The Effects of a FAR Regulation in a Model of Durable Building with Redevelopment: The Case of New York City

Abstract

This paper investigates the partial equilibrium effects of a floor area ratio ("FAR") regulation in a growing city with rising rents. We extend the standard urban land-use model with FAR regulation in the existing literature to a dynamic setting to provide additional insight into development patterns in a city with a regulation which restricts the maximum-allowed FAR. We show that a FAR regulation hastens construction and lowers the density of buildings, and we also find the effects of a FAR regulation on land value to be consistent with the existing literature. We then demonstrate the relationship between a FAR regulation and land value and the relationship between the regulation and development patterns in New York City. Our data shows that all else equal, a lower maximum-allowed FAR is correlated with lower land values, earlier demolition of buildings, and lower density of newly constructed buildings.

1 Introduction

The purpose of this paper is to propose a model which describes the economic effects of introducing a regulation that limits the maximum-allowed floor area ratio ("FAR") on a parcel of land. Our model analyzes the effects of a marginal change on the FAR regulation in a partial equilibrium setting when there is durable housing and redevelopment.

While there are many types of land use regulations, a FAR regulation is intended to deal with various forms of land use externalities arising from high population density and tall buildings. For example, a FAR regulation combined with other regulations, such as total parking space or open space requirements, can be used to i) increase sunlight in the streets; ii) reduce traffic congestion; iii) reduce strain on public infrastructure; and iv) reduce wind-tunnel effects, among other things.

First introduced in New York City in 1961, the regulation of FAR was an attempt to improve on the previous policy of restricting the height and shape of buildings. While height restrictions were intended to regulate the rapid population growth in New York City, FAR restrictions were a much more effective way of directly limiting population density while decentralizing city planning in that this granted developers more freedom to decide on the shape of their buildings.

The literature on the economic impacts of FAR regulation has focused on 1) optimization of FAR regulation policy and 2) measuring the impacts of FAR regulation on land values. Relevant to our paper is a series of papers on measuring the *stringency* of FAR regulation (Brueckner and Sridhar (2012); Brueckner et al (2017); Moon (2018)), defined as the ratio of the theoretical profit-maximizing FAR under no restriction to the maximum-allowed FAR. These papers extend Brueckner's (1987) well-known land use model by including density restriction to examine the cost of FAR regulation to the land developer.

This paper analyzes the effects of a FAR regulation on the profit-maximization problem of the land developer ("developer") who owns the parcel to be developed. The developer decides on the timing and density of a sequence of structures to be built on the parcel under perfect foresight. We then provide an empirical test of the model presented here using data on land value and time of construction of parcels that are in between conversions in New York City. In our empirical exercise, we estimate the elasticity of land value with respect to the maximum-allowed FAR, regress building age at the time of demolition on the maximum-allowed FAR, and regress FAR of newly converted buildings on the maximum-allowed FAR. To do this, we measure the effects of deviation in FAR regulation from their spatially-adjusted means on the deviations in land values, the timing at which a building is demolished, and the density of newly converted buildings from their spatially-adjusted means through fixed effects models. We find that the results are consistent with the predictions of the model in that 1) a regulation that may not be binding today nevertheless is correlated with lower land values and 2) parcels with more stringent FAR regulation tend to be developed earlier and at lower densities.

We make several contributions to the literature on urban land use and density regulation. First, we extend the static, partial equilibrium analysis of the existing literature to a dynamic setting with perfect foresight and multiple conversions of durable housing. Thus, we generalize the analysis of a city in a stationary state to that of a growing city. Second, our empirical results show the importance of understanding the long-term impact of land use regulations on land values and development patterns and the short-term impact of land use regulations even if the policies may seem to be irrelevant today. Finally, our model provides a supply-side module for a spatial general equilibrium model of a growing city with durable housing.

As urban development is an innately dynamic problem with time-varying rents, required maintenance, and land conversion decisions to be made, our model provides additional insight for the study of FAR regulations. In a static model, it is assumed either that the FAR regulation is binding for all buildings or that there is no effect of the regulation on buildings for which the regulation is not binding. In our dynamic model, however, a FAR regulation that may not be binding today nevertheless affects the profit-maximizing program prior to becoming binding. In particular, a marginal relaxation of FAR regulation - given that such a relaxation maintains the current number of building conversions ("conversions") - will result in a sequence of later and higher-density developments for all future buildings. Furthermore, if the FAR regulation is binding for the first and only conversion of land, and if the elasticity of substitution between land

and capital is equal to unity as in the existing literature, then there would be no effect of changing the maximum-allowed FAR on the elasticity of land value with respect to the stringency of the regulation.

We organize the next sections as follows. In section 2, we discuss the development in the two most relevant literatures - partial equilibrium models of urban development in a growing city with durable housing and multiple conversions and partial equilibrium models of FAR restrictions in a static land use model. In section 3, introduce the model by providing an illustrative example for a single conversion case to motivate the problem then present the baseline model of urban development in a growing city with durable housing and multiple conversions. In this section, we also introduce FAR restrictions to the model to analyze the effects of marginal changes in the maximuma–lowed FAR on the optimal development program and on the value of land. In section 4, we discuss how we construct our data. In section 5, we present our empirical strategy and results. In section 6, we provide the concluding remarks.

2 Literature Review

The literature on urban land use model is dense and consists of countless specifications. The first analyses of the housing market were Marshallian, and the housing market was viewed as a market for housing services (Arnott (1987)). This view of the housing market was formalized by Muth (1969), and the housing market literature exploded during the seventies with several main lines of development.

Most relevant to our model is the class of partial equilibrium models of urban development in a growing city with durable housing. One of the first models of this kind was introduced by Richard J. Arnott (Arnott and Lewis (1979); Arnott (1980)). In these models the developer with perfect foresight decides on the timing and the density of a permanently durable housing to be erected on a single plot of vacant land. By working with perfect foresight, Arnott was able to derive important insights not found in models with static or myopic expectations as in the previous literature. For example, in residential location theory the structural density of a building is determined by land rent, but in Arnott's models structural density is determined by land value. Furthermore, the effects of policies will be different based on whether or not the policy was anticipated by developers.

The first paper to introduce multiple conversions with perfect foresight was by Brueckner (1981). Under the assumptions that 1) rents remain constant over time, 2) the quality of the buildings deteriorate over time, and 3) demolition is costless, Brueckner finds that the optimal development program is an infinite sequence of identical buildings. Amin and Capozza (1993) extend the analysis to include growing rents and find that while the qualitative results under multiple conversions is similar to that under a single conversion, the quantitative results differ dramatically.

Also relevant to our study are the stationary, partial equilibrium models by Jan Brueckner (Brueckner and Sridhar (2012); Brueckner et al (2017)) which explore the effects of making incremental changes to the stringency of FAR regulation. These models show that 1) the relaxation of FAR restriction on a property leads to a higher land value and that 2) this increase in land value has a diminishing effect with subsequent relaxation of the FAR restriction.

The series of papers by Brueckner and other related papers also introduced the idea of measuring the *stringency* of land use regulation (Brueckner and Sridhar (2012); Brueckner et al (2017); Brueckner and Singh (2018); Moon (2018)). Measuring the stringency of FAR regulation, as opposed to the cost and magnitude of regulation, allows us to gauge the extent to which the land use regulation causes development patterns and land values to diverge from the theoretical free-market levels in the absence of regulation even if the theoretically optimal levels are unknown, as long as the parameters and the functional forms of the theoretical model are known (see Brueckner and Singh (2018)). In Brueckner's papers, this is accomplished by estimating the elasticity of land value with respect to the maximum-allowed FAR. In particular, he finds that for parcels where the FAR regulation is binding, this elasticity is positive and less than 1 - a marginal increase in the stringency of FAR regulation decreases land value.

Some other papers relevant to the study of FAR regulations explore changes in maximum-allowed FAR over time, optimal FAR regulations, and the effect of FAR regulations congestion. Joshi and Kono (2009) present a model of a growing two-zone city to determine the optimal FAR regulation for mitigating population externalities. Pines and Kono (2012) utilize a closed monocentric city model with unpriced transport congestion to explore the second-best allocation of housing under a spatial-variable excise subsidies or taxes and suggest the possibility of replacing the subsidies or taxes with FAR regulations. They find that while FAR optimal regulations makes urban growth boundary (UGB) useless, they cannot always be used to attain second-best utility. Barr and Cohen (2014) focus on the FAR gradient of New York City during the 20th century to analyze the change in FAR gradient in each borough over time. They show that New York City has largely remained a monocentric city over time and that there is a nonlinear relationship between plot size and the FAR.

Our paper is a union of the partial equilibrium models of urban development with rising rents, durable housing, and multiple conversions under perfect foresight and Brueckner's model of anticipated FAR restriction. We validate our model through an empirical exercise using data from New York City.

3 The Model

3.1 Illustrative Example

We begin with an illustrative example to motivate the problem and provide some insight into the features of our model. While the general model assumes multiple conversions, assume for the purpose of illustration that there is only one conversion of land. The plot of land to be developed is initially vacant, and the developer must decide on the optimal timing and density for the transition of land from rural to urban use. Also assume that the urban housing market is competitive with housing rent which grows at a constant rate, g. The developer's profit-maximization problem can be stated as

$$\max_{t,S} V(t,S) = \int_t^{\infty} p(\tau)\, h(S)\, e^{-\rho\tau} d\tau - rSe^{-\rho t} \tag{3.1}$$

where $V(\cdot)$ is the land value function, p is the rent per unit of housing, S is the units of capital invested per unit of land, r is the price per unit of capital, $h(\cdot)$ is the output of housing per unit of land, t is the time of development, and ρ is the discount rate. The first-order condition with respect to t is

$$p(t)\, h(S) = \rho r S \tag{3.2}$$

with t^* satisfying the equality. Development will take place when the marginal cost of rent foregone is equal to the marginal benefit of delaying construction any further. The first-order condition with respect to S is

$$re^{-\rho t} = h'(S) \int_t^{\infty} p(\tau)\, e^{-\rho\tau} d\tau \tag{3.3}$$

with S^* satisfying the equality. The building will be constructed to the height where the marginal cost of an additional unit of capital invested is equal to the marginal revenue from the extra housing produced from the additional unit of capital.

Now suppose that there is an anticipated FAR regulation imposed prior to the developer entering the market so that the amount of housing one can develop is limited to $\overline{h} \equiv h\left(\overline{S}\right)$ for some $\overline{S} < S^*$. In this case, there is only one first-order condition, and (3.2) becomes

$$p(t) = \frac{\rho r \overline{S}}{h\left(\overline{S}\right)} \tag{3.4}$$

with $\hat{t}$ satisfying the optimality condition. The derivative of this expression with respect to the maximum-allowed FAR is

$$\frac{dp(\hat{t})}{d\overline{S}} = \frac{\rho r \left(h\left(\overline{S}\right) - h'\left(\overline{S}\right)\overline{S}\right)}{\left(h\left(\overline{S}\right)\right)^2} > 0$$

where $h\left(\overline{S}\right) - h'\left(\overline{S}\right)\overline{S} > 0$ by the concavity of $h(\cdot)$. Thus, increasing the stringency of the FAR regulation by decreasing $\overline{S}$ will lead to a lower housing rent at the time of development. A lower housing rent in our context corresponds to an earlier point in time, which implies that development will take place at an earlier time.

Figure 1 illustrates the foregoing.

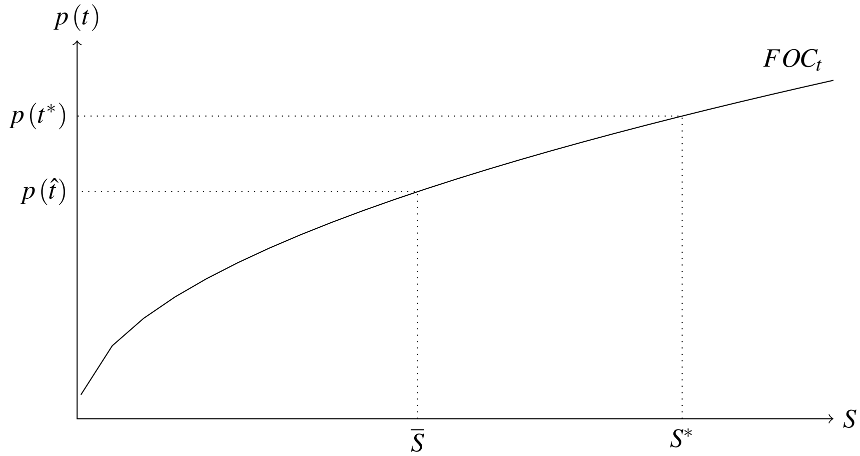


Figure 1: Illustrative Example

Now, given the optimal choice for the timing of construction, the land value is

$$V\left(\hat{t}\left(\overline{S}\right);\overline{S}\right)=\int_{\hat{t}}^{\infty}p\left(\tau\right)h\left(\overline{S}\right)e^{-\rho\tau}d\tau-r\overline{S}e^{-\rho\hat{t}} \tag{3.5}$$

The derivative of land value with respect to the maximum-allowed FAR is

$$\frac{d\hat{V}}{d\overline{S}}=V_{\overline{S}}+V_{\hat{t}}\frac{d\hat{t}}{d\overline{S}} \tag{3.6}$$

where $\hat{V}\equiv V\left(\hat{t}\left(\overline{S}\right);\overline{S}\right)$. But since by the Envelope Theorem $V_{\hat{t}}=0$, we have

$$\begin{aligned}\frac{d\hat{V}}{d\overline{S}}&=V_{\overline{S}}\\&=h'\left(\overline{S}\right)\int_{\hat{t}}^{\infty}p\left(\tau\right)e^{-\rho\tau}d\tau-re^{-\rho\hat{t}}>0\end{aligned} \tag{3.7}$$

so that the FAR regulation decreases the land value. The elasticity of land value with respect to the maximum-allowed FAR is

$$E_{V:\overline{S}}\equiv\frac{V_{\overline{S}}\overline{S}}{V}=\frac{\overline{S}h'\left(\overline{S}\right)\int_{\hat{t}}^{\infty}p\left(\tau\right)e^{-\rho\tau}d\tau-r\overline{S}e^{-\rho\hat{t}}}{h\left(\overline{S}\right)\int_{\hat{t}}^{\infty}p\left(\tau\right)e^{-\rho\tau}d\tau-r\overline{S}e^{-\rho\hat{t}}} \tag{3.8}$$

which is less than one by the concavity of $h(\cdot)$.

We now turn to the general model to explore how the analysis extends when there are multiple conversions.

3.2 Baseline Model

Notational Glossary

$E_{V:\overline{S}}$	Elasticity of land value with respect to the maximum-allowed density
g	Growth rate of rent on housing
$h(S)$	Output of housing per unit of land (FAR)
$\overline{h}$	$\left(\equiv h\left(\overline{S}\right)\right)$ The maximum-allowed density of housing
J	Index for the final building conversion
$p(t)$	Rent on housing at time t per unit of housing
r	Price of a unit of capital
S_j	Capital investment in housing per unit of land at the jth conversion
S_j^*	Optimal density in the absence of FAR regulation
$\overline{S}$	Maximal allowed capital investment in housing
$\hat{S}_j$	Optimal density under FAR regulation
$\tilde{S}_J$	$\left(\equiv \frac{\overline{S}}{S_J^*}\right)$ The reciprocal of stringency of FAR regulation
t_j	Time at the jth conversion
t_j^*	Optimal timing of development in the absence of FAR regulation
$\hat{t}_j$	Optimal timing of development under FAR regulation
V	Value per unit of land
$\hat{V}$	Optimized land value under FAR regulation
β	Output elasticity of land
ρ	Discount rate

This section derives the baseline model describing the relationship between density regulation and land values. We utilize a partial equilibrium model to isolate and analyze the effect of a density regulation on the optimal development program on a single plot of land. A general equilibrium model may show that a density restriction effectively decreases the total supply of land available for development in the form of airspace, thus leading to an overall increase in land value. However, the goal of our model is to show that the immediate effect of a density restriction on a single plot of land - holding the density of all neighboring plots constant - is to lower the value of that plot of land. This allows us to focus on the cost of the FAR restriction to the developer.

Consider a competitive urban land development market with rising housing rent. The problem facing the developer with perfect foresight is to choose the optimal timing and the density for the initial transition of land from rural to urban use and the stream of subsequent building conversions. The rent on housing is determined competitively and is dependent on locational characteristics.

The following are the assumptions used throughout this paper:

Assumptions

Assumption 1. *There is no depreciation of housing.*

Assumption 2. *Demolition for the purpose of reconstruction is costless.*

Assumption 3. *The housing redevelopment market is competitive.*

Assumption 4. *The developer has perfect foresight with respect to future rent on buildings and the FAR restriction.*

Assumption 5. *The rent function is unbounded, is non-decreasing in time, and has the property that* $p(t) \leq p(0)e^{gt}$ *for some* $g < \rho$.

The housing developer's profit-maximization problem can be stated as

$$\max_{\{t_j, S_j\}} V(t_1, S_1, \cdots) = \sum_{j=1}^{\infty} \left\{ \int_{t_j}^{t_{j+1}} p(\tau) h(S_j) e^{-\rho\tau} d\tau - rS_j e^{-\rho t_j} \right\} \tag{3.9}$$

where V is the value per unit of land, $p(\cdot)$ is the rent per unit of housing, S_j is the units of capital invested in the jth future conversion, $h(\cdot)$ is the amount of housing per unit of land (FAR), and r is the price per unit of capital. Note that this value is intermittently calculated between conversions, when the previous building is torn down and the next building is erected. Thus, the jth building's contribution to the total land value is calculated when the plot is vacant. Furthermore, this specification of the model implicitly defines the developer's problem as developing a plot of land which is initially vacant at the beginning of time, denoted t_0[1]. As we shall see in the next section, it is prior to this time t_0 when the density restriction is imposed, thus making the restriction an anticipated regulation.

Figure 2 illustrates the sequence of building conversions for the optimized development program.

The first-order condition for the choice of timing of conversion is

$$(h(S_j) - h(S_{j-1}))\, p(t_j) = \rho r S_j \tag{3.11}$$

where $\left\{t_j^*\right\}_{j=1}^{\infty}$ satisfies the set of optimality conditions and $h(S_0) = 0$. Conversion will take place whenever the rent foregone from waiting an extra period is equal to the benefit of putting off construction for one period. Under our assumptions, there will be an infinite sequence of conversions in the absence of a density restriction.

[1] If we wish to model a scenario where there exists a structure at the beginning of time, the profit-maximization problem would become

$$\max_{\{t_j, S_j\}} V(S_0, t_1, S_1, \cdots) = \int_{t_0}^{t_1} p(\tau) h(S_0) e^{-\rho\tau} d\tau + \sum_{j=1}^{\infty} \left\{ \int_{t_j}^{t_{j+1}} p(\tau) h(S_j) e^{-\rho\tau} d\tau - rS_j e^{-\rho t_j} \right\} \tag{3.10}$$

where $S_0 > 0$ is the density of the initial building. In this paper, we shall assume the former land value function so that the land to be developed is initially vacant.

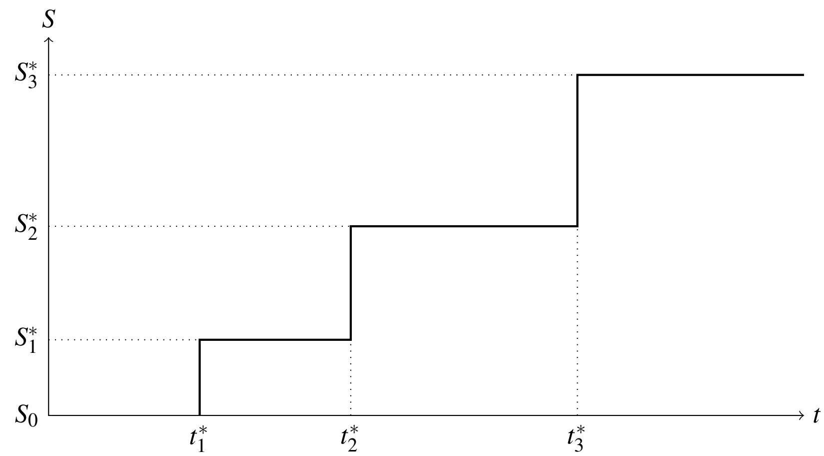


Figure 2: Sequential Development

Proposition 1. *Under Assumptions A1-A5, in the absence of a density restriction there will be an infinite sequence of building conversions.*

Proof. (by contradiction) (from Amin & Capozza (1992)). Consider some nth redevelopment at conversion time t_n and density $h(S_n)$, determined to be the final conversion. But for some S such that $h(S) > h(S_n)$, the variable cost of conversion from density $h(S_n)$ to density $h(S)$ is given by rS. Then, it will be profitable to build to the higher density S if

$$(h(S) - h(S_n)) \int_t^{\infty} p(\tau) e^{-\rho\tau} d\tau \geq rS \tag{3.12}$$

Since by Assumption 5 $p(t)$ is a non-decreasing, unbounded function of t, the above equation will eventually be satisfied. Therefore, without any restriction, housing density will increase over time indefinitely. □

The first-order condition for the choice of housing density is

$$re^{-\rho t_j} = h'(S_j) \int_{t_j}^{t_{j+1}} p(\tau) e^{-\rho\tau} d\tau \tag{3.13}$$

with $\left\{S_j^*\right\}_{j=1}^{\infty}$ satisfying the set of optimality conditions. Each structure will be built up to the density where the price a unit of capital is equal to the present value of the marginal revenue from the investment of capital[2].

[2]The second partial derivatives satisfy the conditions for the solutions to be local maxima, as the land value function is a sum of concave functions with derivatives at the critical points being equal to zero.

3.3 The Effects of an Increase in the Stringency of a FAR Regulation

We now extend the baseline model to treat FAR regulation. Suppose that there is a FAR regulation imposed at some time prior to time t_0 which restricts the density of any future structure to $\overline{h} \equiv h\left(\overline{S}\right)$ for some $\overline{S} > 0$. By Proposition 1, housing density will continue to increase until the FAR restriction is binding for some Jth and final conversion, so that $h\left(\overline{S}\right) < h\left(S_J^*\right)$ (where S_J^* is the unrestricted profit-maximizing density for the Jth conversion). The value function for land now becomes

$$V\left(t_1, S_1, \cdots, t_J, \overline{S}\right) = \sum_{j=1}^{J-1} \left\{ \int_{t_j}^{t_{j+1}} p(\tau) h(S_j) e^{-\rho\tau} d\tau - rS_j e^{-\rho t_j} \right\} + \int_{t_J}^{\infty} p(\tau) h\left(\overline{S}\right) e^{-\rho\tau} d\tau - r\overline{S} e^{-\rho t_J}$$

so that the Jth building is the final structure erected. Thus, there is now a finite number of building conversions if a density regulation is introduced. With this restricted land value function, the set of first-order conditions with respect to the timing of conversion is

$$\begin{cases} \left(h(S_j) - h(S_{j-1})\right) p(t_j) = \rho r S_j & \text{if } j < J \\ \left(h\left(\overline{S}\right) - h(S_{J-1})\right) p(t_J) = \rho r \overline{S} & \text{if } j = J \end{cases} \tag{3.14}$$

and the set of first-order conditions with respect to the density of conversion is

$$\begin{cases} re^{-\rho t_j} = h'(S_j) \int_{t_j}^{t_{j+1}} p(\tau) e^{-\rho\tau} d\tau & \text{if } j < J \\ re^{-\rho t_J} < h'\left(\overline{S}\right) \int_{t_J}^{\infty} p(\tau) e^{-\rho\tau} d\tau & \text{if } j = J \end{cases} \tag{3.15}$$

with $\left\{\hat{t}_j\right\}_{j=1}^{J}$ and $\left\{\hat{S}_j\right\}_{j=1}^{J-1}$ satisfying the optimality conditions, respectively. Note that the second equation of (3.15) is an inequality, because the final structure is built at a density which is less than optimal. Thus, the marginal revenue from an extra unit investment of capital exceeds the marginal cost of an extra unit of investment of capital.

Given the first-order conditions above, we can write the set of comparative statics equations given the optimized values of timing and density in matrix form as

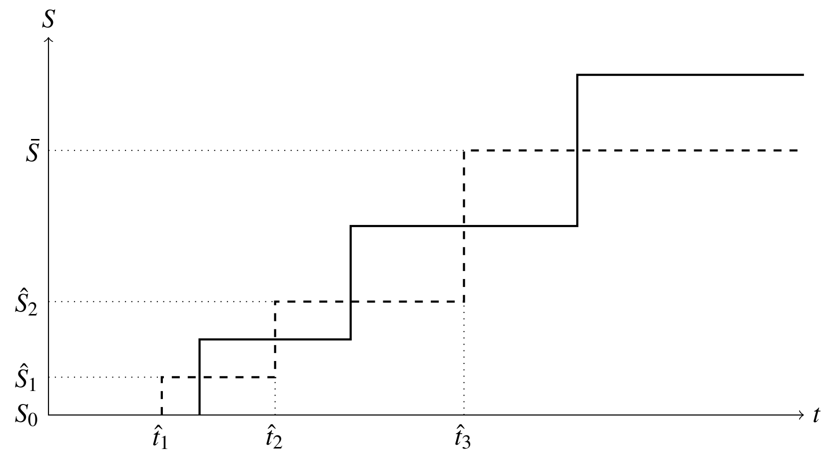


Figure 3: Sequential Development with FAR Restriction

$$\begin{bmatrix} V_{\hat{t}_1\hat{t}_1} & V_{\hat{t}_1\hat{S}_1} & 0 & \cdots & 0 \\ V_{\hat{S}_1\hat{t}_1} & V_{\hat{S}_1\hat{S}_1} & V_{\hat{S}_1\hat{t}_2} & \ddots & 0 \\ 0 & V_{\hat{t}_2\hat{S}_1} & V_{\hat{t}_2\hat{t}_2} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & V_{\hat{S}_{J-1}\hat{t}_J} \\ 0 & 0 & \cdots & V_{\hat{t}_J\hat{S}_{J-1}} & V_{\hat{t}_J\hat{t}_J} \end{bmatrix} \begin{bmatrix} \frac{d\hat{t}_1}{d\bar{S}} \\ \frac{d\hat{S}_1}{d\bar{S}} \\ \frac{d\hat{t}_2}{d\bar{S}} \\ \frac{d\hat{S}_2}{d\bar{S}} \\ \vdots \\ \frac{d\hat{t}_J}{d\bar{S}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ -V_{\hat{t}_J\bar{S}} \end{bmatrix} \tag{3.16}$$

We can now show that all else equal, reducing the density limit so that the FAR regulation is more stringent hastens development and lowers the density of all sequence of buildings, given that the change in the regulation does not change the number of conversions.

Proposition 2. *Under Assumptions A1-A5, a marginal increase in the stringency of the FAR regulation hastens development for all building conversions.*

Proof. See Appendix 1 □

Proposition 3. *Under Assumptions A1-A5, a marginal increase in the stringency of the FAR regulation lowers the density of development for all building conversions.*

Proof. See Appendix 2 □

Figure 3 illustrates the foregoing two Propositions, overlaid on Figure 2.

We now turn to the analysis of the FAR regulation's impact on land value. The developer's optimized program yields a land value of

$$\hat{V} = \sum_{k=1}^{J-1} \left\{ \int_{\hat{t}_k}^{\hat{t}_{k+1}} p(\tau)\, h\left(\hat{S}_k\right) e^{-\rho\tau} d\tau - r\hat{S}_k e^{-\rho \hat{t}_k} \right\} + \int_{\hat{t}_J}^{\infty} p(\tau)\, h\left(\overline{S}\right) e^{-\rho\tau} d\tau - r\overline{S} e^{-\rho \hat{t}_J} \tag{3.17}$$

where $\hat{V} \equiv V\left(\hat{t}_1\left(\overline{S}\right), \hat{S}_1\left(\overline{S}\right), \cdots, \hat{t}_J\left(\overline{S}\right); \overline{S}\right)$ is the maximized land value under the constraint. The derivative of the above with respect to the maximum-allowed FAR is

$$\frac{d\hat{V}}{d\overline{S}} = V_{\overline{S}} + \sum_{k=1}^{J} V_{\hat{t}_k} \frac{d\hat{t}_k}{d\overline{S}} + \sum_{l=1}^{J-1} V_{\hat{S}_l} \frac{d\hat{S}_l}{d\overline{S}} \tag{3.18}$$

where the subscripts on V denote partial derivatives. Thus, an infinitesimal change in $\overline{h}$ affects the timing and density of development for all building conversions prior to the one where the restriction binds. While it is possible for a change in $\overline{S}$ to lead to a change in the number of conversions (from J cycles to $J-1$ or $J+1$ cycles), we shall only treat those cases where the number of conversions remains the same after a change in the FAR restriction.

Since by the Envelope Theorem $\frac{\partial V}{\partial \hat{t}_k} = 0$ and $\frac{\partial V}{\partial \hat{S}_l} = 0 \; \forall \; k \leq J \;\&\; l \leq J-1$, we have

$$\begin{aligned} \frac{d\hat{V}}{d\overline{S}} &= V_{\overline{S}} \\ &= h'\left(\overline{S}\right) \int_{\hat{t}_J}^{\infty} p(\tau)\, e^{-\rho\tau} d\tau - r e^{-\rho \hat{t}_J} > 0 \end{aligned} \tag{3.19}$$

That is, the changes in the timing and density of building conversions where the restriction do not bind does not have a first-order effect on land value. Nevertheless, the FAR regulation lowers the land value overall. As the FAR regulation is anticipated at the outset, all future development is impacted. Since land value is dependent not only on the current best use of the land but also on its future development potential, a lowered future development potential in the form of a FAR restriction which will be binding in the future will lower the current value of land.

If there is a marginal change in the FAR regulation, those plots where the FAR regulation was initially more stringent will experience a greater loss in land value. This can be shown by proving that $\frac{d^2\hat{V}}{d\overline{S}^2} < 0$:

Proposition 4. *Under Assumptions A1-A5, a continued increase in the stringency of the FAR regulation will have an increasing effect on the decrease in land value.*

Proof. See Appendix 3 □

In the following section, we estimate the effect of a percent change in the FAR restriction to the percent change in land value. The following Proposition provides a theoretical basis for the expected estimated value of the empirical exercise:

Proposition 5. *Under Assumptions A1-A5, the elasticity of land value with respect to the maximum-allowed FAR, defined by* $E_{V:\overline{S}} \equiv \frac{V_{\overline{S}}\overline{S}}{V}$ *is less than one.*

Proof. Consider the elasticity of land value with respect to the maximum-allowed capital investment:

$$\begin{aligned} E_{\hat{V}:\overline{S}} \equiv \frac{V_{\overline{S}}\overline{S}}{\hat{V}} &= \frac{\overline{S}h'\left(\overline{S}\right)\int_{\hat{t}_J}^{\infty}p\left(\tau\right)e^{-\rho\tau}d\tau - r\overline{S}e^{-\rho\hat{t}_J}}{\sum_{k=1}^{J-1}\left\{\int_{\hat{t}_k}^{\hat{t}_{k+1}}p\left(\tau\right)h\left(\hat{S}_k\right)e^{-\rho\tau}d\tau - r\hat{S}_k e^{-\rho\hat{t}_k}\right\} + h\left(\overline{S}\right)\int_{\hat{t}_J}^{\infty}p\left(\tau\right)e^{-\rho\tau}d\tau - r\overline{S}e^{-\rho\hat{t}_J}} \\ &= \frac{A}{B+C} \end{aligned} \tag{3.20}$$

where

$$A \equiv \overline{S}h'\left(\overline{S}\right)\int_{\hat{t}_J}^{\infty}p\left(\tau\right)e^{-\rho\tau}d\tau - r\overline{S}e^{-\rho\hat{t}_J} \tag{3.21}$$

$$B \equiv \sum_{k=1}^{J-1}\left\{\int_{\hat{t}_k}^{\hat{t}_{k+1}}p\left(\tau\right)h\left(\hat{S}_k\right)e^{-\rho\tau}d\tau - r\hat{S}_k e^{-\rho\hat{t}_k}\right\} \tag{3.22}$$

$$C \equiv h\left(\overline{S}\right)\int_{\hat{t}_J}^{\infty}p\left(\tau\right)e^{-\rho\tau}d\tau - r\overline{S}e^{-\rho\hat{t}_J} \tag{3.23}$$

Since h is concave, we know that $h'\left(\overline{S}\right)\overline{S} < h\left(\overline{S}\right)$ and $A < C$. Furthermore, since $B > 0$ as it is the value of land for the first $J-1$ conversions, the elasticity of land value with respect to the capital investment limit is less than one. □

4 Data

4.1 Background

New York City is notorious for its complex housing regulations. The first set of zoning regulations were introduced in 1916 and were designed to restrict the height of and set standards for the shape of skyscrapers. Over the years, the 1916 Zoning Resolution was continuously amended in order to adjust for the changing economy, increasing population, and the growth of automobile use. By the 1950s, the 1916 Zoning Resolution came to "[resemble] a torn 'patchwork,' reflecting forty-four tumultuous years of technological, social, and physical change" (Marcus, 1992). As a result, when James Felt was appointed as Chairman of the New York City's Planning Commission in 1956, he put rezoning New York City as his top priority in order to rally public support for the City Planning Commission. That year, Felt commissioned the architectural firm of Voorhees, Walker, Smith, and Smith to conduct a zoning study and propose a new resolution. In 1961, this new Zoning Resolution was approved by the New York City's Board of Estimate to replace the 1916 Zoning

Resolution to come into effect. Rather than regulating the height and shape of skyscrapers as its predecessor did, the 1961 Zoning Resolution simply restricted the FAR of a building, which gave developers more freedom in the shape of buildings while giving planners more direct control over population density. Unfortunately, some of the problems of the 1916 Zoning Resolution are manifest also in the 1961 Zoning Resolution: in its current form, the text of New York City's 1961 Zoning Resolution, colloquially referred to as the "zoning text," is 4,338 mind-numbing pages establishing zoning districts, regulations governing land use and development, and all the exceptions to the rules.

The typical rationale of a land use regulation such as the 1916 and 1961 Zoning Resolutions of New York City is to deal with various forms of land use externalities. For example, some justification used to advance building height and density restrictions include i) to increase sunlight in the streets; ii) to reduce wind-tunnel effects; iii) to reduce traffic congestion; and iv) to reduce the cost of providing certain public services. Optimal regulation requires that these externalities are internalized, such that the marginal social benefit of strengthening the regulation equals the corresponding marginal social cost. But because many of the benefits of regulation are difficult to quantify, most of the empirical literature on land use regulations in economics has focused on its costs. In particular, the majority of research focuses on measuring the effects of regulation on the overall levels of housing prices and quantities (see Gyourko and Molloy (2015) for a survey on this topic).

4.2 Constructing the Data

We construct two sets of data in order to address two aspects of our model: 1) FAR regulation depresses land value for parcels where the regulation is not binding and 2) FAR regulation hastens development.

Testing the first aspect of our model requires us to estimate vacant landvalues, which pose numerous challenges. One of the approaches used in the literature in estimating land values is to utilize teardown data. Dye and McMillen (2007) estimate vacant land values in Chicago based on building sales and demolition permits issued. This is done under the assumption that the sales price of a demolished building is very close to land value. McMillen and O'Sullivan (2013) utilize the active teardown market data in Chicago to test their model which predicts that "hedonic price function coefficients depend on expected time between sale and demolition." Their study shows that the sales price of both teardown and non-teardown properties are affected by structural variables, with the effect being much larger when the estimated teardown probability is low. Gedal and Ellen (2018) analyze 3,800 teardown sales and 4,900 vacant land sales between 2003 and 2009 to estimate vacant land values in New York City. They show that the value of vacant parcels tends to be lower than the value of teardown parcels because of differences in the quality of parcels. They conclude that teardown parcels seem to be more representative of the city as a whole and that vacant land values estimated from teardown sales may be better suited for exercises involving the study of vacant land values. We construct our data on vacant land values based on teardown sales using the methods introduced in Moon (2018), which improves on the methods used in the literature. All of our data comes from New York City's Open Data, which is a joint project between the Mayor's Office of Data Analytics (MODA) and the Department of Information Technology and Telecommunications (DoITT) to provide the public with free access to city related data. The core of our data comes from Primary Land Use Tax Lot Output ("PLUTO"), which contains information on every plot of land in the five boroughs of New York. Some of the relevant information that this dataset provides includes lot area, maximum-allowable FAR, built FAR, the borough in which the property is located, the year in which a property was built, and BBL (borough, tax block, & lot) code. Our extract of the data comes from September 2016 with an initial sample size of 834,182 parcels.

To construct the first set of data, we extract rolling sales records from 2003-2016, provided by the Department of Finance. To clean the data, we remove irregular sales records, some of which may correspond to transactions that were not at arm's length and others to errors, as some properties were transferred at prices as low as $0 or $1. We then match the rolling sales data and the PLUTO data with information on demolition and new building permits from the Department of Buildings. GNamely, we reconcile all the extracts by BBL code for each property, then we isolate all properties that were issued both a demolition permit and a new building permit within a two-year time frame after a sale takes place [3]. This gives us a very accurate measure of vacant land values; a building that is demolished to be converted immediately has a value close to $0, and the value of the parcel that was sold right before demolition largelyconsists of vacant land value [4]. Once we construct this data, we are left with a sample size of 2,720. The significant loss in sample size from the original extract is due to the requirement that the building on the site must be demolished and a redevelopment permit be issued within two years of the demolition in order to be included in the restricted sample of parcels. Many buildings in the original data were issued a demolition permit without a corresponding new building permit, indicating that some properties may have been purchased to be used as a parking lot or held as a vacant lot for development in the future.

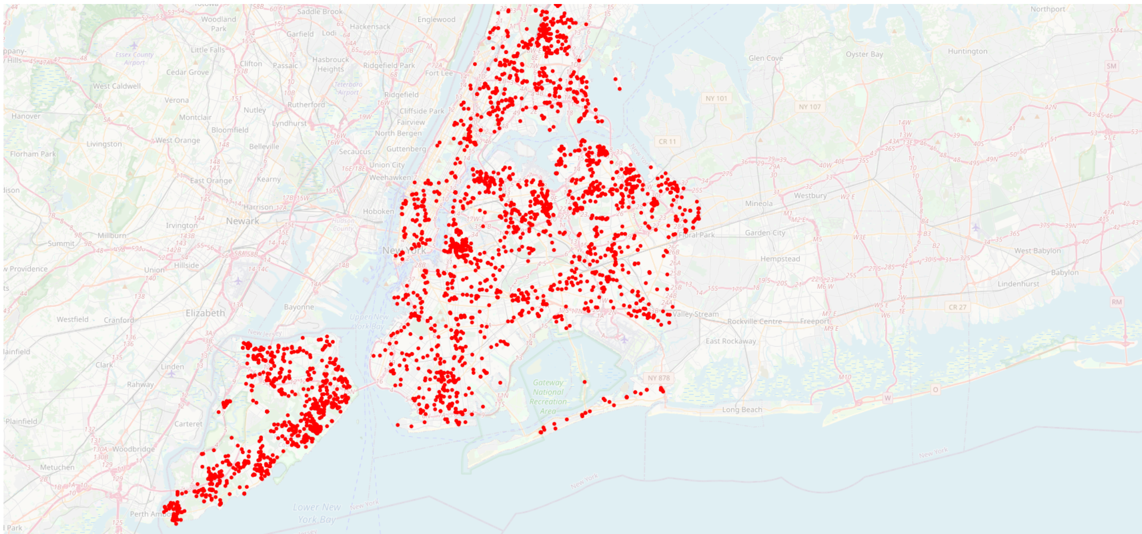
The second set of data is constructed by calculating the age of each building at the time of demolition. In particular, we subtract the year in which a building was erected from the year when a demolition permit was issued at the end of the building's life. This information is used to test the prediction that all else equal, a building with a lower FAR limit will be demolished at an earlier age for a conversion. The idea behind this approach is that if two buildings were constructed at a similar date, comparing the age of the buildings at demolition would allow us to determine the effect of

[3]Note that our methodology also excludes those buildings that received bonus allowable FAR and public buildings. Buildings with bonus FAR are rarely demolished, and public buildings do not have any transaction data.

[4]Such parcels actually may sometimes have negative value because of demolition costs, but we do not take into consideration demolition costs in our paper. Collecting demolition cost data poses a challenge, because it is often unavailable in public data. While Gedal and Ellen (2018) estimate demolition costs based on consultation with contractors in New York City, their estimates are not precise. Even if we were able to estimate demolition costs, Liu et al (2012) show that demolished buildings also have scrap values that may sometimes even exceed the demolition cost - but scrap value is also difficult to estimate for each building. Gedal and Ellen (2013), which is closely related to our research, confirm through an interview with a New York City official that demolition costs consist of the fixed cost of demolishing the floor and ceiling and the variable cost which increases with built FAR. We thus mitigate differences in demolition costs and scrap value by controlling for built FAR in our estimation.

Another reason why we do not consider demolition costs is because demolition costs are relatively low in property development. Rosenthal and Helsley (1994) show that sales prices for teardown properties provide reliable estimates of vacant land value under the assumption that demolition prices are low. Moreover, Gedal and Ellen (2018) show that demolition costs only account for 1.8% of the sales price of teardown properties.

Figure 4: Location of Teardowns



FAR regulation. After constructing this data, we are left with a sample size of 7,398.

Figure 4 shows the locations demolished buildings in our data. As map shows, the data points are well-distributed all over New York; demolitions are not concentrated in certain boroughs, but rather are distributed across all five boroughs[5].

Table 1 shows the summary statistics for the two sets of constructed data. One notable aspect about our data is that the mean Built FAR is lower than the mean maximum-allowed FAR, which indicates that the restriction does not bind for some of the buildings.

5 Empirical Strategy & Results

We address three aspects of the model in our study: 1) the effect of FAR regulation differentials on land value for those properties for which the FAR limit is not binding, 2) the effect of FAR regulation differentials on the timing at which a conversion takes place, and 3) the effect of FAR regulation differentials on the density of newly converted buildings. To do this, we control for spatial heterogeneity across parcels through a fixed effects model and estimate the effect of deviations in the maximum-allowed FAR from the spatially-adjusted means on deviations on land values from their spatially-adjusted means.

[5]To test the randomness of the distribution, we utilize the average nearest neighbor method which compares the average distance between plots in the actual data and the average distance between plots that are randomly distributed across the same area as the actual data. The average nearest neighbor ratio is then calculated by dividing the observed average distance by the expected average distance, where the expected average distance is based on the randomly generated sample. If this ratio is larger than 1, our sample is said to be distributed randomly across space, and if the ratio is less than 1, then our sample is said to be clustered in certain areas. Because properties are already clustered in New York City, our sample is spatially spread out across New York City if the ratio calculated using our sample is larger than the average nearest neighbor ratio of all buildings in New York City. Our results show that the average nearest neighbor ratio for all buildings in New York City is 0.111, and the average nearest neighbor ratio for the teardowns is 0.365, which shows that the teardowns in our data are distributed more randomly across space than all buildings in New York City.

Table 1: Summary Statistics

Vacant Land Value Data		
	Mean	S.D
Lot Area (sqft)	35,387	328,609
Max FAR	2.38	2.13
Built FAR	1.73	2.60
Sale Price ($)	1,405,280	5,260,532
Price Per Square Foot ($)	310	1,053
N	2,720	
Building Age at Demolition Data		
	Mean	S.D.
Year Built	1,931	24.7
Age (years)	76.4	25.6
Max FAR	2.6	2.0
Built FAR	1.2	1.4
N	7,318	

5.1 Estimating the Effect of FAR Regulation Differentials on Land Value

Our model predicts that a marginal increase in the stringency of FAR regulation lowers land value. That is, all else equal a parcel with a higher maximum-allowed FAR will have a higher land value. This is shown in the model by the fact that the elasticity of land value with respect to the maximum-allowed FAR is positive. To identify the effect of FAR regulation differentials on land value, we regress the log of land value on the log of the maximum-allowed FAR. This gives us the estimate for the elasticity of land value with respect to the maximum-allowed FAR. We include a sales year fixed effect and a zip code fixed effect to control for variation in sales prices across different years and for neighborhood characteristics. The equation for this regression is given by

$$\log(V_{ict}) = \alpha_t + \beta_c + \theta \log(FAR_{ict}) + \varepsilon_{ict} \tag{5.1}$$

where V is the land value, FAR is the maximum-allowed FAR, t is the vector of dummies for the year in which building i was sold, and c is the vector of dummies for zip code. Note that this equation does not control for characteristics of the buildings that were sold. This is because in our data, buildings that were sold were going to demolished anyway and their characteristics do not affect land value.

Table 2 gives the estimation results of the correlation between FAR regulation on land value. Column (1) shows the estimation results for the aggregate data, while column (2) shows the estimation results for samples for which the FAR limit is not binding. The results are consistent even for parcels where the FAR is not binding - we find the elasticity of land value with respect to maximum-allowed FAR to be positive, significant, and less than 1 for parcels that have non-binding FAR limits. For these parcels, we find that a 1% increase in the maximum-allowed FAR is associated with a 0.42% increase in land value. We further perform a robustness check by utilizing the clustering method from

Table 2: Effect of FAR Regulation on Land Value

VARIABLES	(1) LN(Land Price/lot area)	(2) LN(Land Price/lot area) MaxFAR>BuiltFAR	(3) LN(Land Price/lot area)	(4) LN(Land Price/lot area) MaxFAR>BuiltFAR
Ln(FAR)	0.432***	0.421***	0.385***	0.323***
	(0.0713)	(0.0881)	(0.122)	(0.104)
Constant	6.528***	7.077***	4.635***	8.570***
	(0.216)	(0.257)	(0.0724)	(0.330)
Zip Code FE	Yes	Yes		
Sales Year FE	Yes	Yes		
Cluster FE			Yes	Yes
Number of zip codes or clusters	167	167	2,321	2,321
Observations	2,720	2,244	2,720	2,244
R-squared	0.585	0.484	0.392	0.325

Robust standard errors in parentheses
*** $p<0.01$, ** $p<0.05$, * $p<0.1$

Brueckner et al. (2017). We cluster the sample based on the borough, block, usage (commercial/residential), and the sale year. This allows us to control for unobserved parcel and building characteristics. The results are shown in columns (3) and (4). Thus, we show that our results remain robust even when we use spatial fixed effects that are finer than zip codes (167 zip codes vs. 2,321 clusters).

5.2 Estimating the Effect of FAR Regulation Differential on Timing of Conversion

To estimate the effect of FAR regulation differentials on the timing of conversion, we regress the age at which a building is demolished on maximum-allowed FAR. Theory predicts that if two buildings are similar in characteristics, the building with a lower maximum-allowed FAR will be converted at an earlier time. If two buildings were constructed at a similar time, have similar neighborhood characteristics, and differ only in the maximum-allowed FAR, the timing of conversion can be estimated by comparing the age at which each of the buildings is demolished.

We include a built year fixed effect based on the decade[6] that a building was erected for two reasons. First, as discussed above, comparing the age of demolition of buildings that were constructed at similar times, allows us to determine the differences in the timing of conversion due to differences in FAR regulation. Second, we assume that the timing at which a building is demolished will heavily depend on the year at which the building was built. Buildings that were constructed at a similar time will have similar characteristics and be constructed with similar materials and with similar technology.

We also include a built FAR fixed effect, with the built FAR of each building rounded to the nearest multiple of 0.5. This allows us to further control for property characteristics, under the assumption that two buildings that were constructed at a similar time to a similar density must also have similar characteristics. Thus, our exercise involves comparing the timing of demolition of two buildings that were built at the same time and at the same density, but differ

[6]Using bins of 3 years and 5 years also yields a similar result.

Table 3: Effect of FAR Regulation on the Timing of Conversion

VARIABLES	(1) Age	(2) Age	(3) Age	(4) Age	(5) Age	(6) Age	(7) Age	(8) Age
FAR	0.708***	0.255***	0.702***	0.247***	0.533***	0.219***	0.559***	0.215***
	(0.0492)	(0.0648)	(0.0493)	(0.0649)	(0.0503)	(0.0571)	(0.0502)	(0.0572)
Built FAR					-0.355***	-0.235***	-0.339***	-0.237***
					(0.0886)	(0.0761)	(0.0865)	(0.0764)
Distance (Distance from ESB)			-0.00893	-0.0179***			0.0306***	-0.0102
			(0.00570)	0.00637)			(0.00953)	(0.00664)
Constant	186.7***	180.2***	186.7***	180.3***	1,839***	1,898***	1,849***	1,896***
	(0.219)	(0.505)	(0.221)	(0.506)	(73.05)	(69.64)	(73.01)	(69.67)
Built Decade FE	Yes	Yes	Yes	Yes	No	No	No	No
Built FAR FE	Yes	No	Yes	No	No	No	No	No
By-Borough Built FAR FE	No	Yes	No	Yes	No	No	No	No
Cluster FE					Yes	Yes	Yes	Yes
Number of clusters					3,381	3,381	3,381	3,381
Observations	7,318	7,318	7,318	7,318	7,318	7,318	7,318	7,318
R-squared	0.915	0.914	0.915	0.914	0.917	0.925	0.917	0.925

Notes: In columns 2 and 4, by-borough built FAR fixed effects were included.
Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

only in their maximum-allowed FAR.

In the simplest case, this estimation equation is given by

$$AGE_{isl} = \alpha_l + \lambda_s + \theta FAR_{isl} + \varepsilon_{isl} \tag{5.2}$$

where l is the vector of dummies for built year and s is the vector of dummies built FAR.

To further account for differences across parcels, we include by-borough fixed effects for the built FAR of buildings prior to conversion[7]. In particular, we interact borough dummies with the built FAR of buildings, rounded to the nearest multiple of 0.5. The resulting estimation equation is

$$AGE_{ibsl} = \alpha_l + \lambda_{bs} + \gamma * X_{ibsl} + \theta FAR_{ibsl} + \varepsilon_{ibsl} \tag{5.3}$$

where b is the vector of dummies for borough and X_{ibsl} includes the distance of building i from the Empire State Building ("ESB").

Table 3 gives the estimation results for the correlation between maximum-allowed FAR and a building's age at demolition. As the theory predicts, the coefficient on *FAR* is positive, indicating that buildings with relatively more stringent regulation are demolished at an earlier time. In particular, if we do not include by-borough built FAR fixed effects, we find that a increasing the maximum-allowed FAR by 1 corresponds to a building being demolished 0.71 of a year later. If we include by-borough built FAR fixed effects, we find that a 1 unit increase in the maximum-allowed

[7]Using by-zip code built FAR fixed effects yields similar results.

FAR corresponds to a building being demolished 0.25 of a year later.

To see the intuition behind this result, consider the single conversion case of Section 3. Recall Equation (3.2):

$$p(t)\,h(S) = \rho r S$$

Equation (3.2) shows that conversion will take place when the rent foregone from waiting for an extra period is equal to the benefit of putting off construction for one period. If the developer decides to build at an earlier time, the building will cost more than it could generate in rents at the margin. Now suppose that we decrease the maximum-allowed FAR. Because of the concavity of $h(\cdot)$, the LHS of (3.2) will decrease by less than the RHS of (3.2). Thus, the time at which the marginal rent will equal the marginal construction cost will come at an earlier time.

Once again, we perform a robustness check by creating clusters based on borough, block, usage, and built year and by excluding the built FAR fixed effect, instead inserting the built FAR directly in the regression as a control. The results are shown in columns (5)-(8). Though the magnitude of the estimation is lowered slightly, the direction and the significance remains the same. \

A limitation of our study is that the data may be censored on the right. That is, if the theory is correct, some properties with relatively loose FAR regulation may be demolished at a later date, which we would not be able to observe in the data. Censoring of data is most likely to lead to a downward bias of our estimates. If we include buildings that are built in the relevant time frame but are demolished at a later date, it is most likely to drive up the estimates we have in Table 3. To address this issue, we conduct a survival analysis, taking into account the censored data. We utilize the Cox proportional hazards model with 2018 as the year when the demolition data is censored. Table 4 shows the results of the survival analysis. The estimation shows that when the maximum-allowed FAR increases by 1, the age of the building at the time of demolition increases by 1.5% to 1.6%. Thus, even when taking into consideration the right-censoring of our data, the qualitative result remains the same.

5.3 Estimating the Effect of FAR Regulation Differential on Density of Conversion

To estimate the effect of FAR regulation differentials on the density of newly converted buildings, we regress the built FAR of new buildings on the maximum-allowed FAR. Our model predicts that if two buildings are similar in characteristics, the building with the lower maximum-allowed FAR will be converted to a lower density building.

As in the previous section, we control for the built year and the old built FAR for similar reasons. That is, buildings that were built in similar years to a similar density must also have similar characteristics. Furthermore, the old built FAR is especially relevant here because as seen in equation (5.1), the old built FAR influences when the developer should convert his building, which in turn affects the density of the new building. Because the model is deterministic, initial parameters are important in accurately determining the optimal choices of timing and density.

Table 4: Effect of FAR Regulation on the Timing of Conversion-Survival

VARIABLES	(1) Age	(2) Age	(3) Age	(4) Age
FAR	0.0154**	0.0158**	0.0152*	0.0157**
	(0.00782)	(0.00791)	(0.00783)	(0.00791)
BuiltFAR		-0.0918**		-0.0753*
		(0.0401)		(0.0417)
Distance (Distance from ESB)			-0.00348**	-0.00326*
			(0.00173)	(0.00169)
Constant	186.7***	180.2***	186.7***	180.3***
	(0.219)	(0.505)	(0.221)	(0.506)
Cluste FE	Yes	Yes	Yes	Yes
Observations	7,318	7,318	7,318	7,318
R-squared	0.915	0.914	0.915	0.914

Notes: In columns 2 and 4, by-borough built FAR fixed effects were included.
Robust standard errors in parentheses+–+
*** p<0.01, ** p<0.05, * p<0.1

Once again, the simplest form of of the estimation equation is given by

$$NBF_{isl} = \alpha_l + \lambda_s + \theta FAR_{isl} + \varepsilon_{isl} \tag{5.4}$$

where NBF is the new built FAR, and the controls are identical to the previous section.

We also include by-borough fixed effects for the old built FAR by interacting borough dummies with the old built FAR variable, which yields the estimation equation

$$NBF_{ibsl} = \alpha_l + \lambda_{bs} + \gamma * X_{ibsl} + \theta FAR_{ibsl} + \varepsilon_{ibsl} \tag{5.5}$$

Table 5 gives the estimation results for the correlation between maximum-allowed FAR and the density of converted buildings. Consistent with the theory, the coefficient on *FAR* is positive, indicating that properties with more stringent FAR regulation have buildings converted to lower densities. In particular, without by-borough built FAR fixed effects, we find that an increase in the maximum-allowed FAR by 1 corresponds to a building being converted to a FAR of approximately 0.37 higher than otherwise. With by-borough built FAR fixed effects, this increase drops down to approximately 0.079.[8]

[8] The estimate of 0.079 higher FAR is lower than we had anticipated. For a 10,000 sqft lot, this is only a difference of 790 sqft extra of development realized for every 10,000 sqft more of development allowed.

Table 5: Effect of FAR Regulation on the Density of Conversion

VARIABLES	(1) New Built FAR	(2) New Built FAR	(3) New Built FAR	(4) New Built FAR
FAR	0.371***	0.0788**	0.369***	0.0789**
	(0.0374)	(0.0329)	(0.0375)	(0.0332)
Distance (Distance from ESB)			-0.00267***	2.92e-05
			(0.000928)	(0.000635)
Constant	-35.19**	-7.004	-35.82**	-6.996
	(17.54)	(11.73)	(17.52)	(11.75)
Built Decade FE	Yes	Yes	Yes	Yes
Old Built FAR FE	Yes	No	Yes	No
By-Borough Built FAR FE	No	Yes	No	Yes
Observations	7,299	7,299	7,299	7,299
R-squared	0.222	0.486	0.222	0.486

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

6 Conclusion

We have presented a model which sheds new light on the effects of FAR regulation in the development of a growing city with rising rents. The model predicts that when the stringency of FAR regulation increases without changing the total number of conversions, buildings will be erected at an earlier time at lower densities. The FAR regulation will also lower land values at an increasing rate as the regulation becomes more stringent. In the case that there is only one conversion of land from rural to urban use where the FAR regulation is binding, the percent amount by which land value is decreased due to an increase in the stringency of regulation depends on the elasticity of substitution between land and capital for the production of housing. If we assume a Cobb-Douglas production function as in the previous literature, the elasticity of land value with respect to the maximum-allowed FAR will be constant at all levels of regulation stringency.

We have also presented an empirical model which tests the results we find in our theoretical model. In particular, we find that all else equal, a more stringent FAR regulation is correlated with lower land values even for buildings where the regulation is not binding, earlier conversion of buildings, and lower densities of conversions.

Appendix 1

Proof of Proposition 2

We would like to prove that the introduction of a density regulation hastens development for all building cycles. To do this, we Cramer's rule on the matrix of the set of comparative statics equations to show that $\frac{d\hat{t}_1}{d\bar{S}} > 0$, $\frac{d\hat{t}_k}{d\bar{S}} > 0$ for any $k \in (0,J)$, and $\frac{d\hat{t}_J}{d\bar{S}} > 0$.

The set of comparative statics equations, in a matrix form, is given by

$$\begin{bmatrix} V_{\hat{t}_1\hat{t}_1} & V_{\hat{t}_1\hat{S}_1} & 0 & \cdots & 0 \\ V_{\hat{S}_1\hat{t}_1} & V_{\hat{S}_1\hat{S}_1} & V_{\hat{S}_1\hat{t}_2} & \ddots & 0 \\ 0 & V_{\hat{t}_2\hat{S}_1} & V_{\hat{t}_2\hat{t}_2} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & V_{\hat{S}_{J-1}\hat{t}_J} \\ 0 & 0 & \cdots & V_{\hat{t}_J\hat{S}_{J-1}} & V_{\hat{t}_J\hat{t}_J} \end{bmatrix} \begin{bmatrix} \frac{d\hat{t}_1}{d\bar{S}} \\ \frac{d\hat{S}_1}{d\bar{S}} \\ \frac{d\hat{t}_2}{d\bar{S}} \\ \frac{d\hat{S}_2}{d\bar{S}} \\ \vdots \\ \frac{d\hat{t}_J}{d\bar{S}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ -V_{\hat{t}_J\bar{S}} \end{bmatrix} \qquad \text{(A2.1)}$$

We show in order that $\frac{d\hat{t}_1}{d\bar{S}} > 0$, $\frac{d\hat{t}_k}{d\bar{S}} > 0$ for any $k \in [1,J]$, and $\frac{d\hat{t}_J}{d\bar{S}} > 0$.

Lemma 1. $\frac{d\hat{t}_1}{d\bar{S}} > 0$.

Proof. Using Cramer's Rule on $(A2.1)$ to find $\frac{d\hat{t}_1}{d\bar{S}}$ yields

$$\frac{d\hat{t}_1}{d\bar{S}} = \frac{\begin{vmatrix} 0 & V_{\hat{t}_1\hat{S}_1} & 0 & \cdots & 0 \\ 0 & V_{\hat{S}_1\hat{S}_1} & V_{\hat{S}_1\hat{t}_2} & \ddots & 0 \\ 0 & V_{\hat{t}_2\hat{S}_1} & V_{\hat{t}_2\hat{t}_2} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & V_{\hat{S}_{J-1}\hat{t}_J} \\ -V_{\hat{t}_J\bar{S}} & 0 & \cdots & V_{\hat{t}_J\hat{S}_{J-1}} & V_{\hat{t}_J\hat{t}_J} \end{vmatrix}}{\begin{vmatrix} V_{\hat{t}_1\hat{t}_1} & V_{\hat{t}_1\hat{S}_1} & 0 & \cdots & 0 \\ V_{\hat{S}_1\hat{t}_1} & V_{\hat{S}_1\hat{S}_1} & V_{\hat{S}_1\hat{t}_2} & \ddots & 0 \\ 0 & V_{\hat{t}_2\hat{S}_1} & V_{\hat{t}_2\hat{t}_2} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & V_{\hat{S}_{J-1}\hat{t}_J} \\ 0 & 0 & \cdots & V_{\hat{t}_J\hat{S}_{J-1}} & V_{\hat{t}_J\hat{t}_J} \end{vmatrix}} \qquad \text{(A2.2)}$$

Applying the Chio pivotal condensation[9] yields

$$\frac{dt_1^*}{d\overline{S}} = \frac{\frac{1}{\left(-V_{\hat{t}_J\overline{S}}\right)^{2J-1-2}} \begin{vmatrix} V_{\hat{t}_J\overline{S}}V_{\hat{t}_1\hat{S}_1} & 0 & \cdots & 0 \\ V_{\hat{t}_J\overline{S}}V_{\hat{S}_1\hat{S}_1} & V_{\hat{t}_J\overline{S}}V_{\hat{S}_1\hat{t}_2} & \ddots & 0 \\ V_{\hat{t}_J\overline{S}}V_{\hat{t}_2\hat{S}_1} & V_{\hat{t}_J\overline{S}}V_{\hat{t}_2\hat{t}_2} & \ddots & \vdots \\ \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & V_{\hat{t}_J\overline{S}}V_{\hat{S}_{J-1}\hat{S}_{J-1}} & V_{\hat{t}_J\overline{S}}V_{\hat{S}_{J-1}\hat{t}_J} \end{vmatrix}}{\det(M)} \tag{A2.3}$$

$$= \frac{\frac{V_{\hat{t}_J\overline{S}}^{2J-2}}{\left(-V_{\hat{t}_J\overline{S}}\right)^{2J-1-2}} \left(V_{\hat{t}_1\hat{S}_1} \cdot V_{\hat{S}_1\hat{t}_2} \cdots V_{\hat{S}_{J-1}\hat{t}_J}\right)}{\det(M)} \tag{A2.4}$$

where the second equality comes from the property of the determinant of a triangular matrix[10] and

$$M \equiv \begin{bmatrix} V_{\hat{t}_1\hat{t}_1} & V_{\hat{t}_1\hat{S}_1} & 0 & \cdots & 0 \\ V_{\hat{S}_1\hat{t}_1} & V_{\hat{S}_1\hat{S}_1} & V_{\hat{S}_1\hat{t}_2} & \ddots & 0 \\ 0 & V_{\hat{t}_2\hat{S}_1} & V_{\hat{t}_2\hat{t}_2} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & V_{\hat{S}_{J-1}\hat{t}_J} \\ 0 & 0 & \cdots & V_{\hat{t}_J\hat{S}_{J-1}} & V_{\hat{t}_J\hat{t}_J} \end{bmatrix} \tag{A2.5}$$

We now show that $(A2.4)$ is positive. We know that $\det(M) < 0$ since it is a negative-definite Hessian matrix of rank $2J-1$ derived from the developer's profit-maximization problem. The numerator of $(A2.4)$ is negative since the only negative term in the numerator is $\left(-V_{\hat{t}_J\overline{S}}\right)^{2J-1-2}$. Since all other terms are cross partial derivatives of the Hessian matrix, they are positive. Therefore, when the maximum-allowed FAR which will be binding in the future increases while maintaining the original number of building conversions, the construction of the first building is delayed. □

Lemma 2. $\frac{d\hat{t}_k}{d\overline{S}} > 0$ *for any* $k \in [2, J-1]$.

[9] Consider a $n \times n$ matrix A. Define a $(n-1)\times(n-1)$ matrix of determinants $B = \left[b_{ij}\right]$ such that $b_{ij} = a_{1,1}a_{i+1,j+1} - a_{1,j+1}a_{i+1,1}$ for $a_{ii} \neq 0$.Then, the Chio pivotal condensation allows us to find $\det(A)$ in terms of $\det(B)$:

$$\det(A) = \frac{\det(B)}{a_{11}^{n-2}}$$

Explicitly,

$$\det(A) = \frac{1}{a_{11}^{n-2}} \begin{vmatrix} \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} & \cdots & \begin{vmatrix} a_{11} & a_{1n} \\ a_{21} & a_{2n} \end{vmatrix} \\ \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} & \cdots & \begin{vmatrix} a_{11} & a_{1n} \\ a_{31} & a_{3n} \end{vmatrix} \\ \vdots & \vdots & \ddots & \vdots \\ \begin{vmatrix} a_{11} & a_{12} \\ a_{n1} & a_{n2} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{13} \\ a_{n1} & a_{n3} \end{vmatrix} & \cdots & \begin{vmatrix} a_{11} & a_{1n} \\ a_{n1} & a_{nn} \end{vmatrix} \end{vmatrix}$$

[10] Let T_n be a triangular matrix of order n. Then, $\det(T_n)$ is equal to the product of all the diagonal elements of T_n.

Proof. Using Cramer's Rule on $(A2.1)$ to find $\frac{d\hat{t}_k}{d\bar{S}}$ for any $j \in [2, J-1]$ yields

$$\frac{d\hat{t}_k}{d\overline{S}} = \frac{\begin{vmatrix} V_{\hat{t}_1\hat{t}_1} & V_{\hat{t}_1\hat{S}_1} & \cdots & \overbrace{0}^{(2k-1)\text{th column}} & \cdots & \cdots & 0 \\ V_{\hat{S}_1\hat{t}_1} & V_{\hat{S}_1\hat{S}_1} & \ddots & 0 & \ddots & \ddots & 0 \\ 0 & V_{\hat{t}_2\hat{S}_1} & \ddots & \vdots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 & \ddots & \ddots & V_{\hat{S}_{J-1}\hat{t}_J} \\ 0 & 0 & \cdots & -V_{\hat{t}_J\overline{S}} & \cdots & V_{\hat{t}_J\hat{S}_{J-1}} & V_{\hat{t}_J\hat{t}_J} \end{vmatrix}}{\det(M)} \tag{A2.6}$$

Once again, applying the Chio pivotal condensation yields

$$\frac{d\hat{t}_k}{d\overline{S}} = \frac{\frac{1}{\left(-V_{\hat{t}_J\overline{S}}\right)^{2J-1-2}}\begin{vmatrix} W & X \\ Y & Z \end{vmatrix}}{\det(M)} \tag{A2.7}$$

where W is a $2k \times 2k$ matrix defined by

$$W \equiv \begin{bmatrix} -V_{\hat{t}_J\overline{S}}V_{\hat{t}_1\hat{t}_1} & -V_{\hat{t}_J\overline{S}}V_{\hat{t}_1\hat{S}_1} & \cdots & 0 \\ -V_{\hat{t}_J\overline{S}}V_{\hat{S}_1\hat{t}_1} & -V_{\hat{t}_J\overline{S}}V_{\hat{S}_1\hat{S}_1} & \ddots & 0 \\ 0 & -V_{\hat{t}_J\overline{S}}V_{\hat{t}_2\hat{S}_1} & \ddots & \vdots \\ \vdots & \ddots & \ddots & -V_{\hat{t}_J\overline{S}}V_{\hat{t}_k\hat{S}_k} \\ 0 & \cdots & -V_{\hat{t}_J\overline{S}}V_{\hat{S}_k\hat{t}_k} & -V_{\hat{t}_J\overline{S}}V_{\hat{S}_k\hat{S}_k} \end{bmatrix}$$

X is a $2k \times (2J-2-2k)$ matrix of zeroes, Y is a $(2J-2-2k) \times 2k$ matrix defined by

$$Y \equiv \begin{bmatrix} 0 & 0 & \cdots & -V_{\hat{t}_J\overline{S}}V_{\hat{t}_{k+1}\hat{S}_k} \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 \end{bmatrix}$$

and Z is a $2J-2-2k \times 2J-2-2k$ matrix defined by

$$Z \equiv \begin{bmatrix} V_{\hat{t}_J\bar{S}}V_{\hat{t}_{k+1}\hat{S}_{k+1}} & 0 & \cdots & \cdots & 0 \\ V_{\hat{t}_J\bar{S}}V_{\hat{S}_{k+1}\hat{S}_{k+1}} & V_{\hat{t}_J\bar{S}}V_{\hat{S}_{k+1}\hat{t}_{k+2}} & \ddots & \ddots & 0 \\ 0 & V_{\hat{t}_J\bar{S}}V_{\hat{t}_{k+2}\hat{t}_{k+2}} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & V_{\hat{t}_J\bar{S}}V_{\hat{S}_{J-1}\hat{S}_{J-1}} & V_{\hat{t}_J\bar{S}}V_{\hat{S}_{J-1}\hat{t}_J} \end{bmatrix}$$

Evaluating the determinant of the block triangular matrix[11] and simplifying yields

$$\frac{d\hat{t}_k}{d\bar{S}} = \frac{\frac{1}{\left(-V_{\hat{t}_J\bar{S}}\right)^{2J-1-2}} \det(W)\cdot\det(Z)}{\det(M)} \tag{A2.8}$$

$$= \frac{\frac{1}{\left(-V_{\hat{t}_J\bar{S}}\right)^{2J-1-2}} \left(-V_{\hat{t}_J\bar{S}}\right)^{2k} \det\left(\hat{W}\right)\cdot\left(V_{\hat{t}_J\bar{S}}\right)^{2J-2-2k}\left(V_{\hat{t}_{k+1}\hat{S}_{k+1}}\cdot V_{\hat{S}_{k+1}\hat{t}_{k+2}}\cdots V_{\hat{S}_{J-1}\hat{t}_J}\right)}{\det(M)} \tag{A2.9}$$

where

$$\hat{W} \equiv \begin{bmatrix} V_{\hat{t}_1\hat{t}_1} & V_{\hat{t}_1\hat{S}_1} & \cdots & 0 \\ V_{\hat{S}_1\hat{t}_1} & V_{\hat{S}_1\hat{S}_1} & \ddots & 0 \\ 0 & V_{\hat{t}_2\hat{S}_1} & \ddots & \vdots \\ \vdots & \ddots & \ddots & V_{\hat{t}_k\hat{S}_k} \\ 0 & \cdots & V_{\hat{S}_k\hat{t}_k} & V_{\hat{S}_k\hat{S}_k} \end{bmatrix}$$

[11]The determinant of a block triangular matrix $M \equiv \begin{pmatrix} X & 0 \\ Y & Z \end{pmatrix}$ is the product of the determinants of its diagonal blocks.

We know that $(A2.9)$ is positive, since $\det(M) < 0$, $\left(-V_{\hat{t}_J\bar{S}}\right)^{2J-1-2} < 0$, and $\det\left(\hat{W}\right) > 0$ since $\hat{W}$ is a negative definite matrix of rank $2k$. As in Lemma 1, this shows that increasing the stringency of a FAR restriction which binds in the future but will not change the number of conversions will hasten the development of all buildings prior to the last building cycle. $\square$

Lemma 3. $\frac{d\hat{t}_J}{d\bar{S}} > 0$.

Proof. Using Cramer's Rule on $(A2.1)$ to find $\frac{d\hat{t}_J}{d\bar{S}}$ yields

$$\frac{d\hat{t}_J}{d\bar{S}} = \frac{\begin{vmatrix} V_{\hat{t}_1\hat{t}_1} & V_{\hat{t}_1\hat{S}_1} & 0 & \cdots & 0 \\ V_{\hat{S}_1\hat{t}_1} & V_{\hat{S}_1\hat{S}_1} & V_{\hat{S}_1\hat{t}_2} & \ddots & 0 \\ 0 & V_{\hat{t}_2\hat{S}_1} & V_{\hat{t}_2\hat{t}_2} & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & V_{\hat{t}_J\hat{S}_{J-1}} & -V_{\hat{t}_J\bar{S}} \end{vmatrix}}{\det(M)} \tag{A2.10}$$

Using the recurrence relation for the determinant for a tridiagonal matrix[12] yields

$$\frac{d\hat{t}_J}{d\bar{S}} = \frac{-V_{\hat{t}_J\bar{S}_J}\det(L)}{\det(M)} \tag{A2.11}$$

where

$$L \equiv \begin{bmatrix} V_{\hat{t}_1\hat{t}_1} & V_{\hat{t}_1\hat{S}_1} & 0 & \cdots & 0 \\ V_{\hat{S}_1\hat{t}_1} & V_{\hat{S}_1\hat{S}_1} & V_{\hat{S}_1\hat{t}_2} & \ddots & 0 \\ 0 & V_{\hat{t}_2\hat{S}_1} & V_{\hat{t}_2\hat{t}_2} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & V_{\hat{t}_{J-1}\hat{S}_{J-1}} \\ 0 & 0 & \cdots & V_{\hat{S}_{J-1}\hat{t}_{J-1}} & V_{\hat{S}_{J-1}\hat{S}_{J-1}} \end{bmatrix} \tag{A2.12}$$

But since matrices L and M are both tridiagonal matrices where $\operatorname{sgn}\det(L) = -\operatorname{sgn}\det(M)$, the above must be postive. That is, when the maximum-allowed FAR increases holding the number of conversions constant, the construction of the final building is delayed. □

[12]Consider a tridiagonal matrix

$$f_n = \begin{bmatrix} a_1 & b_1 & 0 & \cdots & 0 \\ c_1 & a_2 & b_2 & \ddots & 0 \\ 0 & c_2 & a_3 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & b_{n-1} \\ 0 & 0 & \cdots & c_{n-1} & a_n \end{bmatrix}$$

Then, the determinant of matrix f_n can be computed from a recurrence relation given by

$$f_n = a_n f_{n-1} - c_{n-1} b_{n-1} f_{n-2}$$

Appendix 2

Proof of Proposition 3

We would like to prove that the introduction of a FAR regulation reduces the density of buildings for all conversions.

Proof. Consider the set of comparative statics equations as in Appendix 1:

$$\begin{bmatrix} V_{\hat{t}_1\hat{t}_1} & V_{\hat{t}_1\hat{S}_1} & 0 & \cdots & 0 \\ V_{\hat{S}_1\hat{t}_1} & V_{\hat{S}_1\hat{S}_1} & V_{\hat{S}_1\hat{t}_2} & \ddots & 0 \\ 0 & V_{\hat{t}_2\hat{S}_1} & V_{\hat{t}_2\hat{t}_2} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & V_{\hat{S}_{J-1}\hat{t}_J} \\ 0 & 0 & \cdots & V_{\hat{t}_J\hat{S}_{J-1}} & V_{\hat{t}_J\hat{t}_J} \end{bmatrix} \begin{bmatrix} \frac{d\hat{t}_1}{d\bar{S}} \\ \frac{d\hat{S}_1}{d\bar{S}} \\ \frac{d\hat{t}_2}{d\bar{S}} \\ \frac{d\hat{S}_2}{d\bar{S}} \\ \vdots \\ \frac{d\hat{t}_J}{d\bar{S}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ -V_{\hat{t}_J\bar{S}} \end{bmatrix} \tag{A3.1}$$

By Cramer's Rule, we have for any $k \in [1, J-1]$,

$$\frac{d\hat{S}_k}{d\bar{S}} = \frac{\begin{vmatrix} V_{\hat{t}_1\hat{t}_1} & V_{\hat{t}_1\hat{S}_1} & \cdots & \overbrace{0}^{(2k)\text{th column}} & \cdots & \cdots & 0 \\ V_{\hat{S}_1\hat{t}_1} & V_{\hat{S}_1\hat{S}_1} & \ddots & 0 & \ddots & \ddots & 0 \\ 0 & V_{\hat{t}_2\hat{S}_1} & \ddots & \vdots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 & \ddots & \ddots & V_{\hat{S}_{J-1}\hat{t}_J} \\ 0 & 0 & \cdots & -V_{\hat{t}_J\bar{S}} & \cdots & V_{\hat{t}_J\hat{S}_{J-1}} & V_{\hat{t}_J\hat{t}_J} \end{vmatrix}}{\det(M)} \tag{A3.2}$$

Applying the Chio pivotal condensation yields

$$\frac{d\hat{S}_k}{d\bar{S}} = \frac{\frac{1}{\left(-V_{\hat{t}_J\bar{S}}\right)^{2J-1-2}} \begin{vmatrix} W & X \\ Y & Z \end{vmatrix}}{\det(M)} \tag{A3.3}$$

where W is a $(2k-1)\times(2k-1)$ matrix defined by

$$W=\begin{bmatrix} -V_{\hat{t}_J\bar{S}}V_{\hat{t}_1\hat{t}_1} & -V_{\hat{t}_J\bar{S}}V_{\hat{t}_1\hat{S}_1} & \cdots & 0 \\ -V_{\hat{t}_J\bar{S}}V_{\hat{S}_1\hat{t}_1} & -V_{\hat{t}_J\bar{S}}V_{\hat{S}_1\hat{S}_1} & \ddots & 0 \\ 0 & -V_{\hat{t}_J\bar{S}}V_{\hat{t}_2\hat{S}_1} & \ddots & \vdots \\ \vdots & \ddots & \ddots & -V_{\hat{t}_J\bar{S}}V_{\hat{S}_{k-1}\hat{t}_k} \\ 0 & \cdots & -V_{\hat{t}_J\bar{S}}V_{\hat{t}_k\hat{S}_{k-1}} & -V_{\hat{t}_J\bar{S}}V_{\hat{t}_k\hat{t}_k} \end{bmatrix} \tag{A3.4}$$

X is a $(2k-1)\times(2J-2-2k-1)$ matrix of zeroes, Y is a $(2J-2-2k-1)\times(2k-1)$ matrix defined by

$$Y=\begin{bmatrix} 0 & 0 & \cdots & -V_{\hat{t}_J\bar{S}}V_{\hat{S}_k\hat{t}_k} \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 \end{bmatrix} \tag{A3.5}$$

and Z is a $(2J-2-2k-1)\times(2J-2-2k-1)$ matrix define by

$$Z=\begin{bmatrix} V_{\hat{t}_J\bar{S}}V_{\hat{S}_k\hat{t}_{k+1}} & 0 & \cdots & \cdots & 0 \\ V_{\hat{t}_J\bar{S}}V_{\hat{t}_{k+1}\hat{t}_{k+1}} & V_{\hat{t}_J\bar{S}}V_{\hat{t}_{k+1}\hat{S}_{k+1}} & \ddots & \ddots & 0 \\ 0 & V_{\hat{t}_J\bar{S}}V_{\hat{S}_{k+1}\hat{S}_{k+1}} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & V_{\hat{t}_J\bar{S}}V_{\hat{S}_{J-1}\hat{S}_{J-1}} & V_{\hat{t}_J\bar{S}}V_{\hat{S}_{J-1}\hat{t}_J} \end{bmatrix} \tag{A3.6}$$

Evaluating the determinant of the block triangular matrix and simplifying yields

$$\frac{d\hat{S}_k}{d\bar{S}} = \frac{\frac{1}{\left(-V_{\hat{t}_J\bar{S}}\right)^{2J-1-2}}\det(W)\cdot\det(Z)}{\det(M)} \tag{A3.7}$$

$$= \frac{\frac{1}{\left(-V_{\hat{t}_J\bar{S}}\right)^{2J-1-2}}\left(-V_{\hat{t}_J\bar{S}}\right)^{2k-1}\det\left(\hat{W}\right)\cdot\left(V_{\hat{t}_J\bar{S}}\right)^{2J-2-2k-1}\left(V_{\hat{S}_k\hat{t}_{k+1}}\cdot V_{\hat{t}_{k+1}\hat{S}_{k+1}}\cdots V_{\hat{S}_{J-1}\hat{t}_J}\right)}{\det(M)} \tag{A3.8}$$

where

$$\hat{W} \equiv \begin{bmatrix} V_{\hat{t}_1\hat{t}_1} & V_{\hat{t}_1\hat{S}_1} & \cdots & 0 \\ V_{\hat{S}_1\hat{t}_1} & V_{\hat{S}_1\hat{S}_1} & \ddots & 0 \\ 0 & V_{\hat{t}_2\hat{S}_1} & \ddots & \vdots \\ \vdots & \ddots & \ddots & V_{\hat{S}_{k-1}\hat{t}_k} \\ 0 & \cdots & V_{\hat{t}_k\hat{S}_{k-1}} & V_{\hat{t}_k\hat{t}_k} \end{bmatrix} \tag{A3.9}$$

We know that $(A3.8)$ is positive, since $\det(M) < 0$, $\left(-V_{\hat{t}_J\bar{S}}\right)^{2J-1-2} < 0$, $\left(-V_{\hat{t}_J\bar{S}}\right)^{2k-1} < 0$, and $\det\left(\hat{W}\right) < 0$ since $\hat{W}$ is a negative definite matrix of rank $2k-1$. This shows that a FAR restriction which binds in the future but does not change the number of conversions will reduce the density of development for all buildings prior to the final conversion. □

Appendix 3

Proof of Proposition 4

Proposition 4 states that the second derivative of the optimized land value function with FAR regulation is negative. To show this, we simplify $\frac{d^2\hat{V}}{d\overline{S}^2}$ into a ratio of matrices and show that the sign of the matrix are opposites of each other.

Lemma 4. $\frac{d^2\hat{V}}{d\overline{S}^2} = \frac{\det(N)}{\det(M)}$ *where matrix N is defined to be*

$$N \equiv \begin{bmatrix} V_{\hat{t}_1\hat{t}_1} & V_{\hat{t}_1\hat{S}_1} & 0 & 0 & \cdots & 0 \\ V_{\hat{S}_1\hat{t}_1} & V_{\hat{S}_1\hat{S}_1} & V_{\hat{S}_1\hat{t}_2} & 0 & \ddots & 0 \\ 0 & V_{\hat{t}_2\hat{S}_1} & V_{\hat{t}_2\hat{t}_2} & V_{\hat{t}_2\hat{S}_2} & \ddots & 0 \\ 0 & 0 & V_{\hat{S}_2\hat{t}_2} & V_{\hat{S}_2\hat{S}_2} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & V_{\hat{t}_J\overline{S}} \\ 0 & 0 & 0 & \cdots & V_{\overline{S}\hat{t}_J} & V_{\overline{S}\overline{S}} \end{bmatrix}$$

and matrix M is defined as in (A2.5).

Proof. Consider the value function optimized under a FAR restriction:

$$\hat{V} = V\left(\hat{t}_1\left(\overline{S}\right), \hat{S}_1\left(\overline{S}\right), \cdots ; \overline{S}\right) \tag{A4.1}$$

Given (3.18), the second derivative of the above can be written as

$$\begin{aligned} \frac{d^2\hat{V}}{d\overline{S}^2} = & \left[\sum_{k=1}^{J} V_{\hat{t}_k\hat{t}_1}\frac{d\hat{t}_k}{d\overline{S}} + \sum_{l=1}^{J-1} V_{\hat{S}_l\hat{t}_1}\frac{d\hat{S}_l}{d\overline{S}} + V_{\overline{S}\hat{t}_1}\right]\frac{d\hat{t}_1}{d\overline{S}} \\ & + \left[\sum_{k=1}^{J} V_{\hat{t}_k\hat{S}_1}\frac{d\hat{t}_k}{d\overline{S}} + \sum_{l=1}^{J-1} V_{\hat{S}_l\hat{S}_1}\frac{d\hat{S}_l}{d\overline{S}} + V_{\overline{S}\hat{S}_1}\right]\frac{d\hat{S}_1}{d\overline{S}} \\ & + \cdots + \left[\sum_{k=1}^{J} V_{\hat{t}_k\hat{S}_{J-1}}\frac{d\hat{t}_k}{d\overline{S}} + \sum_{l=1}^{J-1} V_{\hat{S}_l\hat{S}_{J-1}}\frac{d\hat{S}_l}{d\overline{S}} + V_{\overline{S}\hat{S}_{J-1}}\right]\frac{d\hat{S}_{J-1}}{d\overline{S}} \\ & + \left[\sum_{k=1}^{J} V_{\hat{t}_k\hat{t}_J}\frac{d\hat{t}_k}{d\overline{S}} + \sum_{l=1}^{J-1} V_{\hat{S}_l\hat{t}_J}\frac{d\hat{S}_l}{d\overline{S}} + V_{\overline{S}\hat{t}_J}\right]\frac{d\hat{t}_J}{d\overline{S}} \\ & + \left[\sum_{k=1}^{J} V_{\hat{t}_k\overline{S}}\frac{d\hat{t}_k}{d\overline{S}} + \sum_{l=1}^{J-1} V_{\hat{S}_l\overline{S}}\frac{d\hat{S}_l}{d\overline{S}} + V_{\overline{S}\overline{S}}\right] \end{aligned} \tag{A4.2}$$

Noting that $V_{\hat{t}_i\hat{t}_j} = V_{\hat{t}_j\hat{t}_i} = 0 \quad \forall i \neq j$ and that $V_{\hat{t}_i\hat{S}_j} = V_{\hat{S}_j\hat{t}_i} = 0 \quad \forall j > i \cup i - j \geq 2$ yields

$$\frac{d^2\hat{V}}{d\overline{S}^2} = \left[V_{\hat{t}_1\hat{t}_1}\frac{d\hat{t}_1}{d\overline{S}} + V_{\hat{S}_1\hat{t}_1}\frac{d\hat{S}_1}{d\overline{S}} + 0 + \cdots + 0\right]\frac{d\hat{t}_1}{d\overline{S}} \tag{A4.3}$$

$$
\begin{aligned}
&+\left[V_{\hat{t}_1\hat{S}_1}\frac{d\hat{t}_1}{d\bar{S}}+V_{\hat{S}_1\hat{S}_1}\frac{d\hat{S}_1}{d\bar{S}}+V_{\hat{t}_2\hat{S}_1}\frac{d\hat{t}_2}{d\bar{S}}+0+\cdots+0\right]\frac{d\hat{S}_1}{d\bar{S}}\\
&+\cdots+\left[0+\cdots+0+V_{\hat{t}_{J-1}\hat{S}_{J-1}}\frac{d\hat{t}_{J-1}}{d\bar{S}}+V_{\hat{S}_{J-1}\hat{S}_{J-1}}\frac{d\hat{S}_{J-1}}{d\bar{S}}+V_{\hat{t}_J\hat{S}_{J-1}}\frac{d\hat{t}_J}{d\bar{S}}+0\right]\frac{d\hat{S}_{J-1}}{d\bar{S}}\\
&+\left[0+\cdots+0+V_{\hat{S}_{J-1}\hat{t}_J}\frac{d\hat{S}_{J-1}}{d\bar{S}}+V_{\hat{t}_J\hat{t}_J}\frac{d\hat{t}_J}{d\bar{S}}+V_{\bar{S}\hat{t}_J}\right]\frac{d\hat{t}_J}{d\bar{S}}\\
&+\left[0+\cdots+0+V_{\hat{t}_J\bar{S}}\frac{d\hat{t}_J}{d\bar{S}}+V_{\overline{SS}}\right]
\end{aligned}
$$

In the matrix form, the above can be written as

$$
\begin{aligned}
\frac{d^2\hat{V}}{d\bar{S}^2}&=\begin{bmatrix}\frac{d\hat{t}_1}{d\bar{S}} & \frac{d\hat{S}_1}{d\bar{S}} & \frac{d\hat{t}_2}{d\bar{S}} & \frac{d\hat{S}_2}{d\bar{S}} & \cdots & 1\end{bmatrix}\\
&\quad\cdot\begin{bmatrix}
V_{\hat{t}_1\hat{t}_1} & V_{\hat{t}_1\hat{S}_1} & 0 & 0 & \cdots & 0\\
V_{\hat{S}_1\hat{t}_1} & V_{\hat{S}_1\hat{S}_1} & V_{\hat{S}_1\hat{t}_2} & 0 & \ddots & 0\\
0 & V_{\hat{t}_2\hat{S}_1} & V_{\hat{t}_2\hat{t}_2} & V_{\hat{t}_2\hat{S}_2} & \ddots & 0\\
0 & 0 & V_{\hat{S}_2\hat{t}_2} & V_{\hat{S}_2\hat{S}_2} & \ddots & \vdots\\
\vdots & \ddots & \ddots & \ddots & \ddots & V_{\hat{t}_J\bar{S}}\\
0 & 0 & 0 & \cdots & V_{\bar{S}\hat{t}_J} & V_{\overline{SS}}
\end{bmatrix}
\begin{bmatrix}\frac{d\hat{t}_1}{d\bar{S}}\\ \frac{d\hat{S}_1}{d\bar{S}}\\ \frac{d\hat{t}_2}{d\bar{S}}\\ \frac{d\hat{S}_2}{d\bar{S}}\\ \vdots\\ 1\end{bmatrix}\\
&=\begin{bmatrix}\frac{dV_{\hat{t}_1}}{d\bar{S}} & \frac{dV_{\hat{S}_1}}{d\bar{S}} & \frac{dV_{\hat{t}_2}}{d\bar{S}} & \frac{dV_{\hat{S}_2}}{d\bar{S}} & \cdots & V_{\overline{SS}}+V_{\hat{t}_J\bar{S}}\frac{d\hat{t}_J}{d\bar{S}}\end{bmatrix}
\begin{bmatrix}\frac{d\hat{t}_1}{d\bar{S}}\\ \frac{d\hat{S}_1}{d\bar{S}}\\ \frac{d\hat{t}_2}{d\bar{S}}\\ \frac{d\hat{S}_2}{d\bar{S}}\\ \vdots\\ 1\end{bmatrix}\\
&=\begin{bmatrix}0 & 0 & 0 & 0 & \cdots & V_{\overline{SS}}+V_{\hat{t}_J\bar{S}}\frac{d\hat{t}_J}{d\bar{S}}\end{bmatrix}
\begin{bmatrix}\frac{d\hat{t}_1}{d\bar{S}}\\ \frac{d\hat{S}_1}{d\bar{S}}\\ \frac{d\hat{t}_2}{d\bar{S}}\\ \frac{d\hat{S}_2}{d\bar{S}}\\ \vdots\\ 1\end{bmatrix}\\
&=\left(V_{\overline{SS}}+V_{\hat{t}_J\bar{S}}\frac{d\hat{t}_J}{d\bar{S}}\right) \qquad \text{(A4.4)}
\end{aligned}
$$

where the third equality holds, since it is a derivative of a parameter along a maximized value function. We can now write $(A4.4)$ as

$$\frac{d^2\hat{V}}{d\bar{S}^2} = V_{\bar{S}\bar{S}} + V_{\hat{t}_J\bar{S}}\frac{d\hat{t}_J}{d\bar{S}}$$
$$= \frac{\left(V_{\bar{S}\bar{S}}\right)\det(M)}{\det(M)} - \frac{\left(V_{\hat{t}_J\bar{S}}\right)^2\det(L)}{\det(M)}$$
$$= \frac{\det(N)}{\det(M)} \tag{A4.7}$$

where the second equality comes from using the recurrence relation for computing the determinant for a tridiagonal matrix and Cramer's Rule on $\frac{d\hat{t}_J}{d\bar{S}}$[13], and the last equality comes from applying the recurrence relation for the determinant of a tridiagonal matrix again.

Since matrices N and M are both negative definite matrices where $\text{rank}(N) = 2J$ and $\text{rank}(M) = 2J-1$, we know that $\text{sgn}\det(N) = -\text{sgn}\det(M)$. Thus, $(A4.7)$ is negative. □

[13]Appendix 1

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