) (b) **HomCB** ($\delta^{hom} = 0.15$)

Figure 5.4: Redistributive effects of an inflation targeting policy under income inequality ($\alpha = \frac{2}{3}$)

Comparing the Figure 5.4 to Figure 5.3, we find that wider income inequality strengthens the result even more. The utilitarian central bank benefits more the households with higher marginal utility and higher responsiveness by stabilizing inflation in the sector of goods consumed more intensively by the poorer or the constrained households. If the central bank mistakenly sets it to be 0.15, the policy would benefit the richer households more than optimal, at the cost of the poorer households' welfare.

Now we conduct experiments on a few more specifications of income inequality. We assume that $(\omega_U,\ \omega_C)=(0.1,\ 0.9)$ for heterogeneous baskets. Since sector sizes are different $(z_j \neq 0.5)$ due to inequality, we compare each case with its homogeneous-basket counterpart $(\omega_U=\omega_C=z_1)$ with the same sector size z_1.

Table 7: Optimal inflation targeting policy under inequality

	$\bar{s}=0$	λ	1
$(\alpha=1)$ z_1	0.50	0.46	0.42
δ^{het}	0.28	0.27	0.26
δ^{hom}	0.23	0.20	0.18
$(\alpha=\frac{2}{3})$ z_1	0.41	0.38	0.35
δ^{het}	0.31	0.29	0.28
δ^{hom}	0.17	0.15	0.14

A strong policy implication of income inequality is drawn in every case of $\bar{s}$: As we introduce a nonlinear production function, the size of sector 1 (z_1) decreases due to the inequality; despite this, δ^{het} increases, whereas δ^{hom} decreases, leading to even wider differences between them. The intuition is that since the hand-to-mouth or the poorer households have higher marginal utility with a higher volatility and are more responsive to real wages, the utilitarian central bank cares disproportionately more about them and redistributes marginal utilities in their favor to maximize the social welfare.

We also find that the dynamics and distribution of non-labor income, such as tax and dividend, are nontrivial. Let us compare $(\alpha,\ \bar{s})=(1,\ 1)$ and $(\frac{2}{3},\ 0)$: Both have similar degrees of inequality at the steady state, sector size, and hence δ^{hom}. However, δ^{het} is smaller for the former, although they are both higher than δ^{hom}; this is attributable to the lump-sum tax on HtM households, which can be regarded as countercyclical non-labor income for them that stabilizes their consumption and labor hours to some degree.

5.2.4 Robustness

We conduct the robustness checks, and the results are robust to the following features: the degrees of heterogeneity in consumption baskets; income inequality; the specifications of whom to tax to finance subsidies; whether monopolistic distortion is eliminated or not at the steady state; and relative degrees of nominal rigidities across sectors. The results are significantly strengthened as income inequality deepens.

6 Conclusion

We analyze optimal monetary policy in a model with households that differ along two dimensions: They consume different baskets of consumption goods and have differential access to financial markets. Households face idiosyncratic real wage risk and non-labor income risk. Imperfect risk-sharing gives monetary policy a new role to address the distributional inefficiencies at the cost of some price instability. Based on a micro-founded welfare criterion, the first-best outcome is not achievable even in the absence of nominal rigidities: Optimal monetary policy targets non-zero output gaps due to new trade-offs, and benefits borrowing-constrained or poorer households more by targeting inflation rates that are weighted toward the goods that are consumed more intensively by the constrained or poorer households and not merely the goods with less flexible prices. This is because the utilitarian central bank benefits more the households with higher marginal utility and higher responsiveness to changes in real wages. If the central bank neglect heterogeneous consumption baskets, the policy would be more beneficial to the richer households than optimal at the cost of the poorer households.

This study focuses on the qualitative aspects of the mechanisms that are newly generated by HetCB, and the new redistributive channel that operates through different price indices across different income levels. But it would be of interest to extend this paper to several dimensions. First, since we abstract from unemployment, it would be an important extension to study the normative implications of the asymmetry in unemployment risk observed in the real world under the heterogeneous consumption baskets framework. Second, we simplified the role of the fiscal sides, but heterogeneous consumption baskets may also have important implications for fiscal policy as we examined shortly in the main text. Monetary and fiscal policy interaction under heterogeneous consumption baskets merits further study. Third, in this study we focused on the differences in the sectors of goods that households consume. Not only that, the differences in the sector households work would also have important implication for monetary policy, because the weight given to each sector by inflation targeting policy would benefit households who work in some sectors at the cost of households who work in other sectors.

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Appendix

A Proofs

A.1 Proof of Proposition 1

Proof. Assume that the efficient allocation can be supported by the market outcome. Then, we have

$$w_t = w_t^E \quad \text{and} \quad q_t = q_t^E$$

Substituting them into the labor market clearing condition and the labor supply schedule of type U households,

$$n_{U,t} = \frac{1-\sigma}{\sigma+\varphi}\left(w_t - q_t\right)$$

Combining the budget constraint and the labor supply schedules of type C households,

$$n_{C,t} = \frac{1-\sigma}{\sigma+\varphi} w_t$$

Labor supply from each type of households are different as long as $q_t^E = a_{1,t} - a_{2,t} \neq 0$ and $\sigma \neq 1$

$$n_{C_t} - n_{U_t} = -\frac{1-\sigma}{\sigma+\varphi} q_t^E$$

and the efficient condition does not hold. This contracts to the assumption. □

A.2 Proof of Proposition 2

Proof. 1) It is obvious by Eq.(2.30) and Eq.(2.31).

2) Assume that closing both output gaps is feasible, $\widetilde{y}_{1,t} = \widetilde{y}_{2,t} = 0$. Then by labor supply schedule of both types of households, type C households' budget constraint, and labor market clearing condition, we have

$$
\begin{aligned}
\widetilde{n}_{C,t} &= \frac{1-\sigma}{\sigma}\frac{1}{1+\varphi}z_2 q_t^E \\
\widetilde{n}_{U,t} &= -\frac{1-\sigma}{\sigma}\frac{1}{1+\varphi}z_1 q_t^E \\
\widetilde{w}_t &= \frac{1-\sigma}{\sigma}\frac{\varphi}{1+\varphi}z_2 q_t^E \\
\widetilde{q}_t &= \frac{1-\sigma}{\sigma}\frac{\varphi}{1+\varphi}q_t^E \\
\widetilde{w}_t - \widetilde{q}_t &= -\frac{1-\sigma}{\sigma}\frac{\varphi}{1+\varphi}z_1 q_t^E
\end{aligned}
$$

If there is no nominal friction in either sector, constant markup leads to $\widetilde{w}_t = 0$ or $\widetilde{w}_t - \widetilde{q}_t = 0$, which contradicts to the solution for $\widetilde{w}_t$ or $\widetilde{w}_t - \widetilde{q}_t$ derived above.

If nominal friction exists in both sectors, we have by the Phillips curve that

$$
\begin{aligned}
\pi_{1,t} &= \beta E_t[\pi_{1,t+1}] + \kappa_1 \frac{1-\sigma}{\sigma}\frac{\varphi}{1+\varphi}z_2 q_t^E \\
&= \kappa_1 \frac{1-\sigma}{\sigma}\frac{\varphi}{1+\varphi}z_2 \sum_{s=0}^{\infty}\beta^s E_t q_{t+s}^E \\
&= \kappa_1 \frac{1-\sigma}{\sigma}\frac{\varphi}{1+\varphi}z_2 \frac{1}{1-\beta\rho_a}q_t^E
\end{aligned}
$$

where we assume $\rho_{a_1} = \rho_{a_2} = \rho_a$. Similarly,

$$
\pi_{2,t} = -\kappa_2 \frac{1-\sigma}{\sigma}\frac{\varphi}{1+\varphi}z_1 \frac{1}{1-\beta\rho_a}q_t^E
$$

However, above solutions contradicts to the definition of relative price:

$$
\begin{aligned}
\widetilde{q}_t - \widetilde{q}_{t-1} + q_t^E - q_{t-1}^E &= \frac{\sigma+\varphi}{\sigma(1+\varphi)}(q_t^E - q_{t-1}^E) \\
&\neq -\frac{1-\sigma}{\sigma}\frac{\varphi}{1+\varphi}\frac{1}{1-\beta\rho_a}(z_1\kappa_2 + z_2\kappa_1)q_t^E = \pi_{2,t} - \pi_{1,t}
\end{aligned}
$$

□

A.3 Proof of Proposition 3

Proof. We follow Woodford (2003) in deriving the welfare-theoretic loss function. Note first that under the assumptions on employment subsidy and government transfers, the steady state is efficient and equitable, $N_{C,t}=N_{U,t}=N=C_{C,t}=C_{U,t}=C=Y=1$ with the wage and the relative price being unity, $W=Q=1$. Thus we have

$$\frac{V_N(N_{U,t})}{U_C(C_{U,t})}=W=1=\frac{W}{Q}=\frac{V_N(N_{c,t})}{U_C(C_{C,t})}$$

Define $h_U\equiv 1-\lambda$ and $h_C\equiv\lambda$. Taking a second-order approximation to the equally weighted sum of both types of households' utilities around the efficient zero-inflation steady state,

$$\begin{aligned}
&\sum_{j=U,C} h_j\mathcal{U}(C_{h,t},N_{h,t})\\
&=\sum_{j=U,C} h_j\left[\begin{array}{c} U_cY\{c_{j,t}+\frac{1-\sigma}{2}c_{j,t}^2\}\\ -V_NN\{n_{j,t}+\frac{1+\varphi}{2}n_{j,t}^2\}\end{array}\right]+t.i.p.+o(||\xi||^3)\\
&=U_cY\left[\begin{array}{c}(1-\lambda)\{\widetilde{c}_{U,t}+\frac{1-\sigma}{2}\widetilde{c}_{U,t}^2+(1-\sigma)c_{U,t}^E\widetilde{c}_{U,t}\}+\lambda\{\widetilde{c}_{C,t}+\frac{1-\sigma}{2}\widetilde{c}_{C,t}^2+(1-\sigma)c_{C,t}^E\widetilde{c}_{C,t}\}\\ -(1-\lambda)\{\widetilde{n}_{U,t}+\frac{1+\varphi}{2}\widetilde{n}_{U,t}^2+(1+\varphi)n_{U,t}^E\widetilde{n}_{U,t}\}-\lambda\{\widetilde{n}_{C,t}+\frac{1+\varphi}{2}\widetilde{n}_{C,t}^2+(1+\varphi)n_{C,t}^E\widetilde{n}_{C,t}\}\end{array}\right]\\
&\quad+t.i.p.+o(||\xi||^3)
\end{aligned}\tag{A.1}$$

Taking a second order approximation to the labor market clearing condition,

$$\begin{aligned}
&\omega(\widetilde{n}_{1,t}+\frac{1}{2}\widetilde{n}_{1,t}^2+n_{1,t}^E\widetilde{n}_{1,t})+(1-\omega)(\widetilde{n}_{2,t}+\frac{1}{2}\widetilde{n}_{2,t}^2+n_{2,t}^E\widetilde{n}_{2,t})\\
&=(1-\lambda)(\widetilde{n}_{U,t}+\frac{1}{2}\widetilde{n}_{U,t}^2+n_{U,t}^E\widetilde{n}_{U,t})+\lambda(\widetilde{n}_{C,t}+\frac{1}{2}\widetilde{n}_{C,t}^2+n_{C,t}^E\widetilde{n}_{C,t})+t.i.p.+o(||\xi||^3)
\end{aligned}\tag{A.2}$$

Let us define $\hat{p}_{j,t}(i)\equiv p_{j,t}(i)-p_{j,t}$. Then, by a second order approximation,

$$\left(\frac{P_{j,t}(i)}{P_{j,t}}\right)^{1-\theta}=e^{(1-\theta)\hat{p}_{j,t}(i)}=1+(1-\theta)\hat{p}_{j,t}(i)+\frac{(1-\theta)^2}{2}\hat{p}_{j,t}^2(i)+o(||\xi||^3)\tag{A.3}$$

Since $\frac{1}{z_j}\int_{\mathcal{I}_j}\big(\frac{P_{j,t}(i)}{P_{j,t}}\big)^{1-\theta}di=1$ by the price aggregator, we integrate Eq.(A.3) to derive

$$E_i^j\{\hat{p}_{j,t}(i)\}=\frac{\theta-1}{2}E_i^j\{\hat{p}_{j,t}^2(i)\}\tag{A.4}$$

Similarly, taking a second order approximation, integrating the result, and substituting Eq.(A.4),

$$\left(\frac{P_{j,t}(i)}{P_{j,t}}\right)^{-\theta} = 1 - \theta\hat{p}_{j,t}(i) + \frac{\theta^2}{2}\hat{p}^2_{j,t}(i) + o(||\xi||^3)$$

$$\frac{1}{z_j}\int_{\mathcal{I}_j}\left(\frac{P_{j,t}(i)}{P_{j,t}}\right)^{-\theta} = 1 - \theta E^j_i\{\hat{p}_{j,t}(i)\} + \frac{\theta^2}{2}E^j_i\{\hat{p}^2_{j,t}(i)\} + o(||\xi||^3)$$

$$= 1 + \frac{\theta}{2}E^j_i\{\hat{p}^2_{j,t}(i)\} + o(||\xi||^3) \tag{A.5}$$

Since $E^j_i\{\hat{p}^2_{j,t}(i)\} = \frac{1}{z_j}\int_{\mathcal{I}_j}\hat{p}^2_{j,t}(i)di = \frac{1}{z_j}\int_{\mathcal{I}_j}\left(p_{j,t}(i) - p_{j,t}\right)^2 di$, and we know that in the first order $p_{j,t} = E^j_i\{p_{j,t}(i)\}$, we derive that $E^j_i\{\hat{p}^2_{j,t}(i)\} = Var^j_i\{p_{j,t}(i)\}$. Substituting this into Eq.(A.5),

$$\frac{1}{z_j}\int_{\mathcal{I}_j}\left(\frac{P_{j,t}(i)}{P_{j,t}}\right)^{-\theta} = 1 + \frac{\theta}{2}Var^j_i\{p_{j,t}(i)\} + o(||\xi||^3) \tag{A.6}$$

Thus we derive the second order approximation to the price dispersion in each sector as

$$d_{j,t} \equiv \log\frac{1}{z_j}\int_{\mathcal{I}_j}\left(\frac{P_{j,t}(i)}{P_{j,t}}\right)^{-\theta} = \frac{\theta}{2}Var^j_i\{p_{j,t}(i)\} + o(||\xi||^3) \tag{A.7}$$

We have $N_{j,t} = \int_{\mathcal{I}_j}\frac{Y_{j,t}(i)}{A_t A_{j,t}}di = \frac{1}{z_j}\frac{Y_{j,t}}{A_t A_{j,t}}\int_{\mathcal{I}_j}\left(\frac{P_{j,t}(i)}{P_{j,t}}\right)^{-\theta}di$ by the relative demand function. Taking a second order approximation and substituting Eq.(A.7), we derive

$$n_{j,t} = y_{j,t} - a_t - a_{j,t} + \log\frac{1}{z_j}\int_{\mathcal{I}_j}\left(\frac{P_{j,t}(i)}{P_{j,t}}\right)^{-\theta}di + o(||\xi||^3)$$

$$= y_{j,t} - a_t - a_{j,t} + \frac{\theta}{2}Var^j_i\{p_{j,t}(i)\} + o(||\xi||^3)$$

$$n^2_{j,t} = y^2_{j,t} + a^2_t + a^2_{j,t} - 2(a_t + a_{j,t})y_{j,t} + 2a_t a_{j,t} + o(||\xi||^3)$$

$$\Rightarrow \widetilde{n}_{j,t} = \widetilde{y}_{j,t} + \frac{\theta}{2}Var^j_i\{p_{j,t}(i)\} + o(||\xi||^3) \tag{A.8}$$

$$\widetilde{n}^2_{j,t} + 2n^E_{j,t}\widetilde{n}_{j,t} = \widetilde{y}^2_{j,t} + 2y^E_{j,t}\widetilde{y}_{j,t} - 2(a_t + a_{j,t})\widetilde{y}_{j,t} + t.i.p. + o(||\xi||^3) \tag{A.9}$$

Substituting Eqs.(A.2), (A.8) and (A.9) into Eq.(A.1), and canceling out the cross terms,

$$(\text{A.1}) = -\frac{U_c Y}{2}\left[\begin{array}{c} \omega\theta Var^1_i\{p_{1,t}(i)\} + (1-\omega)\theta Var^2_i\{p_{2,t}(i)\} \\ +\sigma(1-\lambda)\widetilde{c}^2_{U,t} + \sigma\lambda\widetilde{c}^2_{C,t} \\ +\varphi(1-\lambda)\widetilde{n}^2_{U,t} + \varphi\lambda\widetilde{n}^2_{C,t} \\ +\frac{1}{\eta}\omega(1-\omega)(\widetilde{y}_{1,t} - \widetilde{y}_{2,t})^2 \end{array}\right] + t.i.p. + o(||\xi||^3) \tag{A.10}$$

Let us define $\Delta_t^j \equiv Var_i^j\{P_{j,t}(i)\}$. According to Woodford (2003),

$$\begin{aligned}
\Delta_t^j &= \alpha_j \Delta_{t-1}^j + \frac{\alpha_j}{1-\alpha_j}\pi_{j,t}^2 + o(||\xi||^3) \\
&= \underbrace{\alpha_j^{t+1}\Delta_{-1}^j}_{t.i.p.} + \sum_{k=0}^{t} \alpha_j^{t-k}\frac{\alpha_j}{1-\alpha_j}\pi_{j,t}^2 + o(||\xi||^3)
\end{aligned}$$

and the present valued sum of the cross-sectional price dispersion can be rewritten in terms of present valued sum of squared inflation as

$$\sum_{t=0}^{\infty} \beta^t \Delta_t^j = \frac{\alpha_j}{(1-\alpha_j)(1-\alpha_j\beta)} \sum_{t=0}^{\infty} \beta^t \pi_{j,t}^2 + t.i.p. + o(||\xi||^3) \tag{A.11}$$

Substituting Eq.(A.11) into Eq.(A.10), and summing up the present valued utilities,

$$\begin{aligned}
E_0 \sum_{t=0}^{\infty} \beta^t \sum_{j=U,C} h_j \mathcal{U}(C_{h,t}, N_{h,t}) = -\frac{U_c Y}{2} E_0 \sum_{t=0}^{\infty} \beta^t &\begin{bmatrix} \omega\frac{\theta}{\kappa_1}\pi_{1,t}^2 + (1-\omega)\frac{\theta}{\kappa_2}\pi_{2,t}^2 \\ +\sigma(1-\lambda)\widetilde{c}_{U,t}^2 + \sigma\lambda\widetilde{c}_{C,t}^2 \\ +\varphi(1-\lambda)\widetilde{n}_{U,t}^2 + \varphi\lambda\widetilde{n}_{C,t}^2 \\ +\frac{1}{\eta}\omega(1-\omega)(\widetilde{y}_{1,t}-\widetilde{y}_{2,t})^2 \end{bmatrix} \\
&+ t.i.p. + o(||\xi||^3)
\end{aligned} \tag{A.12}$$

By the final good market clearing condition,

$$\begin{aligned}
&\widetilde{y}_t = (1-\lambda)\widetilde{c}_{U,t} + \lambda\widetilde{c}_{C,t} + o(||\xi||^2) \\
&\Rightarrow (1-\lambda)\widetilde{c}_{U,t}^2 + \lambda\widetilde{c}_{C,t}^2 = \widetilde{y}_t^2 + \lambda(1-\lambda)(\widetilde{c}_{U,t}-\widetilde{c}_{C,t})^2 + t.i.p. + o(||\xi||^3)
\end{aligned} \tag{A.13}$$

By the output aggregator and the labor market clearing condition,

$$\begin{aligned}
&\widetilde{y}_t = \omega\widetilde{y}_{1,t} + (1-\omega)\widetilde{y}_{2,t} = \omega\widetilde{n}_{1,t} + (1-\omega)\widetilde{n}_{2,t} = (1-\lambda)\widetilde{n}_{U,t} + \lambda\widetilde{n}_{C,t} + o(||\xi||^2) \\
&\Rightarrow (1-\lambda)\widetilde{n}_{U,t}^2 + \lambda\widetilde{n}_{C,t}^2 = \widetilde{y}_t^2 + \lambda(1-\lambda)(\widetilde{n}_{U,t}-\widetilde{n}_{C,t})^2 + t.i.p. + o(||\xi||^3)
\end{aligned} \tag{A.14}$$

By the price aggregator,

$$p_{1,t} - p_t = -(1-\omega)q_t - \frac{1-\eta}{2}\omega(1-\omega)q_t^2 + o(||\xi||^3) \tag{A.15}$$

$$p_{2,t} - p_t = \omega q_t - \frac{1-\eta}{2}\omega(1-\omega)q_t^2 + o(||\xi||^3) \tag{A.16}$$

Since we have exact relative demand functions in terms of relative price and aggregate output,

$$y_{1,t} = -\eta(p_{1,t} - p_t) + y_t \tag{A.17}$$

$$y_{2,t} = -\eta(p_{2,t} - p_t) + y_t \tag{A.18}$$

Substituting Eqs.(A.15)-(A.16) into Eqs.(A.17)-(A.18),

$$y_{1,t} = y_t + (1-\omega)\eta q_t + \omega(1-\omega)\frac{\eta(1-\eta)}{2}q_t^2 + o(||\xi||^3) \tag{A.19}$$

$$y_{2,t} = y_t - \omega\eta q_t + \omega(1-\omega)\frac{\eta(1-\eta)}{2}q_t^2 + o(||\xi||^3) \tag{A.20}$$

Subtracting Eq.(A.20) from Eq.(A.19), and rewriting in terms of gaps,

$$(\widetilde{y}_{1,t}-\widetilde{y}_{2,t})^2 = \eta^2\widetilde{q}_t^2 + o(||\xi||^3) \tag{A.21}$$

substituting Eqs.(A.13)-(A.14) and (A.21) into Eq.(A.12),

$$(\text{A.12}) = -\frac{U_cY}{2}E_0\sum_{t=0}^{\infty}\beta^t\begin{bmatrix} \omega\frac{\theta}{\kappa_1}\pi_{1,t}^2 + (1-\omega)\frac{\theta}{\kappa_2}\pi_{2,t}^2 \\ +(\sigma+\varphi)\widetilde{y}_t^2 \\ +\sigma\lambda(1-\lambda)(\widetilde{c}_{U,t}-\widetilde{c}_{C,t})^2 \\ +\varphi\lambda(1-\lambda)(\widetilde{n}_{U,t}-\widetilde{n}_{C,t})^2 \\ +\frac{1}{\eta}\omega(1-\omega)(\widetilde{y}_{1,t}-\widetilde{y}_{2,t})^2 \end{bmatrix} + t.i.p. + o(||\xi||^3)$$

$$= -\frac{U_cY}{2}E_0\sum_{t=0}^{\infty}\beta^t\begin{bmatrix} \omega\frac{\theta}{\kappa_1}\pi_{1,t}^2 + (1-\omega)\frac{\theta}{\kappa_2}\pi_{2,t}^2 \\ +(\sigma+\varphi)\widetilde{y}_t^2 \\ +\sigma\lambda(1-\lambda)(\widetilde{c}_{U,t}-\widetilde{c}_{C,t})^2 \\ +\varphi\lambda(1-\lambda)(\widetilde{n}_{U,t}-\widetilde{n}_{C,t})^2 \\ +\eta\omega(1-\omega)\widetilde{q}_t^2 \end{bmatrix} + t.i.p. + o(||\xi||^3) \tag{A.22}$$

We can simplify the loss function further by deriving the first order relations between distributional variables and aggregate variables using the equilibrium conditions on the household side,

$$(1-\lambda)\widetilde{c}_{U,t} + \lambda\widetilde{c}_{C,t} = (1-\lambda)\widetilde{n}_{U,t} + \lambda\widetilde{n}_{C,t} + o(||\xi||^2)$$

$$\varphi\widetilde{n}_{U,t} + \sigma\widetilde{c}_{U,t} = \widetilde{w}_t - (1-\omega)\widetilde{q}_t + o(||\xi||^2)$$

$$\varphi\widetilde{n}_{C,t} + \sigma\widetilde{c}_{C,t} = \widetilde{w}_t - (1-\omega)\widetilde{q}_t + o(||\xi||^2)$$

$$\widetilde{c}_{C,t} = \widetilde{n}_{C,t} + \widetilde{w}_t - (1-\omega)\widetilde{q}_t + o(||\xi||^2)$$

from which we derive the following relations in the first order:

$$\widetilde{n}_{U,t} = \frac{1-\lambda(1-\sigma)}{(1-\lambda)(1-\sigma)}\widetilde{n}_{C,t} + o(||\xi||^2) \tag{A.23}$$

$$\widetilde{c}_{U,t} = \frac{1-\lambda(1+\varphi)}{(1-\lambda)(1+\varphi)}\widetilde{c}_{C,t} + o(||\xi||^2) \tag{A.24}$$

Since $\widetilde{y}_t = (1-\lambda)\widetilde{c}_{U,t} + \lambda\widetilde{c}_{U,t} = (1-\lambda)\widetilde{n}_{U,t} + \lambda\widetilde{n}_{U,t}$ in the first order,

$$\widetilde{c}_{C,t} = (1+\varphi)\widetilde{y}_t \tag{A.25}$$

$$\widetilde{c}_{U,t} = \frac{1-\lambda(1+\varphi)}{1-\lambda}\widetilde{y}_t \tag{A.26}$$

$$\widetilde{n}_{C,t} = (1-\sigma)\widetilde{y}_t \tag{A.27}$$

$$\widetilde{n}_{U,t} = \frac{1-\lambda(1-\sigma)}{1-\lambda}\widetilde{y}_t \tag{A.28}$$

substituting Eqs.(A.25)-(A.28) into Eq.(A.22), we finally have that

$$\begin{aligned} & E_0\sum_{t=0}^{\infty}\beta^t \sum_{j=U,C} h_j\mathcal{U}(C_{h,t}, N_{h,t}) \\ &= -\frac{U_cY}{2}E_0\sum_{t=0}^{\infty}\beta^t \begin{bmatrix} \omega\frac{\theta}{\kappa_1}\pi_{1,t}^2 + (1-\omega)\frac{\theta}{\kappa_2}\pi_{2,t}^2 \\ +(\sigma+\varphi)\frac{1-\lambda(1-\sigma\varphi)}{1-\lambda}\widetilde{y}_t^2 \\ +\eta\omega(1-\omega)\widetilde{q}_t^2 \end{bmatrix} + t.i.p. + o(||\xi||^3) \end{aligned} \tag{A.29}$$

Note that $\frac{1-\lambda(1-\sigma\varphi)}{1-\lambda}$ is increasing in λ, implying that as the share of the financially constrained households increases, output stabilization becomes relatively more important than price stabilization. If $\alpha_1 = \alpha_2 = \alpha$ and thus $\kappa_1 = \kappa_2 = \kappa$, we can rewrite Eq.(A.29) as

$$(\text{A.29}) = -\frac{U_cY}{2}E_0\sum_{t=0}^{\infty}\beta^t \begin{bmatrix} \frac{\theta}{\kappa}\pi_t^2 + \omega(1-\omega)\frac{\theta}{\kappa}(\pi_{2,t}-\pi_{1,t})^2 \\ +(\sigma+\varphi)\frac{1-\lambda(1-\sigma\varphi)}{1-\lambda}\widetilde{y}_t^2 \\ +\eta\omega(1-\omega)\widetilde{q}_t^2 \end{bmatrix} + t.i.p. + o(||\xi||^3) \tag{A.30}$$

□

A.4 Proof of Proposition 4

Proof. We solve a Ramsey problem of the utilitarian central bank when the price of sector 1 is flexible and the price of sector 2 is sticky, $\alpha_1 = 0$, under **HomCB**. The opposite case will be exactly symmetric under homogeneous consumption baskets. We set up the Lagrangian as:

$$
\begin{aligned}
\mathcal{L}_t = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t & \left[\begin{array}{c} (1-\omega)\frac{\theta}{\kappa_2}\pi_{2,t}^2 + (\sigma+\varphi)\widetilde{y}_t^2 + \eta\omega(1-\omega)\widetilde{q}_t^2 \\ +\sigma\lambda(1-\lambda)(\widetilde{c}_{U,t}-\widetilde{c}_{C,t})^2 + \varphi\lambda(1-\lambda)(\widetilde{n}_{U,t}-\widetilde{n}_{C,t})^2 \end{array} \right] \\
&+E_0 \sum_{t=0}^{\infty} \beta^t \psi_{1,t} \Big\{ \pi_{2,t} - \beta\pi_{2,t+1} + \kappa_2 \widetilde{q}_t \Big\} \\
&+E_0 \sum_{t=0}^{\infty} \beta^t \psi_{2,t} \Big\{ \widetilde{q}_t - \widetilde{q}_{t-1} + q_t^E - q_{t-1}^E - \pi_{2,t} + \pi_{1,t} \Big\} \\
&+E_0 \sum_{t=0}^{\infty} \beta^t \psi_{3,t} \Big\{ \omega\widetilde{y}_{1,t} + (1-\omega)\widetilde{y}_{2,t} - (1-\lambda)\widetilde{n}_{U,t} - \lambda\widetilde{n}_{C,t} \Big\} \\
&+E_0 \sum_{t=0}^{\infty} \beta^t \psi_{4,t} \Big\{ \varphi\widetilde{n}_{U,t} + \sigma\widetilde{c}_{U,t} + (1-\omega)\widetilde{q}_t \Big\} \\
&+E_0 \sum_{t=0}^{\infty} \beta^t \psi_{5,t} \Big\{ \varphi\widetilde{n}_{C,t} + \sigma\widetilde{c}_{C,t} + (1-\omega)\widetilde{q}_t \Big\} \\
&+E_0 \sum_{t=0}^{\infty} \beta^t \psi_{6,t} \Big\{ \widetilde{c}_{C,t} - \widetilde{n}_{C,t} - \widetilde{w}_t + (1-\omega)\widetilde{q}_t \Big\} \\
&+E_0 \sum_{t=0}^{\infty} \beta^t \psi_{7,t} \Big\{ \widetilde{y}_{1,t} - (1-\lambda)\widetilde{c}_{U,t} - \lambda\widetilde{c}_{C,t} - \eta(1-\omega)\widetilde{q}_t \Big\} \\
&+E_0 \sum_{t=0}^{\infty} \beta^t \psi_{8,t} \Big\{ \widetilde{y}_{2,t} - (1-\lambda)\widetilde{c}_{U,t} - \lambda\widetilde{c}_{C,t} + \eta\omega\widetilde{q}_t \Big\} \\
&+E_0 \sum_{t=0}^{\infty} \beta^t \psi_{9,t} \Big\{ \widetilde{y}_t - \omega\widetilde{y}_{1,t} - (1-\omega)\widetilde{y}_{2,t} \Big\}
\end{aligned}
$$

where $\{\psi_{1,t}\}, \cdots, \{\psi_{9,t}\}$ are the Lagrange multipliers.

First order conditions are as follows:

$$
\begin{aligned}
\partial\pi_{1,t} &: 0 = \psi_{2,t} \\
\partial\pi_{2,t} &: 0 = (1-\omega)\theta\pi_{2,t} + \kappa_2(\psi_{1,t} - \psi_{1,t-1}) - \kappa_2\psi_{2,t} \\
\partial\widetilde{y}_{1,t} &: 0 = \omega\psi_{3,t} + \psi_{7,t} - \omega\psi_{9,t} \\
\partial\widetilde{y}_{2,t} &: 0 = (1-\omega)\psi_{3,t} + \psi_{8,t} - (1-\omega)\psi_{9,t} \\
\partial\widetilde{c}_{U,t} &: 0 = \sigma\lambda(1-\lambda)(\widetilde{c}_{U,t}-\widetilde{c}_{C,t}) + \sigma\psi_{4,t} - (1-\lambda)\psi_{7,t} - (1-\lambda)\psi_{8,t} \\
\partial\widetilde{c}_{C,t} &: 0 = -\sigma\lambda(1-\lambda)(\widetilde{c}_{U,t}-\widetilde{c}_{C,t}) + \sigma\psi_{5,t} + \psi_{6,7} - \lambda\psi_{7,t} - \lambda\psi_{8,t} \\
\partial\widetilde{n}_{U,t} &: 0 = \varphi\lambda(1-\lambda)(\widetilde{n}_{U,t}-\widetilde{n}_{C,t}) - (1-\lambda)\psi_{3,t} + \varphi\psi_{4,t} \\
\partial\widetilde{n}_{C,t} &: 0 = -\varphi\lambda(1-\lambda)(\widetilde{n}_{U,t}-\widetilde{n}_{C,t}) - \lambda\psi_{3,t} + \varphi\psi_{5,t} - \psi_{6,t} \\
\partial\widetilde{q}_t &: 0 = \eta\omega(1-\omega)\widetilde{q}_t + \kappa_2\psi_{1,t} + \psi_{2,t} - \beta E_t[\psi_{2,t+1}] + (1-\omega)\psi_{4,t} + (1-\omega)\psi_{5,t} + (1-\omega)\psi_{6,t} \\
&\qquad - \eta(1-\omega)\psi_{7,t} + \eta\omega\psi_{8,t} \\
\partial\widetilde{y}_t &: 0 = (\sigma+\varphi)\widetilde{y}_t + \varphi_{9,t}
\end{aligned}
$$

Simplifying first order conditions, they reduce down to two equations:

$$
(1-\omega)\theta\pi_{2,t} + \kappa_2(\psi_{1,t} - \psi_{1,t-1}) = 0 \tag{A.31}
$$

$$
\kappa_2\psi_{1,t} = -\eta\omega(1-\omega)\widetilde{q}_t + (1-\omega)\left(\widetilde{y}_t - \frac{\sigma\varphi\lambda}{\sigma+\varphi}(\widetilde{c}_{U,t}-\widetilde{c}_{C,t}-\widetilde{n}_{U,t}+\widetilde{n}_{C,t})\right) \tag{A.32}
$$

By using Lagrangian constraints, we rewrite Eq.(A.32) in terms of $\widetilde{q}_t$,

$$
\kappa_2\psi_{1,t} = -(1-\omega)\left(\eta\omega + \frac{1-\omega}{\sigma+\varphi}\frac{1-\lambda(1-\sigma\varphi)}{1-\lambda}\right)\widetilde{q}_t \tag{A.33}
$$

Substituting Eq.(A.33) into Eq.(A.31), and using the Phillips curve in sector 2, we derive a second-order difference equation where $\phi \equiv \frac{1}{\theta}\left(\eta\omega + \frac{1-\omega}{\sigma+\varphi}\frac{1-\lambda(1-\sigma\varphi)}{1-\lambda}\right) > 0$ in this proof:

$$
E_t[\widetilde{q}_{t+1}] - \left(1 + \frac{1}{\beta} + \frac{\kappa_2}{\beta\phi}\right)\widetilde{q}_t + \frac{1}{\beta}\widetilde{q}_{t-1} = 0
$$

Solving the equation, we find $\widetilde{q}_t = \lambda_2\widetilde{q}_{t-1}$ where the two eigenvalues satisfies $0 < \lambda_2 < 1 < \lambda_1$.

Assuming that all the variables are in the steady state initially including $\widetilde{q}_{-1} = 0$, the dynamics under optimal monetary policy achieves efficiency as follows:

$$
\begin{aligned}
&\widetilde{q}_t^{OMP} = \widetilde{w}_t^{OMP} = \pi_{2,t}^{OMP} = \widetilde{y}_t^{OMP} = \widetilde{y}_{1,t}^{OMP} = \widetilde{y}_{2,t}^{OMP} = \widetilde{c}_{U,t}^{OMP} = \widetilde{c}_{C,t}^{OMP} = \widetilde{n}_{U,t}^{OMP} = \widetilde{n}_{C,t}^{OMP} = 0 \\
&\pi_{1,t}^{OMP} = -q_t^E + q_{t-1}^E
\end{aligned}
$$

□

A.5 Proof of Proposition 5

Proof. We solve a Ramsey problem of the utilitarian central bank when the prices of both sectors are sticky to the same degree, $0<\alpha_1=\alpha_2=\alpha$, under **HomCB**. The set-up of Lagrangian is the same as that in the proof of Proposition 6 except that we have $\kappa_1=\kappa_2=\kappa$ now.

Rewriting Lagrangian constraints corresponding to $\{\psi_{1,t}\}$, $\cdots$, $\{\psi_{3,t}\}$,

$$\pi_{1,t}-\beta E_t[\pi_{1,t+1}]=\kappa\widetilde{w}_t \tag{A.34}$$

$$\pi_{2,t}-\beta E_t[\pi_{2,t+1}]=\kappa\widetilde{w}_t-\kappa\widetilde{q}_t \tag{A.35}$$

$$\widetilde{q}_t-\widetilde{q}_{t-1}+q_t^E-q_{t-1}^E=\pi_{2,t}-\pi_{1,t} \tag{A.36}$$

Aggregating Eqs.(A.34)-(A.35) with sector size,

$$\pi_t-\beta E_t[\pi_{t+1}]=\kappa(\widetilde{w}_t-(1-\omega)\widetilde{q}_t) \tag{A.37}$$

Substituting Eqs.(A.48)-(A.51) into eq.(A.37),

$$\pi_t-\beta E_t[\pi_{t+1}]=\kappa(1+\varphi)\widetilde{y}_t \tag{A.38}$$

Rewriting Eqs.(A.52)-(A.53),

$$\pi_t=-\frac{1}{\theta}\frac{1-\lambda(1-\sigma\varphi)}{1-\lambda}(\widetilde{y}_t-\widetilde{y}_{t-1}) \tag{A.39}$$

$$\begin{aligned}0=&-(1-\omega)\theta\pi_{2,t}+\kappa\psi_{3,t}+A(L)\psi_{3,t}-\beta A(L)E_t[\psi_{3,t+1}]+\eta\omega(1-\omega)A(L)\widetilde{q}_t\\&-(1-\omega)\frac{1-\lambda(1-\sigma\varphi)}{1-\lambda}A(L)\widetilde{y}_t\end{aligned} \tag{A.40}$$

Substituting Eq.(A.39) into Eq.(A.38), we derive a second order difference equation where $\phi\equiv\frac{1}{\theta}\frac{1-\lambda(1-\sigma\varphi)}{1-\lambda}>0$ in this proof:

$$E_t[\widetilde{y}_{t+1}]-\left(1+\frac{1}{\beta}+\frac{\kappa(1+\varphi)}{\beta\phi}\right)\widetilde{y}_t+\frac{1}{\beta}\widetilde{y}_{t-1}=0$$

Solving the equation, we find $\widetilde{y}_t=\lambda_2\widetilde{y}_{t-1}$ where the two eigenvalues satisfies $0<\lambda_2<1<\lambda_1$. Assuming that all the variables are in the steady state initially including $\widetilde{y}_{-1}=0$,

$$\widetilde{y}_t^{OMP}=\pi_t^{OMP}=0$$

Subtracting Eq.(A.34) from Eq.(A.35),

$$\pi_{2,t}-\pi_{1,t}=\beta\big(E_t[\pi_{2,t+1}]-E_t[\pi_{1,t+1}]\big)-\kappa\widetilde{q}_t \tag{A.41}$$

Substituting Eq.(A.36) into Eq.(A.41), we derive a second order difference equation:

$$E_t[\widetilde{q}_{t+1}] - \left(1 + \frac{1}{\beta} + \frac{\kappa}{\beta}\right)\widetilde{q}_t + \frac{1}{\beta}\widetilde{q}_{t-1} = \left(\frac{1}{\beta} + 1 - \rho\right)q_t^E - \frac{1}{\beta}q_{t-1}^E$$

Solving the equation,

$$\widetilde{q}_t = -q_t^E + \frac{(\lambda_1 - 1)(1 - \lambda_1)}{\lambda_1 - \rho}\sum_{k=0}^{\infty}\lambda_2^k q_{t-k}^E$$

where the two eigenvalues satisfies $0 < \lambda_2 < 1 < \lambda_1$. The central bank loses control over $\widetilde{q}_t$ if $\alpha_1 = \alpha_2$, because it is affected only by exogenous shocks, q_t^E, moving independently from other variables. Note that this is derived by using only Phillips curves in both sectors and the definition of relative price.

To summarize, the dynamics under optimal monetary policy are given as follows:

$$\begin{aligned}
&\pi_t^{OMP} = \widetilde{y}_t^{OMP} = \widetilde{c}_{U,t}^{OMP} = \widetilde{c}_{C,t}^{OMP} = \widetilde{n}_{U,t}^{OMP} = \widetilde{n}_{C,t}^{OMP} = 0\\
&\widetilde{q}_t^{OMP} = -q_t^E + \frac{(\lambda_1 - 1)(1 - \lambda_1)}{\lambda_1 - \rho}\sum_{k=0}^{\infty}\lambda_2^k q_{t-k}^E\\
&\widetilde{w}_t^{OMP} = -(1-\omega)q_t^E + (1-\omega)\frac{(\lambda_1 - 1)(1 - \lambda_1)}{\lambda_1 - \rho}\sum_{k=0}^{\infty}\lambda_2^k q_{t-k}^E\\
&\widetilde{y}_{1,t}^{OMP} = -\eta(1-\omega)q_t^E + \eta(1-\omega)\frac{(\lambda_1 - 1)(1 - \lambda_1)}{\lambda_1 - \rho}\sum_{k=0}^{\infty}\lambda_2^k q_{t-k}^E\\
&\widetilde{y}_{2,t}^{OMP} = \eta\omega q_t^E - \eta\omega\frac{(\lambda_1 - 1)(1 - \lambda_1)}{\lambda_1 - \rho}\sum_{k=0}^{\infty}\lambda_2^k q_{t-k}^E
\end{aligned}$$

where achieving efficiency is infeasible. Note that as the price converges to flexible price, $\alpha \to 0$, we have $\lambda_1 \to \infty$ and $\lambda_2 \to 0$. Thus relative price, wage and sectoral output gap converge to efficient levels, $\widetilde{q}_t \to 0$, $\widetilde{w}_t \to 0$, $\widetilde{y}_{1,t} \to 0$ and $\widetilde{y}_{2,t} \to 0$. $\square$

A.6 Proof of Proposition 6

Proof. We solve a Ramsey problem of the utilitarian central bank when the prices of both sectors are sticky, but to different degrees, $0<\alpha_1<\alpha_2$, under **HomCB**. The opposite case will be exactly symmetric under homogeneous consumption baskets. We set up the Lagrangian as:

$$
\begin{aligned}
\mathcal{L}_t = \frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t & \left[\begin{array}{c}\omega\frac{\theta}{\kappa_1}\pi_{1,t}^2 + (1-\omega)\frac{\theta}{\kappa_2}\pi_{2,t}^2 + (\sigma+\varphi)\widetilde{y}_t^2 + \eta\omega(1-\omega)\widetilde{q}_t^2 \\ +\sigma\lambda(1-\lambda)(\widetilde{c}_{U,t}-\widetilde{c}_{C,t})^2 + \varphi\lambda(1-\lambda)(\widetilde{n}_{U,t}-\widetilde{n}_{C,t})^2\end{array}\right] \\
+E_0\sum_{t=0}^{\infty}\beta^t\psi_{1,t} & \Big\{\pi_{1,t} - \beta\pi_{1,t+1} - \kappa_1\widetilde{w}_t\Big\} \\
+E_0\sum_{t=0}^{\infty}\beta^t\psi_{2,t} & \Big\{\pi_{2,t} - \beta\pi_{2,t+1} - \kappa_2\widetilde{w}_t + \kappa_2\widetilde{q}_t\Big\} \\
+E_0\sum_{t=0}^{\infty}\beta^t\psi_{3,t} & \Big\{\widetilde{q}_t - \widetilde{q}_{t-1} + q_t^E - q_{t-1}^E - \pi_{2,t} + \pi_{1,t}\Big\} \\
+E_0\sum_{t=0}^{\infty}\beta^t\psi_{4,t} & \Big\{(1-\lambda)\widetilde{c}_{U,t} + \lambda\widetilde{c}_{C,t} - (1-\lambda)\widetilde{n}_{U,t} - \lambda\widetilde{n}_{C,t}\Big\} \\
+E_0\sum_{t=0}^{\infty}\beta^t\psi_{5,t} & \Big\{\varphi\widetilde{n}_{U,t} + \sigma\widetilde{c}_{U,t} - \widetilde{w}_t + (1-\omega)\widetilde{q}_t\Big\} \\
+E_0\sum_{t=0}^{\infty}\beta^t\psi_{6,t} & \Big\{\varphi\widetilde{n}_{C,t} + \sigma\widetilde{c}_{C,t} - \widetilde{w}_t + (1-\omega)\widetilde{q}_t\Big\} \\
+E_0\sum_{t=0}^{\infty}\beta^t\psi_{7,t} & \Big\{\widetilde{c}_{C,t} - \widetilde{n}_{C,t} - \widetilde{w}_t + (1-\omega)\widetilde{q}_t\Big\}
\end{aligned}
$$

where $\{\psi_{1,t}\}, \cdots, \{\psi_{7,t}\}$ are the Lagrange multipliers.

First order conditions are as follows:

$$
\begin{aligned}
\partial\pi_{1,t} &: 0 = \omega\theta\pi_{1,t} + \kappa_1(\psi_{1,t} - \psi_{1,t-1}) + \kappa_1\psi_{3,t} \\
\partial\pi_{2,t} &: 0 = (1-\omega)\theta\pi_{2,t} + \kappa_2(\psi_{2,t} - \psi_{2,t-1}) - \kappa_2\psi_{3,t} \\
\partial\widetilde{c}_{U,t} &: 0 = (\sigma+\varphi)(1-\lambda)\widetilde{y}_t + \sigma\lambda(1-\lambda)(\widetilde{c}_{U,t}-\widetilde{c}_{C,t}) + (1-\lambda)\psi_{4,t} + \sigma\psi_{5,t} \\
\partial\widetilde{c}_{C,t} &: 0 = (\sigma+\varphi)\lambda\widetilde{y}_t - \sigma\lambda(1-\lambda)(\widetilde{c}_{U,t}-\widetilde{c}_{C,t}) + \lambda\psi_{4,t} + \sigma\psi_{6,t} + \psi_{7,t} \\
\partial\widetilde{n}_{U,t} &: 0 = \varphi\lambda(1-\lambda)(\widetilde{n}_{U,t}-\widetilde{n}_{C,t}) - (1-\lambda)\psi_{4,t} + \varphi\psi_{5,t} \\
\partial\widetilde{n}_{C,t} &: 0 = -\varphi\lambda(1-\lambda)(\widetilde{n}_{U,t}-\widetilde{n}_{C,t}) - \lambda\psi_{4,t} + \varphi\psi_{6,t} - \psi_{7,t} \\
\partial\widetilde{w}_t &: 0 = -\kappa_1\psi_{1,t} - \kappa_2\psi_{2,t} - \psi_{5,t} - \psi_{6,t} - \psi_{7,t} \\
\partial\widetilde{q}_t &: 0 = \eta\omega(1-\omega)\widetilde{q}_t + \kappa_2\psi_{2,t} + \psi_{3,t} - \beta E_t[\psi_{3,t+1}] + (1-\omega)\psi_{5,t} + (1-\omega)\psi_{6,t} + (1-\omega)\psi_{7,t}
\end{aligned}
$$

Simplifying first order conditions, they reduce down to four equations where $A(L) \equiv 1 - L$:

$$0 = \omega\theta\pi_{1,t} + \kappa_1 A(L)\psi_{1,t} + \kappa_1\psi_{3,t} \tag{A.42}$$

$$0 = (1-\omega)\theta\pi_{2,t} + \kappa_2 A(L)\psi_{2,t} - \kappa_2\psi_{3,t}) \tag{A.43}$$

$$0 = \kappa_1\psi_{1,t} + \kappa_2\psi_{2,t} - \widetilde{y}_t + \frac{\sigma\varphi\lambda}{\sigma+\varphi}(\widetilde{c}_{U,t} - \widetilde{c}_{C,t} - \widetilde{n}_{U,t} + \widetilde{n}_{C,t}) \tag{A.44}$$

$$0 = \eta\omega(1-\omega)\widetilde{q}_t + \kappa_2\psi_{2,t} + \psi_{3,t} - \beta E_t[\psi_{3,t+1}] - (1-\omega)\left(\widetilde{y}_t - \frac{\sigma\varphi\lambda}{\sigma+\varphi}(\widetilde{c}_{U,t} - \widetilde{c}_{C,t} - \widetilde{n}_{U,t} + \widetilde{n}_{C,t})\right) \tag{A.45}$$

Pre-multiplying Eqs.(A.44)-(A.45) by $A(L)$, and substituting Eqs.(A.42)-(A.43) into them,

$$0 = \omega\theta\pi_{1,t} + (1-\omega)\theta\pi_{2,t} + \kappa_1\psi_{3,t} - \kappa_2\psi_{3,t} + A(L)\left(\widetilde{y}_t - \frac{\sigma\varphi\lambda}{\sigma+\varphi}(\widetilde{c}_{U,t} - \widetilde{c}_{C,t} - \widetilde{n}_{U,t} + \widetilde{n}_{C,t})\right) \tag{A.46}$$

$$\begin{aligned} 0 = &-(1-\omega)\theta\pi_{2,t} + \kappa_2\psi_{3,t} + A(L)\psi_{3,t} - \beta A(L)E_t[\psi_{3,t+1}] + \eta\omega(1-\omega)A(L)\widetilde{q}_t \\ &- (1-\omega)A(L)\left(\widetilde{y}_t - \frac{\sigma\varphi\lambda}{\sigma+\varphi}(\widetilde{c}_{U,t} - \widetilde{c}_{C,t} - \widetilde{n}_{U,t} + \widetilde{n}_{C,t})\right) \end{aligned} \tag{A.47}$$

By using Lagrangian constraints corresponding to $\{\psi_{4,t}\}$, $\cdots$, $\{\psi_{7,t}\}$, the definition of aggregate output gap, goods market clearing condition and labor market clearing condition, we write distributional variables in terms of $\widetilde{w}_t - (1-\omega)\widetilde{q}_t$ or $\widetilde{y}_t$,

$$\widetilde{c}_{C,t} = \frac{1+\varphi}{\sigma+\varphi}(\widetilde{w}_t - (1-\omega)\widetilde{q}_t) = (1+\varphi)\widetilde{y}_t \tag{A.48}$$

$$\widetilde{c}_{U,t} = \frac{1-\lambda(1+\varphi)}{(1-\lambda)(\sigma+\varphi)}(\widetilde{w}_t - (1-\omega)\widetilde{q}_t) = \frac{1-\lambda(1+\varphi)}{1-\lambda}\widetilde{y}_t \tag{A.49}$$

$$\widetilde{n}_{C,t} = \frac{1-\sigma}{\sigma+\varphi}(\widetilde{w}_t - (1-\omega)\widetilde{q}_t) = (1-\sigma)\widetilde{y}_t \tag{A.50}$$

$$\widetilde{n}_{U,t} = \frac{1-\lambda(1-\sigma)}{(1-\lambda)(\sigma+\varphi)}(\widetilde{w}_t - (1-\omega)\widetilde{q}_t) = \frac{1-\lambda(1-\sigma)}{1-\lambda}\widetilde{y}_t \tag{A.51}$$

Substituting Eqs.(A.48)-(A.51) into Eqs.(A.46)-(A.47),

$$0 = (\kappa_2 - \kappa_1)\psi_{3,t} - \theta\pi_t - \frac{1-\lambda(1-\sigma\varphi)}{1-\lambda}A(L)\widetilde{y}_t \tag{A.52}$$

$$\begin{aligned} 0 = &-(1-\omega)\theta\pi_{2,t} + \kappa_2\psi_{3,t} + A(L)\psi_{3,t} - \beta A(L)E_t[\psi_{3,t+1}] + \eta\omega(1-\omega)A(L)\widetilde{q}_t \\ &- (1-\omega)\frac{1-\lambda(1-\sigma\varphi)}{1-\lambda}A(L)\widetilde{y}_t \end{aligned} \tag{A.53}$$

Substituting Eq.(A.52) into Eq.(A.53), we derive a targeting rule

$$\frac{1}{\kappa_2-\kappa_1}\left[\begin{array}{c}\kappa_2\{\theta\pi_t+\frac{1-\lambda(1-\sigma\varphi)}{1-\lambda}A(L)\widetilde{y}_t\}+A(L)\{\theta\pi_t+\frac{1-\lambda(1-\sigma\varphi)}{1-\lambda}A(L)\widetilde{y}_t\}\\-\beta A(L)\{\theta E_t[\pi_{t+1}]+\frac{1-\lambda(1-\sigma\varphi)}{1-\lambda}A(L)E_t[\widetilde{y}_{t+1}]\}\end{array}\right] \tag{A.54}$$
$$=(1-\omega)\theta\pi_{2,t}-\eta\omega(1-\omega)A(L)\widetilde{q}_t+(1-\omega)\frac{1-\lambda(1-\sigma\varphi)}{1-\lambda}A(L)\widetilde{y}_t$$

□

A.7 Proof of Proposition 7

Proof. We follow Woodford (2003) in deriving the welfare-theoretic loss function. Note first that under the assumptions on employment subsidy and government transfers, the steady state is efficient and equitable, $N_{C,t}=N_{U,t}=N=C_{C,t}=C_{U,t}=C=Y=1$ with the wage and the relative price being unity, $W=Q=1$. Thus we have

$$\frac{V_N(N_{U,t})}{U_C(C_{U,t})} = W = 1 = \frac{W}{Q} = \frac{V_N(N_{c,t})}{U_C(C_{C,t})}$$

Define $h_U \equiv 1-\lambda$ and $h_C \equiv \lambda$, and note that we assumed $z_1 \equiv \lambda$ and $z_2 \equiv 1-\lambda$. Taking a second-order approximation to the equally weighted sum of both types of households' utilities around the efficient zero-inflation steady state,

$$\begin{aligned}
&\sum_{j=U,C} h_j \mathcal{U}(C_{h,t}, N_{h,t}) \\
&= \sum_{j=U,C} h_j \begin{bmatrix} U_c Y\{c_{j,t} + \frac{1-\sigma}{2} c_{j,t}^2\} \\ -V_N N\{n_{j,t} + \frac{1+\varphi}{2} n_{j,t}^2\} \end{bmatrix} + t.i.p. + o(||\xi||^3) \\
&= U_c Y \begin{bmatrix} z_1\{\widetilde{c}_{C,t} + \frac{1-\sigma}{2}\widetilde{c}_{C,t}^2 + (1-\sigma)c_{C,t}^E \widetilde{c}_{C,t}\} + z_2\{\widetilde{c}_{U,t} + \frac{1-\sigma}{2}\widetilde{c}_{U,t}^2 + (1-\sigma)c_{U,t}^E \widetilde{c}_{U,t}\} \\ -z_1\{\widetilde{n}_{C,t} + \frac{1+\varphi}{2}\widetilde{n}_{C,t}^2 + (1+\varphi)n_{C,t}^E \widetilde{n}_{C,t}\} - z_2\{\widetilde{n}_{U,t} + \frac{1+\varphi}{2}\widetilde{n}_{U,t}^2 + (1+\varphi)n_{U,t}^E \widetilde{n}_{U,t}\} \end{bmatrix} \\
&\quad + t.i.p. + o(||\xi||^3)
\end{aligned} \tag{A.55}$$

Taking a second order approximation to the labor market clearing condition,

$$\begin{aligned}
&\omega(\widetilde{n}_{1,t} + \frac{1}{2}\widetilde{n}_{1,t}^2 + n_{1,t}^E \widetilde{n}_{1,t}) + (1-\omega)(\widetilde{n}_{2,t} + \frac{1}{2}\widetilde{n}_{2,t}^2 + n_{2,t}^E \widetilde{n}_{2,t}) \\
&= (1-\lambda)(\widetilde{n}_{U,t} + \frac{1}{2}\widetilde{n}_{U,t}^2 + n_{U,t}^E \widetilde{n}_{U,t}) + \lambda(\widetilde{n}_{C,t} + \frac{1}{2}\widetilde{n}_{C,t}^2 + n_{C,t}^E \widetilde{n}_{C,t}) + t.i.p. + o(||\xi||^3)
\end{aligned} \tag{A.56}$$

Let us define $\hat{p}_{j,t}(i) \equiv p_{j,t}(i) - p_{j,t}$. Then, by a second order approximation,

$$\left(\frac{P_{j,t}(i)}{P_{j,t}}\right)^{1-\theta} = e^{(1-\theta)\hat{p}_{j,t}(i)} = 1 + (1-\theta)\hat{p}_{j,t}(i) + \frac{(1-\theta)^2}{2}\hat{p}_{j,t}^2(i) + o(||\xi||^3) \tag{A.57}$$

Since $\frac{1}{z_j}\int_{\mathcal{I}_j} \left(\frac{P_{j,t}(i)}{P_{j,t}}\right)^{1-\theta} di = 1$ by the price aggregator, we integrate Eq.(A.57) to derive

$$E_i^j\{\hat{p}_{j,t}(i)\} = \frac{\theta-1}{2} E_i^j\{\hat{p}_{j,t}^2(i)\} \tag{A.58}$$

Similarly, taking a second order approximation, integrating the result, and substituting Eq.(A.58),

$$\left(\frac{P_{j,t}(i)}{P_{j,t}}\right)^{-\theta} = 1 - \theta\hat{p}_{j,t}(i) + \frac{\theta^2}{2}\hat{p}^2_{j,t}(i) + o(||\xi||^3)$$

$$\frac{1}{z_j}\int_{\mathcal{I}_j}\left(\frac{P_{j,t}(i)}{P_{j,t}}\right)^{-\theta} = 1 - \theta E^j_i\{\hat{p}_{j,t}(i)\} + \frac{\theta^2}{2}E^j_i\{\hat{p}^2_{j,t}(i)\} + o(||\xi||^3)$$

$$= 1 + \frac{\theta}{2}E^j_i\{\hat{p}^2_{j,t}(i)\} + o(||\xi||^3) \tag{A.59}$$

Since $E^j_i\{\hat{p}^2_{j,t}(i)\} = \frac{1}{z_j}\int_{\mathcal{I}_j}\hat{p}^2_{j,t}(i)di = \frac{1}{z_j}\int_{\mathcal{I}_j}\left(p_{j,t}(i) - p_{j,t}\right)^2 di$, and we know that in the first order $p_{j,t} = E^j_i\{p_{j,t}(i)\}$, we derive that $E^j_i\{\hat{p}^2_{j,t}(i)\} = Var^j_i\{p_{j,t}(i)\}$. Substituting this into Eq.(A.59),

$$\frac{1}{z_j}\int_{\mathcal{I}_j}\left(\frac{P_{j,t}(i)}{P_{j,t}}\right)^{-\theta} = 1 + \frac{\theta}{2}Var^j_i\{p_{j,t}(i)\} + o(||\xi||^3) \tag{A.60}$$

Thus we derive the second order approximation to the price dispersion in each sector as

$$d_{j,t} \equiv \log\frac{1}{z_j}\int_{\mathcal{I}_j}\left(\frac{P_{j,t}(i)}{P_{j,t}}\right)^{-\theta} = \frac{\theta}{2}Var^j_i\{p_{j,t}(i)\} + o(||\xi||^3) \tag{A.61}$$

We have $N_{j,t} = \int_{\mathcal{I}_j}\frac{Y_{j,t}(i)}{A_tA_{j,t}}di = \frac{1}{z_j}\frac{Y_{j,t}}{A_tA_{j,t}}\int_{\mathcal{I}_j}(\frac{P_{j,t}(i)}{P_{j,t}})^{-\theta}di$ by the relative demand function. Taking a second order approximation and substituting Eq.(A.61), we derive

$$n_{j,t} = y_{j,t} - a_t - a_{j,t} + \log\frac{1}{z_j}\int_{\mathcal{I}_j}(\frac{P_{j,t}(i)}{P_{j,t}})^{-\theta}di + o(||\xi||^3)$$

$$= y_{j,t} - a_t - a_{j,t} + \frac{\theta}{2}Var^j_i\{p_{j,t}(i)\} + o(||\xi||^3)$$

$$n^2_{j,t} = y^2_{j,t} + a^2_t + a^2_{j,t} - 2(a_t + a_{j,t})y_{j,t} + 2a_ta_{j,t} + o(||\xi||^3)$$

$$\Rightarrow \ \widetilde{n}_{j,t} = \widetilde{y}_{j,t} + \frac{\theta}{2}Var^j_i\{p_{j,t}(i)\} + o(||\xi||^3) \tag{A.62}$$

$$\widetilde{n}^2_{j,t} + 2n^E_{j,t}\widetilde{n}_{j,t} = \widetilde{y}^2_{j,t} + 2y^E_{j,t}\widetilde{y}_{j,t} - 2(a_t + a_{j,t})\widetilde{y}_{j,t} + t.i.p. + o(||\xi||^3) \tag{A.63}$$

Substituting Eqs.(A.56), (A.62) and (A.63) into Eq.(A.1), and canceling out the cross terms,

$$(\text{A.55}) = -\frac{U_cY}{2}\begin{bmatrix} z_1\theta Var^1_i\{p_{1,t}(i)\} + z_2\theta Var^2_i\{p_{2,t}(i)\} \\ +z_1\sigma\widetilde{y}^2_{1,t} + z_2\sigma\widetilde{y}^2_{2,t} \\ +z_1\varphi\widetilde{n}^2_{C,t} + z_2\varphi\widetilde{n}^2_{U,t}\end{bmatrix} + t.i.p. + o(||\xi||^3) \tag{A.64}$$

where $\widetilde{y}_{1,t} \equiv \widetilde{c}_{C,t}$ and $\widetilde{y}_{2,t} \equiv \widetilde{c}_{U,t}$ by goods market clearing condition.

Let us define $\Delta_t^j \equiv Var_i^j\{P_{j,t}(i)\}$. According to Woodford (2003),

$$\begin{aligned}\Delta_t^j &= \alpha_j \Delta_{t-1}^j + \frac{\alpha_j}{1-\alpha_j}\pi_{j,t}^2 + o(||\xi||^3) \\ &= \underbrace{\alpha_j^{t+1}\Delta_{-1}^j}_{t.i.p.} + \sum_{k=0}^{t}\alpha_j^{t-k}\frac{\alpha_j}{1-\alpha_j}\pi_{j,t}^2 + o(||\xi||^3)\end{aligned}$$

and the present valued sum of the cross-sectional price dispersion can be rewritten in terms of present valued sum of squared inflation as

$$\sum_{t=0}^{\infty}\beta^t \Delta_t^j = \frac{\alpha_j}{(1-\alpha_j)(1-\alpha_j\beta)}\sum_{t=0}^{\infty}\beta^t\pi_{j,t}^2 + t.i.p. + o(||\xi||^3) \tag{A.65}$$

Substituting Eq.(A.65) into Eq.(A.64), and summing up the present valued utilities,

$$E_0\sum_{t=0}^{\infty}\beta^t\sum_{j=U,C}h_j\mathcal{U}(C_{h,t},N_{h,t}) = -\frac{U_cY}{2}E_0\sum_{t=0}^{\infty}\beta^t\begin{bmatrix}\frac{z_1\theta}{\kappa_1}\pi_{1,t}^2 + \frac{z_2\theta}{\kappa_2}\pi_{2,t}^2 \\ +z_1\sigma\widetilde{y}_{1,t}^2 + z_2\sigma\widetilde{y}_{2,t}^2 \\ +z_1\varphi\widetilde{n}_{C,t}^2 + z_2\varphi\widetilde{n}_{U,t}^2\end{bmatrix} + t.i.p. + o(||\xi||^3) \tag{A.66}$$

Deriving the relation between labor supply gap of type C households and output gap 1 by the relation between consumption and labor supply of the constrained households,

$$\widetilde{n}_{C,t} = \frac{1-\sigma}{1+\varphi}\widetilde{y}_{1,t} + \frac{1-\sigma}{\sigma}\frac{1}{1+\varphi}z_2q_t^E \tag{A.67}$$

$$\widetilde{n}_{C,t}^2 = \left(\frac{1-\sigma}{1+\varphi}\right)^2\widetilde{y}_{1,t}^2 + 2\left(\frac{1-\sigma}{1+\varphi}\right)^2\frac{z_2}{\sigma}q_t^E\widetilde{y}_{1,t} + t.i.p. \tag{A.68}$$

Substituting Eq.(A.67) into the labor market clearing condition, we derive the relation between labor supply gap of the unconstrained households and output gaps:

$$\widetilde{n}_{U,t} = \frac{z_1}{z_2}\frac{\sigma+\varphi}{1+\varphi}\widetilde{y}_{1,t} + \widetilde{y}_{2,t} - \frac{1-\sigma}{\sigma}\frac{1}{1+\varphi}z_1q_t^E + o(||\xi||^2) \tag{A.69}$$

$$\begin{aligned}\widetilde{n}_{U,t}^2 = {}& \frac{z_1^2}{z_2^2}\left(\frac{\sigma+\varphi}{1+\varphi}\right)^2\widetilde{y}_{1,t}^2 + \widetilde{y}_{2,t}^2 + 2\frac{z_1}{z_2}\left(\frac{\sigma+\varphi}{1+\varphi}\right)\widetilde{y}_{1,t}\widetilde{y}_{2,t} \\ & - 2\frac{1-\sigma}{\sigma}\frac{1}{1+\varphi}\left(\frac{z_1}{z_2}\frac{\sigma+\varphi}{1+\varphi}\widetilde{y}_{1,t}+\widetilde{y}_{2,t}\right)z_1q_t^E + t.i.p. + o(||\xi||^3)\end{aligned} \tag{A.70}$$

substituting Eqs.(A.68) and (A.70) into Eq.(A.66), we can rewrite the loss function in terms of inflation and output gaps only:

$$
(\text{A.66}) = -\frac{U_cY}{2}E_0\sum_{t=0}^{\infty}\beta^t \left[\begin{array}{c}
\underbrace{\frac{z_1\theta}{\kappa_1}}_{\equiv\Gamma_{\pi_1}}\ \pi_{1,t}^2 + \underbrace{\frac{z_2\theta}{\kappa_2}}_{\equiv\Gamma_{\pi_1}}\ \pi_{2,t}^2 \\
+\ \underbrace{z_1\left[\sigma+\left(\frac{1-\sigma}{1+\varphi}\right)^2\varphi+\frac{z_1}{z_2}(\frac{\sigma+\varphi}{1+\varphi})^2\varphi\right]}_{\equiv\Gamma_{y_{11}}}\widetilde{y}_{1,t}^2 \\
+\ \underbrace{2z_1\varphi\frac{\sigma+\varphi}{1+\varphi}}_{\equiv\Gamma_{y_{12}}}\widetilde{y}_{1,t}\widetilde{y}_{2,t}\ +\ \underbrace{z_2(\sigma+\varphi)}_{\equiv\Gamma_{y_{22}}}\widetilde{y}_{2,t}^2 \\
+\ \underbrace{2\varphi\frac{1-\sigma}{1+\varphi}\frac{z_1z_2}{\sigma}\left(\frac{1-\sigma}{1+\varphi}-\frac{z_1}{z_2}\frac{\sigma+\varphi}{1+\varphi}\right)q_t^E}_{\equiv\Gamma_{y_1}}\widetilde{y}_{1,t}\ \underbrace{-\ 2\varphi\frac{1-\sigma}{1+\varphi}\frac{z_1z_2}{\sigma}q_t^E}_{\equiv\Gamma_{y_2}}\widetilde{y}_{2,t}
\end{array}\right]
$$

$$
+\,t.i.p. + o(||\xi||^3)
$$

$$
= -\frac{U_cY}{2}E_0\sum_{t=0}^{\infty}\beta^t \left[\begin{array}{c}
\Gamma_{\pi_1}\pi_{1,t}^2+\Gamma_{\pi_2}\pi_{2,t}^2 \\
+\Gamma_{y_{11}}(\widetilde{y}_{1,t}-x_{1,t}^*)^2+\Gamma_{y_{12}}(\widetilde{y}_{1,t}-x_{1,t}^*)(\widetilde{y}_{2,t}-x_{2,t}^*)+\Gamma_{y_{22}}(\widetilde{y}_{2,t}-x_{2,t}^*)^2
\end{array}\right]
$$

$$
+\ t.i.p. + o(||\xi||^3) \tag{A.71}
$$

$$
\left(\begin{array}{l}
x_{1,t}^* \equiv \dfrac{2\Gamma y_{22}\Gamma y_1-\Gamma y_{12}\Gamma y_2}{\Gamma y_{12}{}^2-4\Gamma y_{11}\Gamma y_{22}} = \dfrac{1-\sigma}{\sigma}\dfrac{\varphi}{\sigma+\varphi}\dfrac{(\sigma-z_2)z_2}{\sigma\varphi+z_2}q_t^E \\
x_{2,t}^* \equiv \dfrac{2\Gamma y_{11}\Gamma y_2-\Gamma y_{12}\Gamma y_1}{\Gamma y_{12}{}^2-4\Gamma y_{11}\Gamma y_{22}} = \dfrac{1-\sigma}{\sigma}\dfrac{\varphi}{\sigma+\varphi}\dfrac{z_1z_2}{\sigma\varphi+z_2}q_t^E
\end{array}\right.
$$

where $\Gamma_{y_{12}}^2-4\Gamma_{y_{11}}\Gamma_{y_{22}} < 0$ holds implying that the contour of the loss function is elliptical with its center being $(x_{1,t}^*,\ x_{2,t}^*)$.

Note that target output gaps shifts according to relative productivity shock, q_t^E, and their directions depends on the value of σ that measures the relative size of the income effect compared to the substitution effect in labor supply and households' preference on consumption smoothing,

$$
x_{1,t}^*\begin{cases} >0, & \text{if } \ z_2<\sigma<1 \\ \leq 0, & \text{otherwise}\end{cases} \quad \text{and} \quad x_{2,t}^*\begin{cases} >0, & \text{if } \ \sigma<1 \\ \leq 0, & \text{otherwise}\end{cases}
$$

□

A.8 Proof of Proposition 8

Proof. We solve a Ramsey problem of the utilitarian central bank when the price of sector 1 is flexible and the price of sector 2 is sticky, $\alpha_1 = 0$, under **HetCB**. We set up the Lagrangian as:

$$\begin{aligned}
\mathcal{L}_t = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t & \begin{bmatrix} \frac{z_2\theta}{\kappa_2}\pi_{2,t}^2 \\ +\Gamma_{y_{11}}(\widetilde{y}_{1,t}^N - x_{1,t}^*)^2 + \Gamma_{y_{12}}(\widetilde{y}_{1,t}^N - x_{1,t}^*)(\widetilde{y}_{2,t} - x_{2,t}^*) + \Gamma_{y_{22}}(\widetilde{y}_{2,t} - x_{2,t}^*)^2 \end{bmatrix} \\
& + E_0 \sum_{t=0}^{\infty} \beta^t \psi_{1,t} \Big\{ \pi_{2,t} - \beta \pi_{2,t+1} + \kappa_2 \widetilde{q}_t \Big\} \\
& + E_0 \sum_{t=0}^{\infty} \beta^t \psi_{2,t} \Big\{ \widetilde{q}_t - \widetilde{q}_{t-1} + q_t^E - q_{t-1}^E - \pi_{2,t} + \pi_{1,t} \Big\} \\
& + E_0 \sum_{t=0}^{\infty} \beta^t \psi_{3,t} \Big\{ z_1 \widetilde{y}_{1,t}^N + z_2 \widetilde{y}_{2,t} - z_2 \widetilde{n}_{U,t} - z_1 \widetilde{n}_{C,t}^N \Big\} \\
& + E_0 \sum_{t=0}^{\infty} \beta^t \psi_{4,t} \Big\{ \varphi \widetilde{n}_{U,t} + \sigma \widetilde{y}_{2,t} + \widetilde{q}_t \Big\}
\end{aligned}$$

where $\{\psi_{1,t}\}, \cdots, \{\psi_{4,t}\}$ are the Lagrange multipliers.

First order conditions are as follows:

$$\begin{aligned}
\partial \pi_{1,t} &: 0 = \psi_{2,t} \\
\partial \pi_{2,t} &: 0 = \frac{z_2\theta}{\kappa_1}\pi_{2,t} + \psi_{1,t} - \psi_{1,t-1} - \psi_{2,t} \\
\partial \widetilde{q}_t &: 0 = \kappa_2 \psi_{1,t} + \psi_{2,t} - \beta E_t[\psi_{2,t+1}] + \psi_{4,t} \\
\partial \widetilde{y}_{2,t} &: 0 = \Gamma_{y_{22}}(\widetilde{y}_{2,t} - x_{2,t}^*) + \frac{\Gamma_{y_{12}}}{2}(\widetilde{y}_{1,t}^N - x_{1,t}^*) + z_2\psi_{3,t} + \sigma\psi_{4,t} \\
\partial \widetilde{n}_{U,t} &: 0 = -z_2\psi_{3,t} + \varphi\psi_{4,t}
\end{aligned}$$

Simplifying first order conditions into one equation:

$$\pi_{2,t} = \frac{z_1}{\theta}\frac{1-\sigma}{\sigma}\frac{\varphi}{\sigma+\varphi}(q_t^E - q_{t-1}^E) - \frac{1}{\theta}\widetilde{y}_{2,t} + \frac{1}{\theta}\widetilde{y}_{2,t-1} \tag{A.72}$$

By using Lagrangian constraints corresponding to $\{\psi_{1,t}\}$, $\{\psi_{3,t}\}$, and $\{\psi_{4,t}\}$,

$$\pi_{2,t} - \beta E_t[\pi_{2,t+1}] - \kappa_2(\sigma+\varphi)\widetilde{y}_{2,t} + \kappa_2 \frac{1-\sigma}{\sigma}\varphi z_1 q_t^E = 0 \tag{A.73}$$

Substituting Eq.(A.72) into Eq.(A.73), we derive a second-order difference equation where $\gamma_1 \equiv$

$\frac{z_1}{\beta}\frac{1-\sigma}{\sigma}\frac{\varphi}{\sigma+\varphi}\big(1+\beta-\beta\rho+\kappa_2\theta(\sigma+\varphi)\big)$ and $\gamma_2 \equiv \frac{z_1}{\beta}\frac{1-\sigma}{\sigma}\frac{\varphi}{\sigma+\varphi}$ in this proof:

$$E_t[\widetilde{y}_{2,t+1}] - \left(1 + \frac{1}{\beta} + \frac{\kappa_2\theta(\sigma+\varphi)}{\beta}\right)\widetilde{y}_{2,t} + \frac{1}{\beta}\widetilde{q}_{t-1} = -\gamma_1 q_t^E + \gamma_2 q_{t-1}^E$$

Solving the equation,

$$\widetilde{y}_{2,t} = \lambda_2\widetilde{y}_{2,t-1} + \frac{z_1}{\beta}\frac{1-\sigma}{\sigma}\frac{\varphi}{\sigma+\varphi}\left(1+\beta-\beta\rho-\frac{1}{\lambda_1}+\kappa_2\theta(\sigma+\varphi)\right)\frac{1}{\lambda_1-\rho}q_t^E - \frac{z_1}{\beta}\frac{1-\sigma}{\sigma}\frac{\varphi}{\sigma+\varphi}\frac{1}{\lambda_1}q_{t-1}^E \tag{A.74}$$

where the two eigenvalues satisfies $0<\lambda_2<1<\lambda_1$. Simplifying further with $\lambda_1+\lambda_2 \equiv 1+\frac{1}{\beta}+\frac{\kappa_2\theta(\sigma+\varphi)}{\beta}$ and $\lambda_1\lambda_2 \equiv \frac{1}{\beta}$,

$$\widetilde{y}_{2,t}^{OMP} = z_1\frac{1-\sigma}{\sigma}\frac{\varphi}{\sigma+\varphi}q_t^E = \widetilde{y}_{2,t}^N \tag{A.75}$$

Substituting Eq.(A.75) into Eq.(A.72),

$$\widetilde{\pi}_{2,t}^{OMP} = 0 \tag{A.76}$$

Solving for the rest variables,

$$\begin{aligned}
\widetilde{n}_{U,t}^{OMP} &= -z_1\frac{1-\sigma}{\sigma+\varphi}q_t^E \\
\widetilde{q}_t^{OMP} &= 0 = \widetilde{q}_t^N \\
\widetilde{\pi}_{1,t}^{OMP} &= -q_t^E + q_{t-1}^E
\end{aligned}$$

We find that optimal policy achieves flexible price (natural) allocation. □

A.9 Proof of Proposition 9

Proof. We solve a Ramsey problem of the utilitarian central bank when the price of sector 2 is flexible and the price of sector 1 is sticky, $\alpha_2 = 0$, under **HetCB**. We set up the Lagrangian as:

$$
\begin{aligned}
\mathcal{L}_t = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t & \left[\begin{array}{c} \frac{z_1\theta}{\kappa_1}\pi_{1,t}^2 \\ +\Gamma_{y_{11}}(\widetilde{y}_{1,t} - x_{1,t}^*)^2 + \Gamma_{y_{12}}(\widetilde{y}_{1,t} - x_{1,t}^*)(\widetilde{y}_{2,t} - x_{2,t}^*) + \Gamma_{y_{22}}(\widetilde{y}_{2,t} - x_{2,t}^*)^2 \end{array} \right] \\
& + E_0 \sum_{t=0}^{\infty} \beta^t \psi_{1,t} \left\{ \pi_{1,t} - \beta\pi_{1,t+1} - \kappa_1 \widetilde{w}_t \right\} \\
& + E_0 \sum_{t=0}^{\infty} \beta^t \psi_{2,t} \left\{ \widetilde{w}_t - \widetilde{q}_t \right\} \\
& + E_0 \sum_{t=0}^{\infty} \beta^t \psi_{3,t} \left\{ \widetilde{q}_t - \widetilde{q}_{t-1} + q_t^E - q_{t-1}^E - \pi_{2,t} + \pi_{1,t} \right\} \\
& + E_0 \sum_{t=0}^{\infty} \beta^t \psi_{4,t} \left\{ z_1 \widetilde{y}_{1,t} + z_2 \widetilde{y}_{2,t} - z_2 \widetilde{n}_{U,t} - z_1 \widetilde{n}_{C,t} \right\} \\
& + E_0 \sum_{t=0}^{\infty} \beta^t \psi_{5,t} \left\{ \varphi \widetilde{n}_{U,t} + \sigma \widetilde{y}_{2,t} \right\} \\
& + E_0 \sum_{t=0}^{\infty} \beta^t \psi_{6,t} \left\{ \varphi \widetilde{n}_{C,t} + \sigma \widetilde{y}_{1,t} - \widetilde{w}_t \right\} \\
& + E_0 \sum_{t=0}^{\infty} \beta^t \psi_{7,t} \left\{ \widetilde{y}_{1,t} - \widetilde{w}_t - \widetilde{n}_{C,t} + \frac{1-\sigma}{\sigma} z_2 q_t^E \right\}
\end{aligned}
$$

where $\{\psi_{1,t}\}, \cdots, \{\psi_{7,t}\}$ are the Lagrange multipliers.

First order conditions are as follows:

$$
\begin{aligned}
\partial \pi_{1,t} &: 0 = \frac{z_1\theta}{\kappa_1}\pi_{1,t} + \psi_{1,t} - \psi_{1,t-1} + \psi_{3,t} \\
\partial \pi_{2,t} &: 0 = -\psi_{3,t} \\
\partial \widetilde{w}_t &: 0 = -\kappa_1 \psi_{1,t} + \psi_{2,t} - \psi_{6,t} - \psi_{7,t} \\
\partial \widetilde{q}_t &: 0 = -\psi_{2,t} + \psi_{3,t} - \beta E_t[\psi_{3,t+1}] \\
\partial \widetilde{y}_{1,t} &: 0 = \Gamma_{y_{11}}(\widetilde{y}_{1,t} - x_{1,t}^*) + \frac{\Gamma_{y_{12}}}{2}(\widetilde{y}_{2,t} - x_{2,t}^*) + z_1\psi_{4,t} + \sigma\psi_{6,t} + \psi_{7,t} \\
\partial \widetilde{y}_{2,t} &: 0 = \Gamma_{y_{22}}(\widetilde{y}_{2,t} - x_{2,t}^*) + \frac{\Gamma_{y_{12}}}{2}(\widetilde{y}_{1,t} - x_{1,t}^*) + z_2\psi_{4,t} + \sigma\psi_{5,t} \\
\partial \widetilde{n}_{U,t} &: 0 = -z_2\psi_{4,t} + \varphi\psi_{5,t} \\
\partial \widetilde{n}_{C,t} &: 0 = -z_1\psi_{4,t} + \varphi\psi_{6,t} - \psi_{7,t}
\end{aligned}
$$

Simplifying first order conditions into one equation:

$$\pi_{1,t} = -\frac{k_0}{z_1\theta}(\widetilde{y}_{1,t} - \widetilde{y}_{1,t-1}) + \frac{k_0}{z_1\theta}(x^*_{1,t} - x^*_{1,t-1}) \tag{A.77}$$

where $k_0 \equiv \frac{1+\varphi}{\sigma+\varphi} z_1[\sigma + (\frac{1-\sigma}{1+\varphi})^2\varphi + \frac{z_1}{z_2}\frac{(\sigma+\varphi)\sigma\varphi}{(1+\varphi)^2}]$ and $k_1 \equiv \sigma + (\frac{1-\sigma}{1+\varphi})^2\varphi + \frac{z_1}{z_2}\frac{(\sigma+\varphi)\sigma\varphi}{(1+\varphi)^2}$.

Simplifying Lagrangian constraints corresponding to $\{\psi_{5,t}\}, \cdots, \{\psi_{7,t}\}$,

$$\widetilde{w}_t = \frac{\sigma+\varphi}{1+\varphi}\widetilde{y}_{1,t} + z_2\frac{1-\sigma}{\sigma}\frac{\varphi}{1+\varphi}q^E_t \tag{A.78}$$

Substituting Eq.(A.78) into the labor market clearing condition,

$$\widetilde{y}_{2,t} = -\frac{z_1}{z_2}\frac{\varphi}{\sigma+\varphi}\widetilde{w}_t + z_1\frac{1-\sigma}{\sigma}\frac{\varphi}{\sigma+\varphi}q^E_t \tag{A.79}$$

Substituting Eq.(A.78) into the Phillips Curve in sector 1,

$$\pi_{1,t} - \beta E_t[\pi_{1,t+1}] - \kappa_1\frac{\sigma+\varphi}{1+\varphi}\widetilde{y}_{1,t} - \kappa_1 z_2\frac{1-\sigma}{\sigma}\frac{\varphi}{1+\varphi}q^E_t = 0 \tag{A.80}$$

Substituting Eq.(A.77) into Eq.(A.80), we derive a second-order difference equation where $\gamma_1 \equiv \frac{z_2}{\beta}\frac{1-\sigma}{\sigma}\frac{\varphi}{\sigma+\varphi}[(1+\beta-\beta\rho)\frac{\sigma-z_2}{\sigma\varphi+z_2} - \frac{\theta\kappa_2}{k_1}(\frac{\sigma+\varphi}{1+\varphi})^2]$ and $\gamma_2 \equiv \frac{z_2}{\beta}\frac{1-\sigma}{\sigma}\frac{\varphi}{\sigma+\varphi}\frac{\sigma-z_2}{\sigma\varphi+z_2}$ in this proof:

$$E_t[\widetilde{y}_{1,t+1}] - \left(1 + \frac{1}{\beta} + \frac{\kappa_1\theta}{\beta k_1}\left(\frac{\sigma+\varphi}{1+\varphi}\right)^2\right)\widetilde{y}_{2,t} + \frac{1}{\beta}\widetilde{y}_{1,t-1} = -\gamma_1 q^E_t + \gamma_2 q^E_{t-1}$$

Solving the equation,

$$\begin{aligned}\widetilde{y}_{1,t} = \lambda_2\widetilde{y}_{1,t-1} &+ \frac{z_2}{\beta}\frac{1-\sigma}{\sigma}\frac{\varphi}{\sigma+\varphi}\frac{1}{\lambda_1-\rho}\left[\left(1+\beta-\beta\rho-\frac{1}{\lambda_1}\right)\frac{\sigma-z_2}{\sigma\varphi+z_2} - \frac{\kappa_1\theta}{k_1}\left(\frac{\sigma+\varphi}{1+\varphi}\right)^2\right]q^E_t \\ &- \frac{z_2}{\beta}\frac{1-\sigma}{\sigma}\frac{\varphi}{\sigma+\varphi}\frac{1}{\lambda_1}\frac{\sigma-z_2}{\sigma\varphi+z_2}q^E_{t-1}\end{aligned} \tag{A.81}$$

where the two eigenvalues satisfies $0 < \lambda_2 < 1 < \lambda_1$. Simplifying further with $\lambda_1 + \lambda_2 \equiv 1 + \frac{1}{\beta} + \frac{\kappa_1\theta}{\beta k_1}(\frac{\sigma+\varphi}{1+\varphi})^2$ and $\lambda_1\lambda_2 \equiv \frac{1}{\beta}$,

$$\begin{aligned}\widetilde{y}^{OMP}_{1,t} &= \underbrace{z_2\frac{1-\sigma}{\sigma}\frac{\varphi}{\sigma+\varphi}\frac{\sigma-z_2}{\sigma\varphi+z_2}q^E_t}_{=x^*_{1,t}} - z_2\frac{1-\sigma}{\sigma}\frac{\varphi}{\sigma+\varphi}\frac{\sigma\varphi+\sigma}{\sigma\varphi+z_2}\frac{(\lambda_1-1)(1-\lambda_2)}{\lambda_1-\rho}\sum_{k=0}^{\infty}\lambda_2^k q^E_{t-k} \\ &\xrightarrow{\text{as } \alpha_1 \to 0} -z_2\frac{1-\sigma}{\sigma}\frac{\varphi}{\sigma+\varphi}q^E_t = \widetilde{y}^N_{1,t}\end{aligned} \tag{A.82}$$

Note that as $\alpha_1 \to 0$, $\lambda_1 \to \infty$ and $\lambda_2 \to 0$.

Substituting Eq.(A.82) into Eq.(A.77),

$$\widetilde{\pi}_{1,t}^{OMP} = \frac{k_0}{z_1\theta} z_2 \frac{1-\sigma}{\sigma} \frac{\varphi}{\sigma+\varphi} \frac{\sigma\varphi+\sigma}{\sigma\varphi+z_2} \frac{(\lambda_1-1)(1-\lambda_2)}{\lambda_1-\rho} \left(q_t^E - (1-\lambda_2) \sum_{k=0}^{\infty} \lambda_2^k q_{t-k}^E \right)$$

Solving for the rest variables,

$$\begin{aligned}
\widetilde{y}_{2,t}^{OMP} &= \underbrace{z_1 \frac{1-\sigma}{\sigma} \frac{\varphi}{\sigma+\varphi} \frac{z_2}{\sigma\varphi+z_2} q_t^E}_{=x_{2,t}^*} + z_1 \frac{1-\sigma}{\sigma} \frac{\varphi}{\sigma+\varphi} \frac{\sigma\varphi}{\sigma\varphi+z_2} \frac{(\lambda_1-1)(1-\lambda_2)}{\lambda_1-\rho} \sum_{k=0}^{\infty} \lambda_2^k q_{t-k}^E \\
&\xrightarrow{\text{as } \alpha_1 \to 0} z_1 \frac{1-\sigma}{\sigma} \frac{\varphi}{\sigma+\varphi} q_t^E = \widetilde{y}_{2,t}^N \\
\widetilde{n}_{U,t}^{OMP} &= -z_1 \frac{1-\sigma}{\sigma+\varphi} \left(\frac{z_2}{\sigma\varphi+z_2} q_t^E + \frac{\sigma\varphi}{\sigma\varphi+z_2} \frac{(\lambda_1-1)(1-\lambda_2)}{\lambda_1-\rho} \sum_{k=0}^{\infty} \lambda_2^k q_{t-k}^E \right) \\
&\xrightarrow{\text{as } \alpha_1 \to 0} -z_1 \frac{1-\sigma}{\sigma+\varphi} q_t^E = \widetilde{n}_{U,t}^N \\
\widetilde{n}_{C,t}^{OMP} &= z_2 \frac{1-\sigma}{\sigma+\varphi} \left(\frac{\varphi+z_2}{\sigma\varphi+z_2} q_t^E - \frac{\varphi-\sigma\varphi}{\sigma\varphi+z_2} \frac{(\lambda_1-1)(1-\lambda_2)}{\lambda_1-\rho} \sum_{k=0}^{\infty} \lambda_2^k q_{t-k}^E \right) \\
&\xrightarrow{\text{as } \alpha_1 \to 0} z_2 \frac{1-\sigma}{\sigma+\varphi} q_t^E = \widetilde{n}_{C,t}^N \\
\widetilde{w}_t^{OMP} &= \widetilde{q}_t^{OMP} = z_2 \frac{1-\sigma}{\sigma} \frac{\varphi}{1+\varphi} \frac{\sigma\varphi+\sigma}{\sigma\varphi+z_2} \left(q_t^E - \frac{(\lambda_1-1)(1-\lambda_2)}{\lambda_1-\rho} \sum_{k=0}^{\infty} \lambda_2^k q_{t-k}^E \right) \\
&\xrightarrow{\text{as } \alpha_1 \to 0} 0 = \widetilde{w}_t^N = \widetilde{q}_t^N
\end{aligned}$$

Thus the market outcome under optimal monetary policy fails to obtain efficiency. Note that under optimal monetary policy we have

$$\frac{\widetilde{y}_{2,t} - x_{2,t}^*}{\widetilde{y}_{1,t} - x_{1,t}^*} = -\frac{z_1}{z_2} \frac{\varphi}{1+\varphi}$$

Rearranging the terms,

$$\varphi z_1 (\widetilde{y}_{1,t} - x_{1,t}^*) + (1+\varphi) z_2 (\widetilde{y}_{2,t} - x_{2,t}^*) = 0 \tag{A.83}$$

It is trivial to prove that flexible price allocation is achievable, thus the latter is sub-optimal.

□

A.10 Proof of Proposition 10

Proof. We solve a Ramsey problem of the utilitarian central bank when the prices of both sectors are sticky to the same degree, $0 < \alpha_1 < \alpha_2 = \alpha$, under **HetCB**. The set-up of Lagrangian is the same as that in the proof of Proposition 11 except that we have $\kappa_1 = \kappa_2 = \kappa$ now:

$$
\begin{aligned}
\mathcal{L}_t = & \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{c} \frac{z_1 \theta}{\kappa_1} \pi_{1,t}^2 + \frac{z_2 \theta}{\kappa_2} \pi_{2,t}^2 \\ + \Gamma_{y_{11}} (\widetilde{y}_{1,t} - x_{1,t}^*)^2 + \Gamma_{y_{12}} (\widetilde{y}_{1,t} - x_{1,t}^*)(\widetilde{y}_{2,t} - x_{2,t}^*) + \Gamma_{y_{22}} (\widetilde{y}_{2,t} - x_{2,t}^*)^2 \end{array} \right] \\
& + E_0 \sum_{t=0}^{\infty} \beta^t \psi_{1,t} \left\{ \pi_{1,t} - \beta \pi_{1,t+1} - \kappa_1 \widetilde{w}_t \right\} \\
& + E_0 \sum_{t=0}^{\infty} \beta^t \psi_{2,t} \left\{ \pi_{2,t} - \beta \pi_{2,t+1} - \kappa_2 \widetilde{w}_t + \kappa_2 \widetilde{q}_t \right\} \\
& + E_0 \sum_{t=0}^{\infty} \beta^t \psi_{3,t} \left\{ \widetilde{q}_t - \widetilde{q}_{t-1} + q_t^E - q_{t-1}^E - \pi_{2,t} + \pi_{1,t} \right\} \\
& + E_0 \sum_{t=0}^{\infty} \beta^t \psi_{4,t} \left\{ z_1 \widetilde{y}_{1,t} + z_2 \widetilde{y}_{2,t} - z_2 \widetilde{n}_{U,t} - z_1 \widetilde{n}_{C,t} \right\} \\
& + E_0 \sum_{t=0}^{\infty} \beta^t \psi_{5,t} \left\{ \varphi \widetilde{n}_{U,t} + \sigma \widetilde{y}_{2,t} - \widetilde{w}_t + \widetilde{q}_t \right\} \\
& + E_0 \sum_{t=0}^{\infty} \beta^t \psi_{6,t} \left\{ \varphi \widetilde{n}_{C,t} + \sigma \widetilde{y}_{1,t} - \widetilde{w}_t \right\} \\
& + E_0 \sum_{t=0}^{\infty} \beta^t \psi_{7,t} \left\{ \widetilde{y}_{1,t} - \widetilde{w}_t - \widetilde{n}_{C,t} + \frac{1-\sigma}{\sigma} z_2 q_t^E \right\}
\end{aligned}
$$

where $\{\psi_{1,t}\}$, $\cdots$, $\{\psi_{7,t}\}$ are the Lagrange multipliers.

First order conditions are as follows:

$$
\begin{aligned}
\partial \pi_{1,t} &: 0 = \frac{z_1 \theta}{\kappa_1} \pi_{1,t} + \psi_{1,t} - \psi_{1,t-1} + \psi_{3,t} \\
\partial \pi_{2,t} &: 0 = \frac{z_2 \theta}{\kappa_1} \pi_{2,t} + \psi_{2,t} - \psi_{2,t-1} - \psi_{3,t} \\
\partial \widetilde{w}_t &: 0 = -\kappa_1 \psi_{1,t} - \kappa_2 \psi_{2,t} - \psi_{5,t} - \psi_{6,t} - \psi_{7,t} \\
\partial \widetilde{q}_t &: 0 = \kappa_2 \psi_{2,t} + \psi_{3,t} - \beta E_t[\psi_{3,t+1}] + \psi_{5,t} \\
\partial \widetilde{y}_{1,t} &: 0 = \Gamma_{y_{11}} (\widetilde{y}_{1,t} - x_{1,t}^*) + \frac{\Gamma_{y_{12}}}{2} (\widetilde{y}_{2,t} - x_{2,t}^*) + z_1 \psi_{4,t} + \sigma \psi_{6,t} + \psi_{7,t} \\
\partial \widetilde{y}_{2,t} &: 0 = \Gamma_{y_{22}} (\widetilde{y}_{2,t} - x_{2,t}^*) + \frac{\Gamma_{y_{12}}}{2} (\widetilde{y}_{1,t} - x_{1,t}^*) + z_2 \psi_{4,t} + \sigma \psi_{5,t} \\
\partial \widetilde{n}_{U,t} &: 0 = -z_2 \psi_{4,t} + \varphi \psi_{5,t} \\
\partial \widetilde{n}_{C,t} &: 0 = -z_1 \psi_{4,t} + \varphi \psi_{6,t} - \psi_{7,t}
\end{aligned}
$$

Rewriting Lagrangian constraints corresponding to $\{\psi_{1,t}\}, \cdots, \{\psi_{3,t}\}$,

$$\pi_{1,t} - \beta E_t[\pi_{1,t+1}] = \kappa \widetilde{w}_t \tag{A.84}$$

$$\pi_{2,t} - \beta E_t[\pi_{2,t+1}] = \kappa \widetilde{w}_t - \kappa \widetilde{q}_t \tag{A.85}$$

$$\widetilde{q}_t - \widetilde{q}_{t-1} + q_t^E - q_{t-1}^E = \pi_{2,t} - \pi_{1,t} \tag{A.86}$$

Subtracting Eq.(A.90) from Eq.(A.85),

$$\pi_{2,t} - \pi_{1,t} = \beta\big(E_t[\pi_{2,t+1}] - E_t[\pi_{1,t+1}]\big) - \kappa \widetilde{q}_t \tag{A.87}$$

Substituting Eq.(A.86) into Eq.(A.87), we derive a second order difference equation:

$$E_t[\widetilde{q}_{t+1}] - \left(1 + \frac{1}{\beta} + \frac{\kappa}{\beta}\right)\widetilde{q}_t + \frac{1}{\beta}\widetilde{q}_{t-1} = \left(\frac{1}{\beta} + 1 - \rho\right) q_t^E - \frac{1}{\beta} q_{t-1}^E$$

Solving the equation,

$$\widetilde{q}_t = -q_t^E + \frac{(\lambda_1 - 1)(1 - \lambda_1)}{\lambda_1 - \rho} \sum_{k=0}^{\infty} \lambda_2^k q_{t-k}^E$$

where the two eigenvalues satisfies $0 < \lambda_2 < 1 < \lambda_1$. The central bank loses control over $\widetilde{q}_t$ if $\alpha_1 = \alpha_2$, because it is affected only by exogenous asymmetric shocks, q_t^E independently from other variables. Note that this is derived by using only Phillips curves in both sectors and the definition of relative price.

Rewriting Eqs.(A.96)-(A.97),

$$z_1\theta\pi_{1,t} + z_2\theta\pi_{2,t} + \left(z_1 + \frac{z_1}{z_2}\frac{\sigma\varphi}{1+\varphi}\right) A(L)(\widetilde{y}_{1,t} - x_{1,t}^*) + z_2 A(L)(\widetilde{y}_{2,t} - x_{2,t}^*) = 0 \tag{A.88}$$

$$\kappa\psi_{3,t} + A(L)\psi_{3,t} - \beta A(L) E_t[\psi_{3,t+1}] = z_2\theta\pi_{2,t} + \frac{z_1\varphi}{1+\varphi} A(L)(\widetilde{y}_{1,t} - x_{1,t}^*) + z_2 A(L)(\widetilde{y}_{2,t} - x_{2,t}^*) \tag{A.89}$$

A targeting rule Eq.(A.88) closes the model, and Eq.(A.89) only determines $\psi_{3,t}$ if $\alpha_1 = \alpha_2$. $\square$

A.11 Proof of Proposition 11

Proof. We solve a Ramsey problem of the utilitarian central bank when the prices of both sectors are sticky, but to different degrees, $0<\alpha_1<\alpha_2$, under **HetCB**. We set up the Lagrangian as:

$$\begin{aligned}
\mathcal{L}_t = \frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t & \left[\begin{array}{c} \frac{z_1\theta}{\kappa_1}\pi_{1,t}^2 + \frac{z_2\theta}{\kappa_2}\pi_{2,t}^2 \\ +\Gamma_{y_{11}}(\widetilde{y}_{1,t}-x^*_{1,t})^2 + \Gamma_{y_{12}}(\widetilde{y}_{1,t}-x^*_{1,t})(\widetilde{y}_{2,t}-x^*_{2,t}) + \Gamma_{y_{22}}(\widetilde{y}_{2,t}-x^*_{2,t})^2 \end{array}\right] \\
&+E_0\sum_{t=0}^{\infty}\beta^t\psi_{1,t}\Big\{\pi_{1,t}-\beta\pi_{1,t+1}-\kappa_1\widetilde{w}_t\Big\} \\
&+E_0\sum_{t=0}^{\infty}\beta^t\psi_{2,t}\Big\{\pi_{2,t}-\beta\pi_{2,t+1}-\kappa_2\widetilde{w}_t+\kappa_2\widetilde{q}_t\Big\} \\
&+E_0\sum_{t=0}^{\infty}\beta^t\psi_{3,t}\Big\{\widetilde{q}_t-\widetilde{q}_{t-1}+q_t^E-q_{t-1}^E-\pi_{2,t}+\pi_{1,t}\Big\} \\
&+E_0\sum_{t=0}^{\infty}\beta^t\psi_{4,t}\Big\{z_1\widetilde{y}_{1,t}+z_2\widetilde{y}_{2,t}-z_2\widetilde{n}_{U,t}-z_1\widetilde{n}_{C,t}\Big\} \\
&+E_0\sum_{t=0}^{\infty}\beta^t\psi_{5,t}\Big\{\varphi\widetilde{n}_{U,t}+\sigma\widetilde{y}_{2,t}-\widetilde{w}_t+\widetilde{q}_t\Big\} \\
&+E_0\sum_{t=0}^{\infty}\beta^t\psi_{6,t}\Big\{\varphi\widetilde{n}_{C,t}+\sigma\widetilde{y}_{1,t}-\widetilde{w}_t\Big\} \\
&+E_0\sum_{t=0}^{\infty}\beta^t\psi_{7,t}\Big\{\widetilde{y}_{1,t}-\widetilde{w}_t-\widetilde{n}_{C,t}+\frac{1-\sigma}{\sigma}z_2q_t^E\Big\}
\end{aligned}$$

where $\{\psi_{1,t}\}$, $\cdots$, $\{\psi_{7,t}\}$ are the Lagrange multipliers.

First order conditions are as follows:

$$\begin{aligned}
\partial\pi_{1,t} &: 0 = \frac{z_1\theta}{\kappa_1}\pi_{1,t}+\psi_{1,t}-\psi_{1,t-1}+\psi_{3,t} \\
\partial\pi_{2,t} &: 0 = \frac{z_2\theta}{\kappa_1}\pi_{2,t}+\psi_{2,t}-\psi_{2,t-1}-\psi_{3,t} \\
\partial\widetilde{w}_t &: 0 = -\kappa_1\psi_{1,t}-\kappa_2\psi_{2,t}-\psi_{5,t}-\psi_{6,t}-\psi_{7,t} \\
\partial\widetilde{q}_t &: 0 = \kappa_2\psi_{2,t}+\psi_{3,t}-\beta E_t[\psi_{3,t+1}]+\psi_{5,t} \\
\partial\widetilde{y}_{1,t} &: 0 = \Gamma_{y_{11}}(\widetilde{y}_{1,t}-x^*_{1,t})+\frac{\Gamma_{y_{12}}}{2}(\widetilde{y}_{2,t}-x^*_{2,t})+z_1\psi_{4,t}+\sigma\psi_{6,t}+\psi_{7,t} \\
\partial\widetilde{y}_{2,t} &: 0 = \Gamma_{y_{22}}(\widetilde{y}_{2,t}-x^*_{2,t})+\frac{\Gamma_{y_{12}}}{2}(\widetilde{y}_{1,t}-x^*_{1,t})+z_2\psi_{4,t}+\sigma\psi_{5,t} \\
\partial\widetilde{n}_{U,t} &: 0 = -z_2\psi_{4,t}+\varphi\psi_{5,t} \\
\partial\widetilde{n}_{C,t} &: 0 = -z_1\psi_{4,t}+\varphi\psi_{6,t}-\psi_{7,t}
\end{aligned}$$

Simplifying first order conditions, they reduce down to four equations where $A(L) \equiv 1 - L$:

$$0 = z_1\theta\pi_{1,t} + \kappa_1 A(L)\psi_{1,t} + \kappa_1\psi_{3,t} \tag{A.90}$$

$$0 = z_2\theta\pi_{2,t} + \kappa_2 A(L)\psi_{2,t} - \kappa_2\psi_{3,t}) \tag{A.91}$$

$$\begin{aligned} 0 = \kappa_1\psi_{1,t} + \kappa_2\psi_{2,t} - \left(\frac{z_2}{\varphi} - z_1\right)\frac{\varphi}{z_2(\sigma+\varphi)}\left[\frac{\Gamma_{y_{12}}}{2}(\widetilde{y}_{1,t} - x^*_{1,t}) + \Gamma_{y_{22}}(\widetilde{y}_{2,t} - x^*_{2,t})\right] \\ - \frac{1+\varphi}{\sigma+\varphi}\left[\Gamma_{y_{11}}(\widetilde{y}_{1,t} - x^*_{1,t}) + \frac{\Gamma_{y_{22}}}{2}(\widetilde{y}_{2,t} - x^*_{2,t})\right] \end{aligned} \tag{A.92}$$

$$0 = \kappa_2\psi_{2,t} + \psi_{3,t} - \beta E_t[\psi_{3,t+1}] - \frac{1}{\sigma+\varphi}\left[\frac{\Gamma_{y_{12}}}{2}(\widetilde{y}_{1,t} - x^*_{1,t}) + \Gamma_{y_{22}}(\widetilde{y}_{2,t} - x^*_{2,t})\right] \tag{A.93}$$

Pre-multiplying Eqs.(A.92)-(A.93) by $A(L)$, and substituting Eqs.(A.90)-(A.91) into them,

$$0 = z_1\theta\pi_{1,t} + z_2\theta\pi_{2,t} + \kappa_1\psi_{3,t} - \kappa_2\psi_{3,t} + \left(z_1 + \frac{z_1}{z_2}\frac{\sigma\varphi}{1+\varphi}\right)A(L)(\widetilde{y}_{1,t} - x^*_{1,t}) + z_2 A(L)(\widetilde{y}_{2,t} - x^*_{2,t}) \tag{A.94}$$

$$0 = -z_2\theta\pi_{2,t} + \kappa_2\psi_{3,t} + A(L)\psi_{3,t} - \beta A(L)E_t[\psi_{3,t+1}] - \frac{z_1\varphi}{1+\varphi}A(L)(\widetilde{y}_{1,t} - x^*_{1,t}) - z_2 A(L)(\widetilde{y}_{2,t} - x^*_{2,t}) \tag{A.95}$$

Simplifying further,

$$\psi_{3,t} = \frac{1}{\kappa_2 - \kappa_1}\left[z_1\theta\pi_{1,t} + z_2\theta\pi_{2,t} + \left(z_1 + \frac{z_1}{z_2}\frac{\sigma\varphi}{1+\varphi}\right)A(L)(\widetilde{y}_{1,t} - x^*_{1,t}) + z_2 A(L)(\widetilde{y}_{2,t} - x^*_{2,t})\right] \tag{A.96}$$

$$\kappa_2\psi_{3,t} + A(L)\psi_{3,t} - \beta A(L)E_t[\psi_{3,t+1}] = z_2\theta\pi_{2,t} + \frac{z_1\varphi}{1+\varphi}A(L)(\widetilde{y}_{1,t} - x^*_{1,t}) + z_2 A(L)(\widetilde{y}_{2,t} - x^*_{2,t}) \tag{A.97}$$

Substituting Eq.(A.96) into Eq.(A.97), we derive a targeting rule

$$\begin{aligned} &\frac{\kappa_2}{\kappa_2 - \kappa_1}\left[z_1\theta\pi_{1,t} + z_2\theta\pi_{2,t} + \left(z_1 + \frac{z_1}{z_2}\frac{\sigma\varphi}{1+\varphi}\right)A(L)(\widetilde{y}_{1,t} - x^*_{1,t}) + z_2 A(L)(\widetilde{y}_{2,t} - x^*_{2,t})\right] \\ &+ \frac{1}{\kappa_2 - \kappa_1}\left[\begin{array}{c} \left[z_1\theta\pi_{1,t} + z_2\theta\pi_{2,t} + \left(z_1 + \frac{z_1}{z_2}\frac{\sigma\varphi}{1+\varphi}\right)A(L)(\widetilde{y}_{1,t} - x^*_{1,t}) + z_2 A(L)(\widetilde{y}_{2,t} - x^*_{2,t})\right] \\ -\left[z_1\theta\pi_{1,t-1} + z_2\theta\pi_{2,t-1} + \left(z_1 + \frac{z_1}{z_2}\frac{\sigma\varphi}{1+\varphi}\right)A(L)(\widetilde{y}_{1,t-1} - x^*_{1,t-1}) + z_2 A(L)(\widetilde{y}_{2,t-1} - x^*_{2,t-1})\right] \end{array}\right] \\ &- \frac{\beta}{\kappa_2 - \kappa_1}\left[\begin{array}{c} E_t\left[z_1\theta\pi_{1,t+1} + z_2\theta\pi_{2,t+1} + \left(z_1 + \frac{z_1}{z_2}\frac{\sigma\varphi}{1+\varphi}\right)A(L)(\widetilde{y}_{1,t+1} - x^*_{1,t+1}) + z_2 A(L)(\widetilde{y}_{2,t+1} - x^*_{2,t+1})\right] \\ -E_{t-1}\left[z_1\theta\pi_{1,t} + z_2\theta\pi_{2,t} + \left(z_1 + \frac{z_1}{z_2}\frac{\sigma\varphi}{1+\varphi}\right)A(L)(\widetilde{y}_{1,t} - x^*_{1,t}) + z_2 A(L)(\widetilde{y}_{2,t} - x^*_{2,t})\right] \end{array}\right] \\ &= z_2\theta\pi_{2,t} + \frac{z_1\varphi}{1+\varphi}A(L)(\widetilde{y}_{1,t} - x^*_{1,t}) + z_2 A(L)(\widetilde{y}_{2,t} - x^*_{2,t}) \end{aligned}$$

$\square$

B Heterogeneous Consumption Baskets

We provide the system of equations and some derivations of the equilibrium in the efficient allocation and the decentralized model under **HetCB** ($\omega_U = 0,\ \omega_C = 1$).

B.1 Efficient Allocation

We derive the economy's efficient allocation by solving a social planner's problem that maximizes the weighted sum of utility of both types of households, subject to the resource and technology constraints

$$\max_{\{C_{h,t}, N_{h,t}, Y_{j,t}(i)\}} \left\{ \varpi_U (1-\lambda) \left[\frac{C_{U,t}^{1-\sigma}}{1-\sigma} - \frac{N_{U,t}^{1+\varphi}}{1+\varphi} \right] + \varpi_C \lambda \left[\frac{C_{C,t}^{1-\sigma}}{1-\sigma} - \frac{N_{C,t}^{1+\varphi}}{1+\varphi} \right] \right\}$$

$$\begin{aligned} s.t. \qquad \lambda C_{C,t} &= \left(\int_{\mathcal{I}_1} \left(\frac{1}{z_1} \right)^{\frac{1}{\theta}} Y_{1,t}(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} \\ (1-\lambda) C_{U,t} &= \left(\int_{\mathcal{I}_2} \left(\frac{1}{z_2} \right)^{\frac{1}{\theta}} Y_{2,t}(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} \\ (1-\lambda) N_{U,t} + \lambda N_{C,t} &= \int_{\mathcal{I}_1} \frac{Y_{1,t}(i)}{A_t A_{1,t}} di + \int_{\mathcal{I}_2} \frac{Y_{2,t}(i)}{A_t A_{2,t}} di \end{aligned}$$

where $\{\varpi_h\}$ denotes Pareto weights. First order conditions with respect to $C_{h,t}$, $N_{h,t}$, and $Y_{j,t}(i)$ are given by

$$\varpi_C C_{C,t}^{-\sigma} = \mu_1 \tag{B.1}$$

$$\varpi_U C_{U,t}^{-\sigma} = \mu_2 \tag{B.2}$$

$$\varpi_C N_{C,t}^{\varphi} = \mu_3 \tag{B.3}$$

$$\varpi_U N_{U,t}^{\varphi} = \mu_3 \tag{B.4}$$

$$\mu_1 Y_{1,t}^{\frac{1}{\theta}}\ z_1^{-\frac{1}{\theta}} Y_{1,t}(i)^{-\frac{1}{\theta}} = \mu_3 \frac{1}{A_t A_{1,t}} \tag{B.5}$$

$$\mu_2 Y_{2,t}^{\frac{1}{\theta}}\ z_2^{-\frac{1}{\theta}} Y_{2,t}(i)^{-\frac{1}{\theta}} = \mu_3 \frac{1}{A_t A_{2,t}} \tag{B.6}$$

where μ_1, μ_2 and μ_3 are Lagrange multipliers. According to the last two conditions, $Y_{j,t}(i)$ should have a common value, $Y_{j,t}(i) = \frac{Y_{j,t}}{z_j}$, implying no output dispersion within sector in the efficient allocation.

Simplifying further, the efficient allocation is characterized by

$$N_{C,t}^{E\ \varphi} = C_{C,t}^{E\ -\sigma} A_t A_{1,t} \tag{B.7}$$

$$N_{U,t}^{E\ \varphi} = C_{U,t}^{E\ -\sigma} A_t A_{2,t} \tag{B.8}$$

$$\frac{N_{C,t}^{E}}{N_{U,t}^{E}} = \left(\frac{\varpi_C}{\varpi_U}\right)^{-\varphi} \tag{B.9}$$

$$\lambda C_{C,t}^{E} = Y_{1,t}^{E} \tag{B.10}$$

$$(1-\lambda) C_{U,t}^{E} = Y_{2,t}^{E} \tag{B.11}$$

$$(1-\lambda) N_{U,t}^{E} + \lambda N_{C,t}^{E} = \frac{Y_{1,t}^{E}}{A_t A_{1,t}} + \frac{Y_{2,t}^{E}}{A_t A_{2,t}} \tag{B.12}$$

where E stands for "*Efficient*".

Since the efficient allocation is affected by relative Pareto weights, $\frac{\varpi_C}{\varpi_U}$, we assume that a social planner is utilitarian ($\varpi_U = \varpi_C$). Then, the log-linearized system of equations of the efficient allocation around the deterministic efficient zero-inflation steady state is given by

$$n_{C,t}^{E} = n_{U,t}^{E} (\equiv n_t^{E}) \tag{B.13}$$

$$\varphi n_{C,t}^{E} + \sigma c_{C,t}^{E} = a_t + a_{1,t} \tag{B.14}$$

$$\varphi n_{U,t}^{E} + \sigma c_{U,t}^{E} = a_t + a_{2,t} \tag{B.15}$$

$$c_{C,t}^{E} = y_{1,t}^{E} \tag{B.16}$$

$$c_{U,t}^{E} = y_{2,t}^{E} \tag{B.17}$$

$$n_t^{E} = z_1 (y_{1,t}^{E} - a_t - a_{1,t}) + z_2 (y_{2,t}^{E} - a_t - a_{2,t}) \tag{B.18}$$

The dynamics of variables expressed in terms of exogenous processes are given by

$$n_t^E = n_{C,t}^E = n_{U,t}^E = \underbrace{\frac{1-\sigma}{\sigma+\varphi}}_{+/-}\left(a_t + n_1 a_{1,t} + n_2 a_{2,t}\right) \tag{B.19}$$

$$y_{1,t}^E = c_{C,t}^E = \underbrace{\left(\frac{1}{\sigma} - \frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}\right)}_{+/+} a_t \underbrace{+\left(\frac{1}{\sigma} - \frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}n_1\right)}_{+/+} a_{1,t} \underbrace{-\frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}n_2}_{-/+} a_{2,t} \tag{B.20}$$

$$y_{2,t}^E = c_{U,t}^E = \underbrace{\left(\frac{1}{\sigma} - \frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}\right)}_{+/+} a_t \underbrace{-\frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}n_1}_{-/+} a_{1,t} \underbrace{+\left(\frac{1}{\sigma} - \frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}n_2\right)}_{+/+} a_{2,t} \tag{B.21}$$

$$n_{1,t}^E = \underbrace{\left(\frac{1-\sigma}{\sigma} - \frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}\right)}_{+/-} a_t \underbrace{+\left(\frac{1-\sigma}{\sigma} - \frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}n_1\right)}_{+/-} a_{1,t} \underbrace{-\frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}n_2}_{-/+} a_{2,t} \tag{B.22}$$

$$n_{2,t}^E = \underbrace{\left(\frac{1-\sigma}{\sigma} - \frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}\right)}_{+/-} a_t \underbrace{-\frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}n_1}_{-/+} a_{1,t} + \underbrace{\left(\frac{1-\sigma}{\sigma} - \frac{\varphi}{\sigma}\frac{1-\sigma}{\sigma+\varphi}n_2\right)}_{+/-} a_{2,t} \tag{B.23}$$

where the signs are when $\sigma < 1$ and $\sigma > 1$, respectively. The implied wage and relative price are derived as $w_t^E = a_t + a_{1,t}$ and $q_t^E = a_{1,t} - a_{2,t}$, so we identify heterogeneous real wages, $w_{C,t}^E (= w_t^E) = a_t + a_{1,t}$ and $w_{U,t}^E (= w_t^E - q_t^E) = a_t + a_{2,t}$, in the efficient allocation.

B.1.1 Steady State

By assuming $A = A_1 = A_2 = 1$, we have symmetric steady state as follows.

$$C_C = C_U = N_C = N_U = 1$$
$$Y_1 = N_1 = \lambda \text{ and } Y_2 = N_2 = 1 - \lambda$$

B.2 Sticky-Price Allocation

We present the system of equations that characterize the first-order approximation of the equilibrium of the model under sticky-price.

- Consumption baskets (Goods market clearing condition)

$$\widetilde{c}_{C,t} = \widetilde{y}_{1,t} \tag{B.24}$$
$$\widetilde{c}_{U,t} = \widetilde{y}_{2,t} \tag{B.25}$$

- Euler equation

$$\widetilde{y}_{2,t} - E_t[\widetilde{y}_{2,t+1}] = -\frac{1}{\sigma}\left(\widetilde{i}_t - E_t[\pi_{2,t+1}] - r_t^E\right) \tag{B.26}$$

where $r_t^E \equiv \sigma(E_t[y_{2,t+1}^E] - y_{2,t}^E)$

- Labor supply schedule of type U households

$$\varphi\widetilde{n}_{U,t} + \sigma\widetilde{y}_{2,t} = \widetilde{w}_t - \widetilde{q}_t \tag{B.27}$$

- Labor supply schedule of type C households

$$\varphi\widetilde{n}_{C,t} + \sigma\widetilde{y}_{1,t} = \widetilde{w}_t \tag{B.28}$$

- Budget constraint of type C households

$$\widetilde{w}_t + \widetilde{n}_{C,t} = \widetilde{y}_{1,t} + \frac{1-\sigma}{\sigma} z_2 q_t^E \tag{B.29}$$

- Labor market clearing condition

$$z_1\widetilde{y}_{1,t} + z_2\widetilde{y}_{2,t} = z_1\widetilde{n}_{C,t} + z_2\widetilde{n}_{U,t} \tag{B.30}$$

- Phillips curve in sector 1

$$\pi_{1,t} = \beta E_t[\pi_{1,t+1}] + \frac{(1-\alpha_1\beta)(1-\alpha_1)}{\alpha_1}\widetilde{w}_t \tag{B.31}$$

- Phillips curve in sector 2

$$\pi_{2,t} = \beta E_t[\pi_{2,t+1}] + \frac{(1-\alpha_2\beta)(1-\alpha_2)}{\alpha_2}\left(\widetilde{w}_t - \widetilde{q}_t\right) \tag{B.32}$$

- Real marginal cost in sector 1

$$\widetilde{w}_t = \frac{\sigma+\varphi}{1+\varphi}\widetilde{y}_{1,t} + z_2\frac{\varphi}{1+\varphi}\frac{1-\sigma}{\sigma}q_t^E \tag{B.33}$$

- Real marginal cost in sector 2

$$\widetilde{w}_t - \widetilde{q}_t = \varphi\frac{z_1}{z_2}\frac{\sigma+\varphi}{1+\varphi}\widetilde{y}_{1,t} + (\sigma+\varphi)\widetilde{y}_{2,t} - z_1\frac{\varphi}{1+\varphi}\frac{1-\sigma}{\sigma}q_t^E \tag{B.34}$$

- Relative price

$$\widetilde{q}_t - \widetilde{q}_{t-1} + q_t^E - q_{t-1}^E = \pi_{2,t} - \pi_{1,t} \tag{B.35}$$

- Monetary policy

$$\widetilde{i}_t = \phi_{\pi_1}\pi_{1,t} + \phi_{\pi_2}\pi_{2,t} + \phi_{y_1}y_{1,t} + \phi_{y_2}y_{2,t} + \nu_t \tag{B.36}$$

- Exogenous processes

$$a_t = \rho_a a_{t-1} + \sigma_a \varepsilon_t^a \tag{B.37}$$

$$a_{1,t} = \rho_{a_1} a_{1,t-1} + \sigma_{a_1} \varepsilon_t^{a_1} \tag{B.38}$$

$$a_{2,t} = \rho_{a_2} a_{2,t-1} + \sigma_{a_2} \varepsilon_t^{a_2} \tag{B.39}$$

$$\nu_t = \rho_\nu a_{t-1} + \sigma_\nu \varepsilon_t^\nu \tag{B.40}$$

B.2.1 Steady State

Steady state is symmetric due to fiscal specifications. Note that the steady state is efficient.

$$\begin{aligned}
&C_C = C_U = N_C = N_U = A = A_1 = A_2 = 1 \\
&Y_1 = N_1 = \lambda \text{ and } \; Y_2 = N_2 = 1-\lambda \\
&W = Q = 1 \\
&D = \frac{1}{\theta} \text{ and } \; T_U = -\frac{1}{\theta(1-\lambda)}
\end{aligned}$$

B.3 Flexible-Price Allocation

The system of equations of the equilibrium of the model under flexible-price is the same except that sectoral Phillips curves are replaced with constant markup or zero real marginal cost gap, $\widetilde{w}_t^N = \widetilde{w}_t^N - \widetilde{q}_t^N = 0$, where N stands for *natural* or flexible-price allocation. Thus we present the first-order approximation to the solutions under flexible-price as functions of exogenous processes or $q_t^E = a_{1,t} - a_{2,t}$.

$$\widetilde{y}_{1,t}^N = -\frac{1-\sigma}{\sigma} \frac{\varphi}{\sigma+\varphi} z_2 q_t^E \tag{B.41}$$

$$\widetilde{y}_{2,t}^N = \frac{1-\sigma}{\sigma} \frac{\varphi}{\sigma+\varphi} z_1 q_t^E \tag{B.42}$$

$$\widetilde{n}_{C,t}^N = \frac{1-\sigma}{\sigma+\varphi} z_2 q_t^E \tag{B.43}$$

$$\widetilde{n}_{U,t}^N = -\frac{1-\sigma}{\sigma+\varphi} z_1 q_t^E \tag{B.44}$$

$$\widetilde{w}_t^N = 0 \tag{B.45}$$

$$\widetilde{q}_t^N = 0 \tag{B.46}$$

B.3.1 Steady State

Steady state is the same as in the model sticky-price, and thus efficient.

$$
\begin{aligned}
&C_C = C_U = N_C = N_U = A = A_1 = A_2 = 1 \\
&Y_1 = N_1 = \lambda \text{ and } \ Y_2 = N_2 = 1-\lambda \\
&W = Q = 1 \\
&D = \frac{1}{\theta} \text{ and } \ T_U = -\frac{1}{\theta(1-\lambda)}
\end{aligned}
$$

B.4 Asymmetric redistribution of inflationary pressure

The effects of sectoral output gaps and adjustment terms on dynamics of sectoral inflation are asymmetric as shown in the Phillips curves rewritten in terms of sectoral output gaps:(1) inflation in sector 1 is affected only by output gap 1, while (2) inflation in sector 2 is affected by both output gaps; (3) a relative productivity shock q_t^E has the opposite consequences in each sector. (1) and (2) imply the redistribution of inflationary pressure across sectors as the labor demand is redistributed across households, and (3) is due to the lack of risk-sharing. We analyze the asymmetry in inflation dynamics in order.

$$
\begin{aligned}
\pi_{1,t} &= \beta E_t[\pi_{1,t+1}] + \kappa_1 \left(\frac{\sigma+\varphi}{1+\varphi} \widetilde{y}_{1,t} + z_2 \frac{\varphi}{1+\varphi} \frac{1-\sigma}{\sigma} q_t^E \right) \\
\pi_{2,t} &= \beta E_t[\pi_{2,t+1}] + \kappa_2 \left(\frac{z_1}{z_2} \varphi \frac{\sigma+\varphi}{1+\varphi} \widetilde{y}_{1,t} + (\sigma+\varphi) \widetilde{y}_{2,t} - z_1 \frac{\varphi}{1+\varphi} \frac{1-\sigma}{\sigma} q_t^E \right)
\end{aligned}
$$

The real wages in the Phillips curves are those in equilibrium that explain how demand or output gaps affect marginal costs through the labor market and eventually inflation. Thus the relations of real wages and outputs in equilibrium are derived by labor supply relations and budget constraints of households, goods market clearing conditions, labor market clearing condition and production function.

We begin with sector 1 in which the real marginal cost coincides with type C households' real wage gap, $\widetilde{w}_t$. According to type C households' labor supply schedule, their real wage is the ratio between marginal disutility of labor supply and marginal utility of consumption, or in log, real wage is marginal disutility of labor supply less marginal utility of consumption in equilibrium. Since their labor supply has a perfect correlation with their consumption, $n_{C,t} = \frac{1-\sigma}{1+\varphi} y_{1,t}$ considering the redistribution of labor demand by $\frac{\sigma+\varphi}{1+\varphi} y_{1,t}$, we find that the real wage w_t is associated only with output 1. This is expressed in gaps by

$$\begin{aligned}\widetilde{w}_t &= \varphi\widetilde{n}_{C,t} - (-\sigma\widetilde{c}_{C,t}) = \varphi\left(1 - \frac{\sigma+\varphi}{1+\varphi}\right)\widetilde{y}_{1,t} + \sigma\widetilde{y}_{1,t} + (\textit{adjustment term})\\ &= \frac{\sigma+\varphi}{1+\varphi}\widetilde{y}_{1,t} + (\textit{adjustment term})\end{aligned}$$

Next, consider sector 2 in which the real marginal cost coincides with type U households' real wage gap, $\widetilde{w}_t - \widetilde{q}_t$. Analogously, type U households' real wage is their marginal disutility of labor supply less marginal utility of consumption. Since labor demand is redistributed from type C to type U, labor supply of the latter is affected by type C households consumption, $y_{1,t}$, as well as their own consumption, $y_{2,t}$. Thus the real wage $w_t - q_t$ is associated with both output 1 and 2.This is expressed in gaps by

$$\begin{aligned}\widetilde{w}_t - \widetilde{q}_t &= \varphi\widetilde{n}_{U,t} - (-\sigma\widetilde{c}_{U,t}) = \varphi\big(\widetilde{y}_{2,t} - \frac{z_1}{z_2}(\widetilde{n}_{C,t} - \widetilde{y}_{1,t})\big) - (-\sigma c_{U,t})\\ &= \frac{z_1}{z_2}\varphi\frac{\sigma+\varphi}{1+\varphi}\widetilde{y}_{1,t} + (\sigma+\varphi)\widetilde{y}_{2,t} + (\textit{adjustment term})\end{aligned}$$

Suppose an increase in output gap 1. On the one hand, HtM households' real wage should increase to support a higher consumption in equilibrium, which is in turn associated with an increase in labor hours assuming $\sigma < 1$. As marginal utility of consumption decreases and marginal disutility of labor increases, their real wage would increase in equilibrium, leading to higher inflation in sector 1. On the other hand, type U households' labor hours also increase as labor demand is redistributed. This raises their marginal disutility of labor and real wage in equilibrium, with inflation in sector 2 also increasing. Now suppose an increase in output gap 2. Since HtM households are not affected by sector 2, and the labor demanded by sector 2 is filled, in effect, by type U households, their marginal disutility of labor increases, inducing inflation in sector 2 to rise.

We summarize the analysis as follows:

- Each output gap poses inflationary pressure by a factor "$\sigma+\varphi$": φ and $-\sigma$ reflects marginal disutility of labor gap and marginal utility of consumption gap, with the difference between them being the real wage gap in equilibrium, through which inflationary pressure is created in each sector.

- (*Marginal utility of consumption channel: "σ"*) An increase in each sectoral output gap lowers marginal utility of consumption gap of households who consume goods from that sector intensively, creating inflationary pressure on its own sector: output gap 1 (output gap 2) affects type C (type U) households' marginal utility of consumption gap creating inflationary pressure on sector 1 (sector 2) by a factor σ.[65]

[65]Unlike the marginal disutility of labor supply channel, there is no redistibution of inflationary pressure,

- (*Marginal disutility of Labor supply channel: "φ"*) Since outputs are produced by labor hours that is a source of disutility, an increase in each sectoral output gap can raise marginal disutility of labor supply of each type of households creating inflationary pressure. How much each inflationary pressure is distributed to each sector is determined by how labor demanded by each sector is distributed to each type of household. As labor demand is redistributed from sector 1 to sector 2 by $\frac{\sigma+\varphi}{1+\varphi}y_{1,t}$, inflationary pressure is also redistributed by a factor φ adjusted by sector size.

Lastly, note that inefficient distribution of inflation occurs, which is represented by the adjustment terms in the Phillips curves that show up as a result of the impossibility. They are similar to cost-push shocks in that they add stochasticity to inflation dynamics even under zero output gaps and hence divine coincidence no longer holds, but different in that the former always disappears as we aggregate sectoral inflation with the economic size of each sector, because they put inflationary pressure on each sector in the opposite direction but in the same size as much as the consequences of the lack of risk-sharing (before adjusted for sectoral size). Let us take as an example the case of a positive shock on sector-specific productivity $a_{1,t}$ as seen in Section 2.5.3 and Section 2.5.4. Due to financial constraints, type C households have to work more and type U households have to work less than under efficient allocation. Since marginal disutility of labor supply gap is higher (lower) for type C (type U) households, their real wage gap that equals to real marginal cost, $\widetilde{w}_t$ ($\widetilde{w}_t - \widetilde{q}_t$), and inflation in the sector of consumption, $\pi_{1,t}$ ($\pi_{2,t}$), are higher (lower) in equilibrium in the absence of risk-sharing, implying that inefficient distribution of labor supply translates to inefficient distribution of inflationary pressure across sectors. As a result, inflation dynamics in both sectors are amplified if $\sigma < 1$, or subdued if $\sigma > 1$, considering that the shock leads to a negative output gap in sector 1 and a positive output gap in sector 2 due to nominal rigidity.

B.5 Wage Elasticity of Labor Hours

Using the example of a household that makes a static decision on consumption and labor supply given the wage with utility function and budget constraint below in Section 3.3.1, we derive the wage elasticity of labor hours as below:

$$\varepsilon_{N,W} \equiv \frac{\partial N}{\partial W}\frac{W}{N} = \frac{1-\sigma(\frac{WN+\frac{\partial M}{\partial W}W}{WN+M})}{\varphi+\sigma(\frac{WN}{WN+M})} = \frac{1-\sigma(\frac{WN+\varepsilon_{M,W}M}{WN+M})}{\varphi+\sigma(\frac{WN}{WN+M})}$$

Since HtM households depend entirely on wage income ($M=0,\ \varepsilon_{M,W} < 0$), their wage elasticity

because consumption sectors of each type of household is completely differennt. But if we introduce the general case of heterogeneous consumption baskets where households have common share of consumption, there will be redistribution of inflationary pressure across households depending on who consumes more intensively.

of labor hours would be $\frac{1-\sigma}{\varphi+\sigma}$. However, the unconstrained households have other sources of income, dividend, which is countercyclical ($M > 0,\ \varepsilon_{M,W} < 0$). If $\sigma > \frac{WN+M}{WN+\varepsilon_{M,W}M}(> 1)$, the unconstrained households' wage elasticity of labor hours would be smaller in absolute terms than that of HtM ($|\varepsilon_{N,W,\text{type C}}| = |\frac{1-\sigma}{\varphi+\sigma}| > |\varepsilon_{N,W,\text{type U}}|$). If not, wage elasticity of labor hours is higher for the unconstrained households. But in this case, consumption volatility gets more important as labor hours gets relatively less volatile than that of consumption with $\varepsilon_{C,W} > \varepsilon_{N,W}$.

C Homogeneous Consumption Baskets

We provide the system of equations and some derivations of the equilibrium in the efficient allocation and the decentralized model under **HomCB** ($\omega_U = \omega_C = \frac{1}{2}$).

C.1 Efficient Allocation

As both types of households are of the same preference consuming homogeneous consumption baskets and under the same economic constraints, the first-best is that consumption and labor supply are equalized across all the households as if there is a representative household:

$$c_t^E \equiv c_{C,t}^E = c_{U,t}^E \tag{C.1}$$

$$c_{1,t}^E \equiv c_{C,1,t}^E = c_{U,1,t}^E \tag{C.2}$$

$$c_{2,t}^E \equiv c_{C,2,t}^E = c_{U,2,t}^E \tag{C.3}$$

$$n_t^E \equiv n_{C,t}^E = n_{U,t}^E \tag{C.4}$$

The log-linearized system of equations of the efficient allocation around the deterministic efficient zero-inflation steady state is given by

$$\omega(c_{1,t}^E - c_t^E) + (1-\omega)(c_{2,t}^E - c_t^E) = 0 \tag{C.5}$$

$$\varphi n_t^E + \sigma c_t^E + \frac{1}{\eta}(c_{1,t}^E - c_t^E) = a_t + a_{1,t} \tag{C.6}$$

$$\varphi n_t^E + \sigma c_t^E + \frac{1}{\eta}(c_{2,t}^E - c_t^E) = a_t + a_{2,t} \tag{C.7}$$

$$n_t^E + a_t + \omega a_{1,t} + (1-\omega)a_{2,t} = c_t^E \tag{C.8}$$

The dynamics of variables expressed in terms of exogenous processes are given by

$$c_t^E = \frac{1+\varphi}{\sigma+\varphi}\big(a_t + \omega a_{1,t} + (1-\omega)a_{2,t}\big) \tag{C.9}$$

$$c_{1,t}^E = c_t^E + (1-\omega)\eta(a_{1,t} - a_{2,t}) \tag{C.10}$$

$$c_{2,t}^E = c_t^E - \omega\eta(a_{1,t} - a_{2,t}) \tag{C.11}$$

$$n_t^E = \frac{1-\sigma}{\sigma+\varphi}\big(a_t + \omega a_{1,t} + (1-\omega)a_{2,t}\big) \tag{C.12}$$

C.1.1 Steady State

We assume $A = A_1 = A_2 = 1$, and have symmetric steady state as follows.

$$C_C = C_U = N_C = N_U = 1$$
$$Y_1 = N_1 = C_1 = C_{C,1} = C_{U,1} = \omega$$
$$Y_2 = N_2 = C_2 = C_{C,2} = C_{U,2} = 1 - \omega$$

C.2 Sticky-Price Allocation

We present the system of equations that characterize the first-order approximation of the equilibrium of the model under sticky-price.

- Euler equation

$$\widetilde{c}_{U,t} - E_t[\widetilde{c}_{U,t+1}] = -\frac{1}{\sigma}\left(\widetilde{i}_t - (\omega E_t[\pi_{1,t+1}] + (1-\omega) E_t[\pi_{2,t+1}]) - r_t^E\right) \tag{C.13}$$

  where $r_t^E \equiv \sigma(E_t[c_{U,t+1}^E] - c_{U,t}^E)$

- Labor supply schedule of type U households

$$\varphi \widetilde{n}_{U,t} + \sigma \widetilde{c}_{U,t} = \widetilde{w}_t - (1-\omega)\widetilde{q}_t \tag{C.14}$$

- Labor supply schedule of type C households

$$\varphi \widetilde{n}_{C,t} + \sigma \widetilde{y}_{1,t} = \widetilde{w}_t - (1-\omega)\widetilde{q}_t \tag{C.15}$$

- Budget constraint of type C households

$$\widetilde{w}_t - (1-\omega)\widetilde{q}_t + \widetilde{n}_{C,t} = \widetilde{c}_{C,t} \tag{C.16}$$

- Labor market clearing condition

$$\omega \widetilde{y}_{1,t} + (1-\omega)\widetilde{y}_{2,t} = \lambda \widetilde{n}_{C,t} + (1-\lambda)\widetilde{n}_{U,t} \tag{C.17}$$

- Phillips curve in sector 1

$$\pi_{1,t} = \beta E_t[\pi_{1,t+1}] + \frac{(1-\alpha_1\beta)(1-\alpha_1)}{\alpha_1}\widetilde{w}_t \tag{C.18}$$

- Phillips curve in sector 2

$$\pi_{2,t} = \beta E_t[\pi_{2,t+1}] + \frac{(1-\alpha_2\beta)(1-\alpha_2)}{\alpha_2}\big(\widetilde{w}_t - \widetilde{q}_t\big) \tag{C.19}$$

- Real marginal cost in sector 1

$$\widetilde{w}_t = (\sigma+\varphi)\widetilde{y}_t + (1-\omega)\widetilde{q}_t \tag{C.20}$$

- Real marginal cost in sector 2

$$\widetilde{w}_t - \widetilde{q}_t = (\sigma+\varphi)\widetilde{y}_t - \omega\widetilde{q}_t \tag{C.21}$$

- Relative price

$$\widetilde{q}_t - \widetilde{q}_{t-1} + q_t^E - q_{t-1}^E = \pi_{2,t} - \pi_{1,t} \tag{C.22}$$

- Monetary policy

$$\widetilde{i}_t = \phi_{\pi_1}\pi_{1,t} + \phi_{\pi_2}\pi_{2,t} + \phi_{y_1}y_{1,t} + \phi_{y_2}y_{2,t} + \nu_t \tag{C.23}$$

- Exogenous processes

$$a_t = \rho_a a_{t-1} + \sigma_a \varepsilon_t^a \tag{C.24}$$

$$a_{1,t} = \rho_{a_1} a_{1,t-1} + \sigma_{a_1} \varepsilon_t^{a_1} \tag{C.25}$$

$$a_{2,t} = \rho_{a_2} a_{2,t-1} + \sigma_{a_2} \varepsilon_t^{a_2} \tag{C.26}$$

$$\nu_t = \rho_\nu a_{t-1} + \sigma_\nu \varepsilon_t^\nu \tag{C.27}$$

C.2.1 Steady State

Steady state is symmetric due to fiscal specifications. Note that the steady state is efficient.

$$\begin{aligned}
&C_C = C_U = N_C = N_U = A = A_1 = A_2 = 1 \\
&Y_1 = N_1 = C_1 = C_{C,1} = C_{U,1} = \omega \\
&Y_2 = N_2 = C_2 = C_{C,2} = C_{U,2} = 1-\omega \\
&W = Q = 1 \\
&D = \frac{1}{\theta} \text{ and } T_U = -\frac{1}{\theta(1-\lambda)}
\end{aligned}$$

C.3 Flexible-Price Allocation

The system of equations of the equilibrium of the model under flexible-price is the same except that sectoral Phillips curves are replaced with constant markup or zero real marginal cost gap, $\widetilde{w}_t^N = \widetilde{w}_t^N - \widetilde{q}_t^N = 0$, where N stands for *natural* or flexible-price allocation. Unlike the **HetCB** case, flexible-price allocation under **HomCB** achieves the efficient allocation closing both output gaps and labor supply gaps.

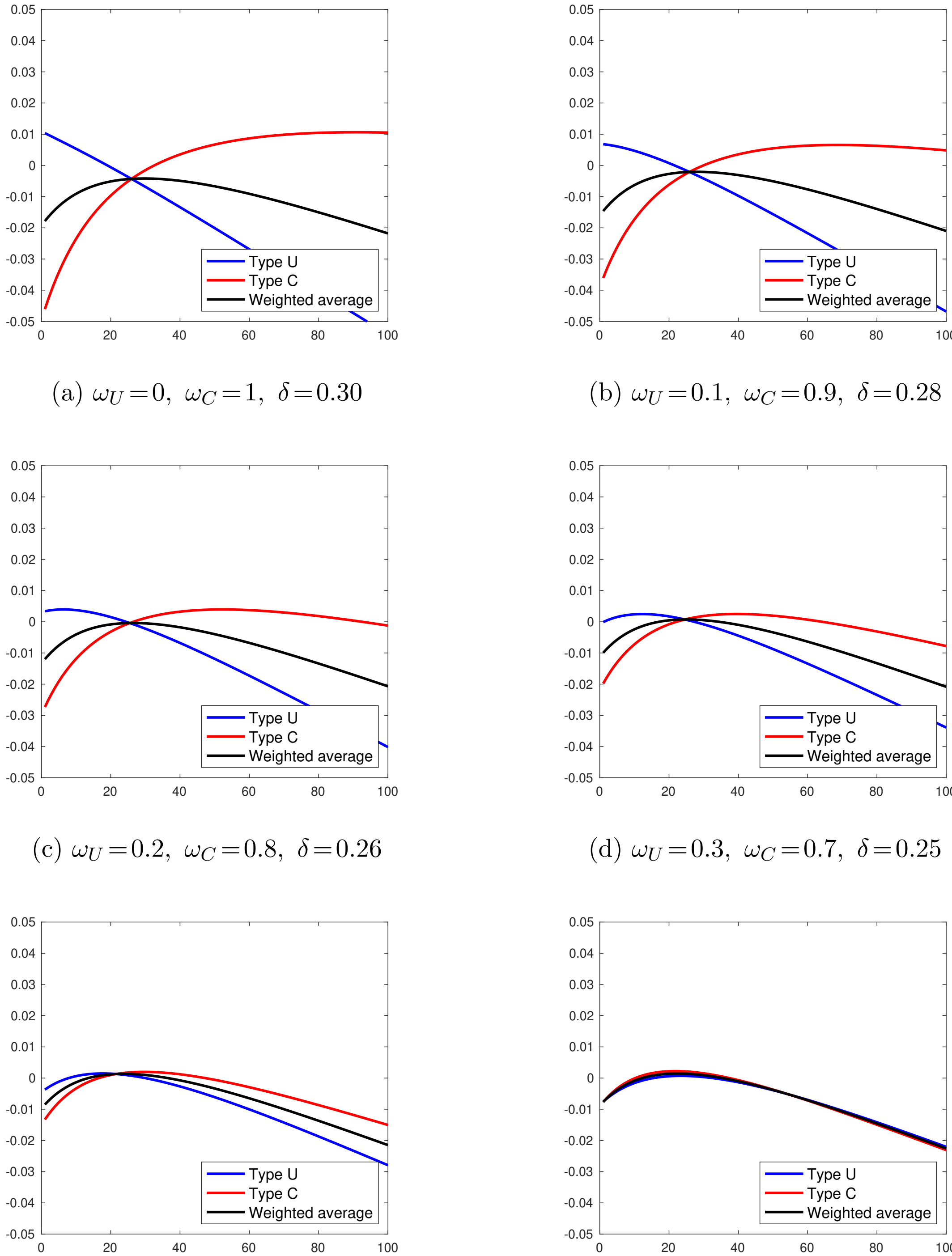
$$\widetilde{y}_{1,t}^N = \widetilde{y}_{2,t}^N = \widetilde{n}_{C,t}^N = \widetilde{n}_{U,t}^N = \widetilde{w}_t^N = \widetilde{q}_t^N = 0 \tag{C.28}$$

C.3.1 Steady State

Steady state is the same as in the model sticky-price, and thus efficient.

$$\begin{aligned}
&C_C = C_U = N_C = N_U = A = A_1 = A_2 = 1\\
&Y_1 = N_1 = \lambda \text{ and } \; Y_2 = N_2 = 1-\lambda\\
&W = Q = 1\\
&D = \frac{1}{\theta} \text{ and } \; T_U = -\frac{1}{\theta(1-\lambda)}
\end{aligned}$$

D Figures



(a) $\omega_U = 0,\ \omega_C = 1,\ \delta = 0.30$

(b) $\omega_U = 0.1,\ \omega_C = 0.9,\ \delta = 0.28$

(c) $\omega_U = 0.2,\ \omega_C = 0.8,\ \delta = 0.26$

(d) $\omega_U = 0.3,\ \omega_C = 0.7,\ \delta = 0.25$

(e) $\omega_U = 0.4,\ \omega_C = 0.6,\ \delta = 0.26$

(f) $\omega_U = 0.5,\ \omega_C = 0.5,\ \delta = 0.22$

Figure D.1: Redistributive effects of inflation targeting policy (under no inequality)