Learning Rival's Information in Interdependent Value Auctions

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Abstract

We study a simple auction model with interdependent values in which bidders can learn their rival's information in the first-price and second-price auctions. We characterize unique symmetric equilibrium strategies—both learning and bidding strategies—for the two auction formats. While bidders learn rival's signals with higher probabilities in the first-price auction, they earn higher rent in the second-price auction. We also show that when the learning cost is small or when bidders' signals are less correlated and values are less private, the revenue in the second-price auction is higher than in the first-price auction. The revenue ranking is reserved otherwise.

1 Introduction

In August 2013, the three Korean mobile network operators—SK Telecom, KT, and LG Uplus—competed for long-term evolution (LTE) wireless spectrum bands in the latest spectrum auction in Korea. Since KT was lagging behind its competitors in the LTE market at that moment, it was imperative for the company to get an extra spectrum block that could have been combined with its existing block to provide the LTE service. The first thing it did to prepare a bid in the auction was to form a task force that worked intensively to estimate the values of the spectrum blocks to its rivals as well as the company itself. Also, the task force advised the bid team throughout the bidding process and, to do so, they paid close

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attention to what the true values of the rival companies could be.[1] In the end, KT won the desired block by outbidding a rival company by less than a couple of million dollars in the contest where the three companies ended up paying more than two billion dollars in total.

We believe that the above story illustrates just one of many instances where bidders try to acquire information about rivals. While the information acquisition in auctions has been an important issue in the auction literature, there are few studies that investigate the acquisition of information about rival bidders.[2] Learning rival's information is important in two aspects: first, the learners can estimate the value of an auctioned object more precisely, gaining an *informational advantage*; second, they can better predict the bidding strategy of their rival, gaining a *strategic advantage*. Notice that the first aspect becomes important to the extent that one's value depends on his rival's information, that is, the values are *interdependent*. In the current paper, we study how the two aspects of learning work together to affect the bidders' incentive to learn their rival's information in standard auctions—first-price and second-price auctions—with interdependent values and thereby affect the performance of the two auction formats.

To this end, we consider a simple model in which there are two bidders, who are ex ante symmetric, competing for a single object. Each bidder is informed of a binary signal which is correlated with the other's signal. The value of the object for each bidder is given as a linear combination of his own signal and his rival's one with more weight on the former.[3] The weight assigned to the rival's signal measures the degree of value interdependence, capturing the private and common values as two polar cases. Our model involves a simple time-line: Initially, for a given auction format, each bidder decides whether to learn the other bidder's signal by incurring a cost. This decision is unobservable, i.e., the information acquisition is *covert*. Next, bidders simultaneously decide how much to bid based on their information. Lastly, the winner is announced and trade occurs according to the auction format.

This is the main setup for our study and we denote it I^2. In the paper, we also consider an alternative setup, called I^1, in which bidders are informed of no prior signal and can incur a cost to learn their own signal. This setup has been studied in the auction literature (for instance, Persico (2000) and Shi (2012) among others) and will be used as a benchmark to compare the results from the main setup.

[1] The auction format was a variant of the simultaneous ascending auction, followed by a one-shot, seal-bid stage.

[2] For a short while, we will review the literature on information acquisition in auctions and mechanism design in general.

[3] This implies that whoever holds a higher signal has a higher value for the object.

As opposed to the benchmark case, our main setup, I^2, has a couple of important implications: Since bidders are informed of prior signals, they will have multi-dimensional information in the bidding stage upon learning their rival's information. Moreover, the learning decision of each bidder is dependent on his prior signal, which is a key element of our analysis. Although the simplicity of our setup leaves a question of generalization, it is instrumental for obtaining clear intuition about how the possibility of acquiring rival's information affects bidders' learning and bidding behavior through the two channels—informational and strategic advantages, and this allows us to conduct various comparative statics analyses.

For the analysis, we characterize a unique (mixed-strategy) symmetric equilibrium, consisting of learning and bidding strategies, for the two auction formats. In the second-price auction, we show that no bidder learns the rivals' signal and a bidder with a higher prior signal—called *strong bidder*—always wins against the rival with a low signal—called *weak bidder*. In the first-price auction, some interesting properties emerge: First, despite binary signals, the number of bidder types in the bidding stage can increase substantially and is determined endogenously as a result of learning decision. Second, bidders' learning behavior varies with their prior signals. In particular, a strong bidder learns the rival's signal with higher probability than does a weak bidder. Third, a weak bidder may bid more aggressively than a strong bidder, even though the former has a lower value than the latter, as opposed to the standard first-price auction without learning possibilities.

The learning and bidding behavior in equilibrium can be explained by the aforementioned advantages. Clearly, the informational advantage is important in any auction format. In contrast, the strategic advantage is more important in the first-price auction where bidders wish to shade their bids, which makes a (correct) prediction of the rival's strategy important since the optimal shading depends on it. For this reason, the equilibrium strategy involves higher learning probabilities in the first-price auction than in the second-price auction.[4] Note also that in the first-price auction, the strategic advantage is more valuable to a strong bidder, who has a greater value and can thus shade his bid more when facing a weak rival. This makes a strong bidder more likely to learn the rival's signal than does a weak bidder. This does not mean, however, the ordering of bid distributions is monotone in bidder signal. In fact, due to the multi-dimensionality of bidder information, a weaker bidder who learns that his rival has a high signal may bid more aggressively than a strong bidder who learns

[4]Note that the strategic advantage exists even in I^1, since the learning of one's own signal helps predict the rival's (correlated) signal and thereby his bidding strategy. Hence, bidders in the first-price auction learn their own signal with higher probability than in the second-price auction under I^1 as well.

that his rival has a low signal.

For the payoff consequence of the equilibrium learning behavior, note that learning the rival's signal reduces "privateness" of bidders' own information, which leads to a lower rent for them in the first-price auction than in the second-price auction. However, the effect of learning on the seller revenue is more subtle, and we show that the first-price auction yields a higher revenue than the second-price auction when a weak bidder is more likely to learn, which holds true when the learning cost is small, signals are less correlated, and/or values are more interdependent. Otherwise, the second-price auction is revenue-superior. To understand this, note first that the learning probability decreases in the learning cost regardless of prior signals, which is intuitive. Note also that unlike a strong bidder, the benefit of learning for a weak bidder mostly derives from the informational advantage—that is, finding out whether his value is higher than that his prior signal would indicate. This advantage is thus more valuable when the values are more interdependent and/or when the signals are less correlated, inducing the weak bidder to learn with higher probability. This in turn induces the strong bidder to bid more competitively, since the strong bidder expects his weak rival to be informed of his high signal and bid more aggressively.[5] Consequently, the first-price auction is revenue-superior to the second-price auction when the learning probability of a weak bidder becomes high due to smaller learning cost, lower signal correlation, and/or more interdependent values.[6]

Several papers have studied the problem of information acquisition in auctions, but most of them have focused on the problem of learning bidders' own signal. In the private value setup, Stegeman (1996) and Shi (2012) study the second-price auction and the optimal auctions, respectively. In the interdependent values setup, Milgrom (1981) studies second-price auction and Matthews (1984) focuses on first-price common value auctions. Hausch and Li (1993) compare first-price and second-price auctions in a common value setting and show that the seller's revenue is higher in the second-price auction than in the first-price auction. Persico (2000) shows that the incentive to acquire information about the value of the object is stronger in the first-price auction than in the second-price auction when bidders' valuations are affiliated. In all of these studies, however, bidders are uninformed prior to learning and the possibility of learning rival's information is not considered.

[5] There is also a direct effect of the weak bidder's learning on his own bidding behavior, which is ambiguous, however, since he will bid more or less aggressively depending on the signal he has learned.

[6] The welfare comparison under I^1 is straightforward: bidders in the first-price auction are more likely to be private informed, receiving a higher rent and giving lower revenue to the seller.

Closely related to our study, Bergemann and Välimäki (2005) discuss the possibility that bidders engage in costly "espionage" in the first-price auction—which refers to the activity of learning other bidder's information. Fang and Morris (2006) and Tian and Xiao (2010) study how the outcomes of the standard auctions are affected when bidders observe their rival's information. Those papers, however, consider the private value setup as opposed to our paper. Fang and Morris (2006) show that in a private value setting, when each of two bidders observes an imperfect signal about the rival's valuation, the first-price auction provides more incentive to acquire information and so reduces the seller's revenue than the second-price auction. However, they assume that the signal about the rival's valuation is not acquired but exogenously given. Tian and Xiao (2010) extend Fang and Morris (2006) by endogenizing bidders' information acquisition.[7] To the best of our knowledge, our work is the first one studying the information acquisition of rival's signal in an interdependent value setup.

The paper is organized as follows. We introduce our model in Section 2. Section 3 analyzes the first-price and the second-price auctions under the setup I^1, as a benchmark. Section 4 characterizes equilibrium for our main setup I^2 in the two auction formats. The comparison between the outcomes of two auctions is provided in Section 5. Section 6 concludes the paper. Proofs are provided in the appendix unless stated otherwise.

2 The Model

Suppose that there is a single object to be auctioned off to two bidders, 1 and 2. Each bidder $i = 1, 2$ is initially informed of a signal s_i, which takes one of two values, 0 and 1. This signal will sometimes be referred to as bidder i's *prior signal*. We assume that $\text{Prob}(s_i = 0) = \text{Prob}(s_i = 1) = \frac{1}{2}$ for each $i = 1, 2$, and the two signals are correlated as follows: for all $i, j = 1, 2$ with $i \neq j$, and for all $m, m' \in \{0, 1\}$ with $m \neq m'$,

$$\text{Prob}(s_j = m | s_i = m) = 1 - \text{Prob}(s_j = m' | s_i = m) = \alpha \in (\tfrac{1}{2}, 1).$$

Hence, a higher α means a higher correlation between the signals.

[7] Tian and Xiao (2010) compare two specifications: *ex ante* and *interim* information acquisition where bidders can learn their rivals' valuations before and after observing their own valuations, respectively. See also Li and Tian (2008) for an analysis of the second-price auction.

The value of the object to each bidder $i = 1, 2$ is given as

$$v_i(s_i, s_j) = \beta s_i + (1-\beta)s_j, \quad \beta \in [\tfrac{1}{2}, 1],$$

that is, values are *interdependent* in the sense that each bidder's value depends on the other's signal as well as his own (unless $\beta = 1$). Note that when $\beta = \frac{1}{2}$, bidders have a common value, and when $\beta \in (1/2, 1]$, whoever has a higher signal has a higher value for the object. For this reason, bidders with a low prior signal will be often called *weak* bidders and those with a high prior signal *strong* bidders. Note that as β decreases, the relative impact of the other's signal on one's value increases, that is, the values become more interdependent. This implies that the knowledge of rival's signal becomes more important for the estimation of one's own value. Note also that as β decreases, the value difference between weak and strong bidders becomes smaller.

We consider two auction formats, first-price and second-price auctions. In both auctions, a bidder who submits a higher bid wins the object, while the winner pays the highest (i.e., his own) bid in the first-price auction and the second-highest (i.e., the rival's) bid in the second-price auction. Ties are broken randomly.

In each auction, our model of information acquisition consists of two stages; the *learning stage* and the subsequent *bidding stage*. In the learning stage, each bidder i decides whether to learn the rival's signal $s_j, j \neq i$, by incurring cost $k > 0$. We assume that whether each bidder has acquired information is unobservable to his rival.[8] In the bidding stage, the two bidders submit bids in a given auction format, based on the information they have acquired in the learning stage. The model described so far is henceforth referred to as the setup I^2. We also consider a benchmark setup, called I^1, which is identical to I^2 except that each bidder i is initially informed of no signals and decides to learn s_i at the learning stage by incurring cost $c > 0$. This setup has been studied by the previous literature, such as Persico (2000) and Shi (2012), and is used to compare the results from our main setup I^2.

The information each bidder i holds at the bidding stage is called bidder i's *type* and denoted by t_i, which can be both s_i and s_j, or only s_i, or none of the two signals, depending on the information setup as well as bidder i's learning decision in that setup. To simplify notation, for any $m, m' \in \{0, 1\}$, we let $t_i = mm'$ and $t_i = m$ indicate that bidder i is informed of $(s_i, s_j) = (m, m')$ and $s_i = m$, respectively, while $t_i = U$ indicates that bidder

[8]This is a model of *covert information acquisition*, which captures a situation where one's activity of information acquisition is not readily detectable to others, as is plausible in many cases.

i is uninformed of both signals (under I^1). We let $\overline{\Omega}_i$ denote the set of all possible types under each setup $I^n, n = 1, 2$: $\overline{\Omega}_1 = \{U, 0, 1\}$ and $\overline{\Omega}_2 = \{0, 1, 00, 01, 10, 11\}$. Let v_t denote the expected value of the object to each bidder conditional on his type being t:

$$\begin{aligned}
v_{11} &= \mathbb{E}[v_i(s_i, s_j)|(s_i, s_j) = (1, 1)] = 1 = 1 - v_{00}, \\
v_1 &= \mathbb{E}[v_i(s_i, s_j)|s_i = 1] = \beta + (1 - \beta)\alpha = 1 - v_0, \\
v_{10} &= \mathbb{E}[v_i(s_i, s_j)|(s_i, s_j) = (1, 0)] = \beta = 1 - v_{01}, \quad \text{and} \\
v_U &= \mathbb{E}[v_i(s_i, s_j)] = 1/2.
\end{aligned}$$

Note that $v_{00} \leq v_0 \leq v_{01} \leq v_U \leq v_{10} \leq v_1 \leq v_{11}$, where the inequalities become strict for $\beta \in (\frac{1}{2}, 1)$. Let $v(t, t')$ denote the expected value of the object to a bidder conditional on his own type being t and his rival's type being t'. Clearly, for any m, m' and t,

$$v(m, m') = v(mm', t) = v(t, mm') = v_{mm'},\; v(m, U) = v_m, \text{ and } v(U, U) = v_U.$$

Note also that

$$v(U, 0) = \mathbb{E}[v_i(s_i, s_j)|s_j = 0] = \beta(1 - \alpha) \text{ and } v(U, 1) = \mathbb{E}[v_i(s_i, s_j)|s_j = 1] = \beta\alpha + (1 - \beta).$$

We will sometimes write v_{U0} and v_{U1} to denote $v(U, 0)$ and $v(U, 1)$, respectively.

Define the *allocative surplus* as the expected value that bidders receive from the object allocation. In our setup of binary signals, the allocative surplus is higher if and only if each bidder i with $s_i = 1$ is more likely to win the object against the rival with $s_j = 0$. The maximum allocative surplus is achieved when the former always wins against the latter and equals $\frac{1}{2}\alpha + (1 - \alpha)\beta$.[9] The *total surplus* is equal to the allocative surplus minus the (expected) cost of learning.

Throughout the paper, we focus on the symmetric sequential equilibrium—henceforth referred to symmetric equilibrium or more simply equilibrium—, allowing for mixed strategies. The equilibrium strategy consists of learning strategy and bidding strategy. Under I^1, the equilibrium learning strategy is represented by π_U, the probability that each uninformed bidder i learns s_i. Similarly, under I^2, the equilibrium learning strategy is represented by π_0 and π_1, the probabilities that each bidder i learns s_j, $j \neq i$, conditional on his prior signal

[9] The maximum allocation surplus is obtained a bidder with the high signal wins whenever bidders have different signal.

being $s_i = 0$ and 1, respectively. Given the learning strategies, we let $\Omega \subset \overline{\Omega}$ denote the set of all bidder types that arise with positive probability in equilibrium.[10] The equilibrium bidding strategy is represented by a profile of bid distributions $\{H_t\}_{t\in\Omega}$, where $H_t(b)$ is the probability that a type-t bidder submits a bid less than or equal to $b \in \mathbb{R}_+$. Let E_t denote the support of the equilibrium bid distribution H_t with $int(E_t)$ denoting its interior, and let $\overline{b}_t := \sup E_t$ and $\underline{b}_t := \inf E_t$. The equilibrium payoff for each type in the bidding stage is denoted by Γ_t. Note that this payoff does not account for the information acquisition cost.

In a sequential equilibrium, bidders must behave optimally in both learning and bidding stages. First, the learning strategy must be optimal, comparing the learning cost and the payoff increase that accrues in the bidding stage from learning.[11] Specifically, bidders learn with a positive probability when the payoff increase from learning is no smaller than the learning cost, and strictly randomizes when they are equal. In the bidding stage, each bidder must bid optimally given his information (or type) and his belief on the rival's learning and bidding strategies. By definition of the sequential equilibrium, we impose the consistency requirement on the bidders' belief. This requirement is only slightly stronger than imposing the Bayes rule alone, in that it requires each bidder to believe that his rival follows the equilibrium learning and bidding strategies even after the bidder himself deviates from the equilibrium strategy in the learning stage.

3 Analysis of Setup I^1

In this section, we study the first-price and second-price auctions under the benchmark setup I^1. All proofs in this section are provided in the Supplementary Appendix.

We first characterize a unique symmetric equilibrium for the second-price auction. In the characterization, we focus on the range of cost c that permits bidders to learn with a positive probability.[12]

[10] For instance, if $\pi_U \in (0, 1)$ under I^1, then $\Omega = \{U, 0, 1\}$. Likely, if $\pi_0 = 0$ and $\pi_1 \in (0, 1)$ under I^2, then $\Omega = \{0, 1, 10, 11\}$.

[11] Because of the assumption that the learning decision is unobservable to the other bidder, learning an additional information can never hurt the bidder in the bidding state since he can simply ignore it. Kim (2008) shows, however, that a bidder can get worse off with having an additional information when the learning decision is observable, since it can cause a rival's adverse response.

[12] One can easily check that if $c = 0$, then $\pi_U = 1$, and if $c \geq \overline{c}$, then $\pi_U = 0$ in the unique equilibrium in both second-price and first-price auctions. It is well known that there are asymmetric equilibria in the (symmetric) second-price auction without information acquisition. In our model, there also exist asymmetric equilibria in which bidder i learns his signal and bids v_{11} if $s_i = 1$ and v_{00} if $s_i = 0$, and bidder $j \neq i$ bids v_U without learning his signal, where $i, j = 1, 2$ and $i \neq j$.

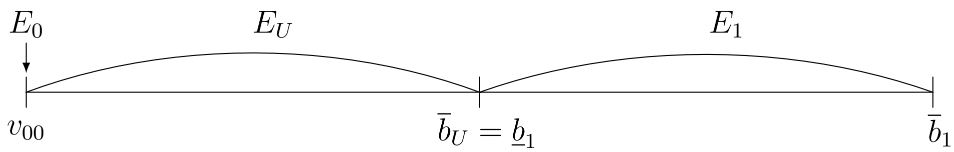


Figure 1: **Bid supports of the first-price auction under I^1 when $\pi_{\mathbf{U}} > \mathbf{0}$.**

Proposition 1 (Second-Price Auction)**.** *Suppose that $c \in (0, \overline{c})$, where $\overline{c} := \frac{v_1 - v_U}{2}$. Then, there exists a unique symmetric equilibrium of the second-price auction under I^1 in which*

(i) $\pi_U = \frac{v_1 - v_U - 2c}{v_1 - v_U} \in (0, 1)$*, which is increasing in α and β while decreasing in c;*

(ii) each bidder of type $t \in \Omega = \{0, U, 1\}$ bids $b_0 = v_{00} < b_U = v_U < b_1 = v_{11}$;

(iii) the payoff for each bidder is equal to $\Gamma_U = \frac{1}{2}\pi_U(v_{U0} - v_{00})$, which is decreasing in c.

To explain Part (ii) first, an uninformed bidder bids the ex ante expected value of the object, while each bidder who is informed of his own signal bids what would be the object value if his rival had the same signal. This strategy is consistent with the well known equilibrium characterization for the second-price auction in the standard interdependent value setup—e.g., Milgrom and Weber (1982)—where bidders' information is exogenously given. To explain Part (i), observe that each bidder i's learning yields a positive gain only when he learns $s_i = 1$ while his rival is uniformed and bids v_U in equilibrium, since if the rival is informed of $s_j = 0$ or 1, bidder i obtains the same payoff whether or not he is informed. Thus, the benefit from learning is proportional to $v_1 - v_U = \beta + (1 - \beta)\alpha - \frac{1}{2}$, capturing the informational advantage. This benefit increases as α or β increases—that is, signals are more correlated or values become more private—, explaining the effect of these parameters on π_U. Also, π_U is increasing as the learning cost c decreases, as intuitively clear. Lastly, the indifference between learning and not learning implies that the equilibrium payoff of each bidder i equals Γ_U, the payoff of an uninformed bidder. This explains Part (iii).

We next characterize the symmetric equilibrium for the first-price auction:

Proposition 2 (First-Price Auction)**.** *Suppose that $c \in (0, \overline{c})$. Then, there exists a unique equilibrium of the first-price auction under I^1 in which*

(i) $\pi_U \in (0, 1)$ *solves the equation*

$$(1 - \pi_U)(1 - \alpha\pi_U) = (v_1 - 2c - \pi_U v_1)(2 - \pi_U) \tag{1}$$

and is increasing in α and β while decreasing in c;

(ii) $v_{00} = \underline{b}_0 = \overline{b}_0 = \underline{b}_U < \overline{b}_U = \underline{b}_1 < \overline{b}_1$ *(refer to Figure 1), where*

$$\underline{b}_1 = \frac{(1-\pi_U)v_1 - 2c}{1-\alpha\pi_U} \quad \text{and} \quad \overline{b}_1 = v_1 - 2c - \pi_U v_{U0}; \tag{2}$$

(iii) the payoff for each bidder is equal to $\Gamma_U = \frac{1}{2}\pi_U(v_{U0} - v_{00})$, *which is decreasing in* c.

The equilibrium characterized in Proposition 2 is similar to the one for the second-price auction in Proposition 1, except that bidders are randomizing their bidding strategies as well as learning strategies. Figure 1 depicts the support of bid distribution for each type of bidders, E_t for $t \in \Omega \equiv \{U, 0, 1\}$, as given in Part *(ii)*. Note that different bidder types bid in distinct intervals, which will appear under I^2 as well.

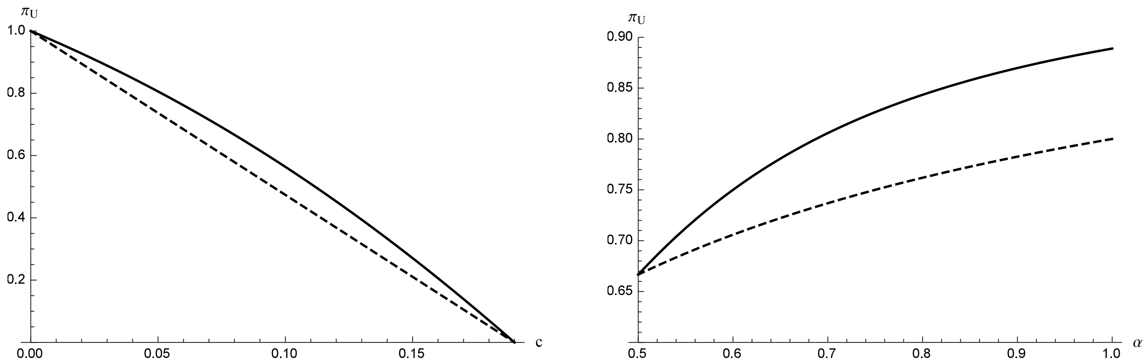
We now compare the outcomes of the first-price and second-price auctions.

Proposition 3. *Under* I^1*, the learning probability, the bidders' payoff and the allocative surplus are higher in the first-price auction than in the second-price auction, while the total surplus and the seller's revenue are higher in the second-price auction than in the first-price auction.*

The first-price auction induces bidders to learn their (own) signals with higher probability than does the second-price auction, as depicted in Figure 2(a), while the learning probabilities are strictly positive under both auctions. The higher probability that bidders are privately informed of their signals translates into more information rent of bidders, which makes each bidder's payoff higher in the first-price auction than in the second-price auction (even after accounting for the learning cost).

The higher learning probability in the first-price auction is reminiscent of the result established by Persico (2000), showing (in the setup with continuum signals) that bidders acquire more accurate signals in the first-price auction than in the second-price auction. Intuitively, this is because the strategic advantage is more important in the first-price auction, as explained in the introduction. Note that this advantage becomes greater when signals are more correlated, because the learning of one's own signal conveys more accurate information about the other's (correlated) signal and thus his bidding strategy. This explains why the discrepancy between the learning probabilities in the two auction formats increases as α increases, as shown in Figure 2(b).

To compare the total surplus in the two auctions, recall that the total surplus is equal to the allocative surplus minus the sum of the two bidders' (expected) learning cost. The latter



(a) Learning probability (according to c) (b) Learning probability (according to α)

Figure 2: **Comparison of learning probabilities.** Primitive values: $\alpha = 0.7$, $\beta = 0.6$, $c = 0.05$. The solid and the dashed lines represent the first- and the second-price auctions, respectively.

cost $2\pi_U c$ is proportional to the learning probability. The allocative surplus also increases in the learning probability, because when bidders are informed of their signals with higher probability, each bidder i with $s_i = 1$ is more likely to win against the rival bidder j with $s_j = 0$. Thus, both the learning cost and the allocative surplus increase going from the second-price to first-price auction. It turns out that the former increases more than the latter, so that the total surplus becomes higher in the second-price auction. Lastly, the observations so far imply that the seller's revenue, which equals the total surplus minus bidders' payoff, is higher in the second-price auction than in the first-price auction.

4 Analysis of Main Setup I^2

We now turn to the analysis of I^2 in which each bidder i is initially informed of s_i and decides whether or not to learn s_j, $j \neq i$. In this setup, we ask how bidders with different prior signals learn their rival's signal, and how it affects their payoffs, the seller's revenue, and the total surplus across the two auction formats. As we will show, the answers to these questions depend on the magnitude of learning cost (i.e., k) and the degrees of signal correlation and value interdependence (i.e., α and β). Similar to the previous setup, our intuition behind the results will come from understanding a combined effect of the informational and strategic advantages on bidders' incentive or disincentive to learn their rival's signal.

4.1 Second-Price Auction

The following theorem shows that the second-price auction induces no learning.

Theorem 1. *In the second-price auction under I^2, there exists a unique symmetric equilibrium in which*

(i) $\pi_1 = \pi_0 = 0$*;*

(ii) each bidder of type $t \in \Omega = \{0, 1\}$ *bids* $b_0 = v_{00} < b_1 = v_{11}$*;*

(iii) each bidder's payoff is $\frac{1}{2}(1-\alpha)(v_{10} - v_{00})$*, which is decreasing in* α*, increasing in* β *and independent of* k*.*

Proof. See Appendix A.1. ■

With no bidder learning the rival's signal, the equilibrium bidding strategy is identical to that of the standard setup without the information acquisition possibility. The property of this strategy is that the winning bidder's payment is weakly lower than his (true) value, while the winning bid is weakly higher than the losing bidder's (true) value.[13] This implies, in contrast with I^1 case, that neither the winning bidder nor the losing bidder can gain from the informational advantage. Hence, $\pi_1 = \pi_0 = 0$.[14]

Note also that given the equilibrium learning and bidding strategies, each bidder i with $s_i = 1$ wins for sure against the rival with $s_j = 0$. This implies that each bidder i's payoff is $\frac{1}{2}(1-\alpha)(v_{10} - v_{00})$, where $\frac{1}{2}(1-\alpha)$ is the probability that signal profile is $(s_i, s_j) = (1, 0)$, v_{10} is bidder i's value, and v_{00} is his payment. It also implies that the allocation is fully efficient, achieving the maximum allocative surplus $\frac{1}{2}\alpha + (1-\alpha)\beta$. The total surplus also achieves the first-best, since no bidder incurs the learning cost.

4.2 First-Price Auction

Turning to the analysis of the first-price auction under I^2, the following proposition provides the pattern of information acquisition in equilibrium:

Proposition 4. *In the first-price auction under I^2, the following results hold:*

(i) There exists a unique symmetric equilibrium with $\pi_1 = \pi_0 = 0$ *if and only if* $k \geq \overline{k}_1 := \alpha(1-\alpha)(v_{11} - v_{00})$*;*

[13] For instance, a bidder i with $s_i = 1$ has value v_{11} or v_{10} when the rival bidder has $s_j = 1$ or $s_j = 0$, respectively, in which case bidder i pays $b_1 = v_{11}$ (conditional upon winning) or $b_0 = v_{00}$.

[14] Of course, proving the uniqueness of equilibrium in the symmetric class requires further work.

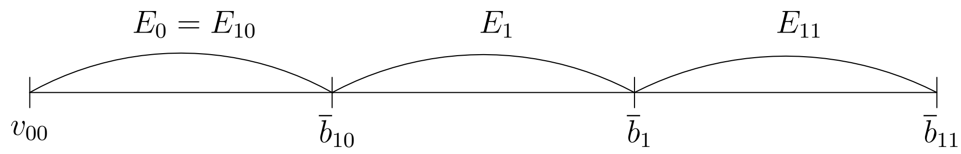


Figure 3: **Bid supports of the first-price auction under I^2 when $\pi_1 > 0 = \pi_0$**

(ii) $\pi_0, \pi_1 < 1$ *in any symmetric equilibrium;*

(iii) There is no symmetric equilibrium with $\pi_1 = 0 < \pi_0$*, so* π_1 *must be positive if* $k < \overline{k}_1$.

Proof. See Appendix B.1. ■

Note that no bidder chooses to learn his rival's signal if the learning cost is above the threshold $\overline{k}_1$. With the learning cost below this threshold, strong bidders—i.e., those with a high prior signal—are learning with a positive probability, while weak bidders—i.e., those with a low prior signal—may not. It suggests that strong bidders are more prone to learn their rival's signal. Indeed, as we will show later, for values of k lower than $\overline{k}_1$, strong bidders are learning with higher probability than weak bidders, while weak bidders may not learn at all. This is because strong bidders capture a greater benefit from the strategic advantage of learning their rival's signal. The benefit comes from being able to shade their bids against a weak rival, so it decreases as signals become more correlated (that is, the rival is less likely to be weak). Thus, they choose not to learn the rival's signal if α is so high that $\overline{k}_1 = \alpha(1-\alpha)(v_{11} - v_{00})$ is smaller than k.

We proceed with a more detailed analysis of the equilibrium in which learning occurs with positive probabilities (i.e., at least one of π_1 and π_0 is positive). In all equilibrium characterizations below, the support of bid distribution for each type of bidder is a connected interval: that is, $E_t = [\underline{b}_t, \overline{b}_t]$ for all $t \in \Omega$, while the interval may be degenerate (i.e., $\underline{b}_t = \overline{b}_t$). Similar to Proposition 2, our statement of the results below will only provide the upper and lower bounds of the bid supports. The equilibrium bid distribution, $H_t(\cdot)$, can then be derived in a straightforward manner using the fact that the payoff for each type $t \in \Omega$ remains constant over the interval E_t.

We first provide a characterization of symmetric equilibrium in which only strong bidders learn with a positive probability. Note that a weak bidder becomes of type $t = 0$ in this case, but a strong bidder can be of type $t = 1$, $t = 10$ or $t = 11$ when he does not learn the rival's signal, learns that it is low, or learns that it is high, respectively. We thus have $\Omega = \{0, 1, 10, 11\}$.

Proposition 5. *In any symmetric equilibrium with $\pi_1 > 0 = \pi_0$ for the first-price auction under I^2, the following results hold:*

(i) π_1 *solves the equation*

$$\frac{v_{11} - v_{01}}{k} = \frac{1}{1-\alpha} + \frac{1}{\alpha(1-\pi_1)} - \frac{\alpha v_{01}}{k[\alpha + (1-\alpha)\pi_1]}; \tag{3}$$

(ii) $v_{00} = \underline{b}_0 = \underline{b}_{10} < \bar{b}_0 = \bar{b}_{10} = \underline{b}_1 < \bar{b}_1 = \underline{b}_{11} < \bar{b}_{11}$ *(refer to Figure 3), where*

$$\bar{b}_{10} = v_{11} - \frac{k}{(1-\pi_1)\alpha} - \frac{k}{1-\alpha}, \quad \bar{b}_1 = v_{11} - \frac{k}{(1-\pi_1)\alpha}, \quad \bar{b}_{11} = v_{11} - \frac{k}{\alpha}; \tag{4}$$

(iii) This equilibrium exists only if $k \in [\bar{k}_0, \bar{k}_1)$, *where* $\bar{k}_0$ *is the solution of*

$$k = (1-\alpha)\pi_1(v_{01} - \bar{b}_0). \tag{5}$$

Proof. See Appendix B.2. ∎

To explain Part (*ii*) first, Figure 3 illustrates the supports of equilibrium bid distributions. Observe that the bid support of strong bidder shifts upward as he learns that the rival is strong (i.e., E_{11} lies above E_1), and likewise, it shifts downward as he learns that the rival is weak (i.e., E_{10} lies below E_1). This reflects the informational advantage. The less aggressive bidding of type $t = 10$ bidder is also a consequence of the strategic advantage: each strong bidder who learns that his rival has a low signal revises downwardly his inference of the rival's bidding strategy and shades his bid further.

Let Γ_t denote the equilibrium payoff of type t bidder, where $t \in \Omega$. The learning probability π_1 in Part (*i*) is chosen to make bidder i with $s_i = 1$ indifferent between learning and not learning; that is,

$$\Gamma_1 = (1-\alpha)\Gamma_{10} + \alpha\Gamma_{11} - k, \tag{6}$$

where the right hand side comes from the fact that when bidder i learns the rival's signal s_j, it will be $s_j = 0$ with probability $1-\alpha$ and $s_j = 1$ with probability α. Rearranging (6), we obtain the expression (3).

To understand Part (*iii*), note that if a weak bidder deviates to learn his rival's signal, then he could lower his bid to the lowest level v_{00} upon learning that it is low or he could raise his bid to $\bar{b}_{10}$ upon learning that it is high, which results in the (post) deviation payoff equal to the right hand side of (5). If $k < \bar{k}_0$, then this payoff exceeds the learning cost, so

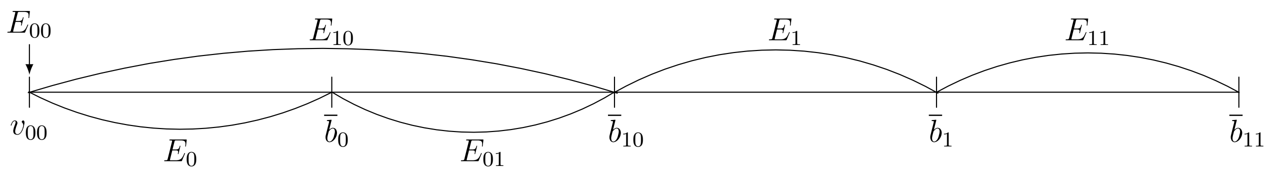


Figure 4: **Bid supports of the first-price auction under I^2 when $\pi_1, \pi_0 > 0$**

the equilibrium where only strong bidders are learning cannot be sustained.

We next provide a characterization of equilibrium in which both strong and weak bidders learn with positive probabilities. Note that we have $\Omega = \overline{\Omega}_2 = \{0, 1, 00, 01, 10, 11\}$.

Proposition 6. *In any symmetric equilibrium with $0 < \pi_0, \pi_1 < 1$ for the first-price auction under I^2, the following results hold:*

(i) π_1 solves the equation

$$\frac{v_{11} - v_{01}}{k} = \frac{1}{1-\alpha} + \frac{1}{\alpha(1-\pi_1)} - \frac{1}{(1-\alpha)\pi_1}, \tag{7}$$

while

$$\pi_0 = \frac{v_{01} - \frac{k}{(1-\alpha)\pi_1} - \frac{k}{\alpha}}{v_{10} - \frac{k}{\alpha}} < \pi_1; \tag{8}$$

(ii) $v_{00} = \underline{b}_{00} = \overline{b}_{00} = \underline{b}_0 = \underline{b}_{10} < \overline{b}_0 = \underline{b}_{01} < \overline{b}_{01} = \overline{b}_{10} = \underline{b}_1 < \overline{b}_1 = \underline{b}_{11} < \overline{b}_{11}$ (refer to Figure 4), where

$$\overline{b}_0 = \frac{k}{\alpha},\ \overline{b}_{10} = v_{01} - \frac{k}{(1-\alpha)\pi_1},\ \overline{b}_1 = v_{11} - \frac{k}{\alpha(1-\pi_1)},\ \overline{b}_{11} = v_{11} - \frac{k}{\alpha}; \tag{9}$$

(iii) This equilibrium exists only if $k < \overline{k}_0$, where k_0 is defined by (5).

Proof. See Appendix B.3. ■

Figure 4 illustrates the supports of equilibrium bid distributions. Note that E_{11} lies above E_1 and E_{10} lies below E_1, as was the case with $\pi_1 > 0 = \pi_0$. With the learning cost below the threshold $\overline{k}_0$, weak bidders also learn with positive probability π_0. They then adopt a more (less, resp.) aggressive bidding strategy when their rival's signal turns out to be high (low, resp.)—i.e., E_{01} (E_{00}, resp.) lies above (below, resp.) E_0. This reflects the informational advantage. It is worth noting that the support E_{01} overlaps with the upper segment of the support E_{10}. This is because type $t = 10$ bidder entertains the possibility of competing against a weak rival of type $t = 0$ or $t = 01$, while type $t = 01$ bidder is certain about competing against a strong rival of type $t = 10$ or $t = 1$. Hence, type $t = 10$ bidder tends to bid less aggressively than does type $t = 01$, even though the former has a higher value than the latter.[15] This pattern reflects the strategic advantage of learning from the perspective of strong bidder who wants to shade his bid more upon learning that his rival is weak.

The following theorem summarizes the equilibrium characterization in Proposition 4 through Proposition 6 and establishes the existence of such an equilibrium.

Theorem 2. *Under I^2, there exists a unique symmetric equilibrium of the first-price auction in which*

(i) for $k \geq \overline{k}_1$, $\pi_1 = \pi_0 = 0$;

(ii) for $k \in [\overline{k}_0, \overline{k}_1)$, $\pi_0 = 0$ and $\pi_1 \in (0, 1)$ is given as the solution of (3), which is decreasing in k and increasing in β;

(iii) for $k < \overline{k}_0$, $\pi_1 \in (0, 1)$ is given as the solution of (7), which is decreasing in k and increasing in α and β, while $\pi_0 \in (0, 1)$ is given as (8) and decreasing in k, α and β.

(iv) each bidder's payoff is $\frac{1}{2}(1-\alpha)(v_{10}-\overline{b}_{10})$, which is increasing in k and β and decreasing in α for $k < \overline{k}_1$.[16]

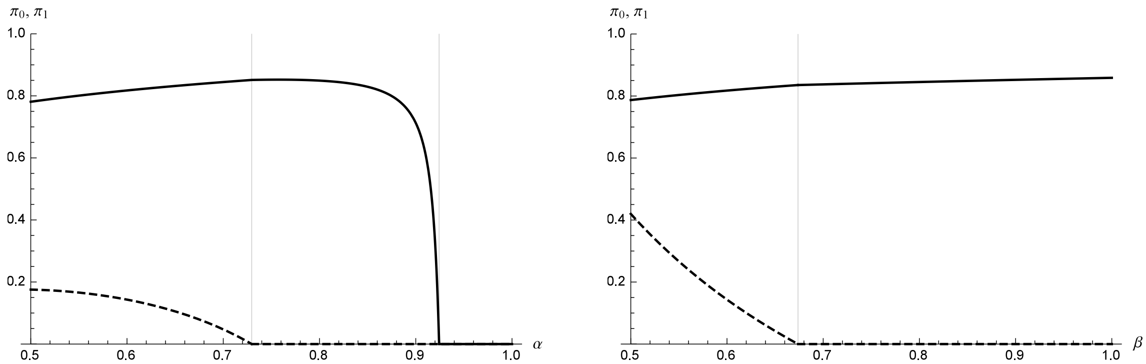
Proof. See Appendix B.4. ■

It is intuitively clear that the learning probabilities decrease in k.[17] To understand the effect of α on the learning probabilities, recall that under I^1, a higher α—i.e., higher correlation between signals—allows one to make more accurate inference of the other's signal and bidding strategy by learning his own signal, which gives a greater incentive to learn the latter signal. Under I^2, however, the higher correlation means that the prior signal each bidder initially holds is more informative of his rival's signal, so bidders expect less informational or strategic gain from learning their rival's signal. While π_0 is decreasing in α as a consequence, it gives strong bidders an incentive to maintain, or even slightly increase,

[15] A numerical analysis shows that the bid distribution of type $t = 10$ does not necessarily first-order stochastically dominates that of type $t = 01$. In fact, the latter distribution can dominate the former for some parameter values.

[16] The term $\overline{b}_{10}$ is given by (4) for $k \in [\overline{k}_0, \overline{k}_1)$ and (9) for $k < \overline{k}_0$. It is straightforward to check that in the case $k \geq \overline{k}_1$, the bidders' payoff is increasing in β and decreasing in α while being constant in k.

[17] Interestingly, as k converges to zero, π_1 converges to 1 but π_0 converges to $\frac{v_{01}}{v_{10}} < 1$, as can be seen from (8). In fact, the limit of bidding strategies combined with these learning probabilities constitutes an equilibrium when the learning cost is zero. However, there are other equilibria with zero learning cost. Our result thus provides a selection of equilibrium at $k = 0$ that is robust to perturbation of the learning cost.



(a) Changes of π_0 and π_1 according to α

(b) Changes of π_0 and π_1 according to β

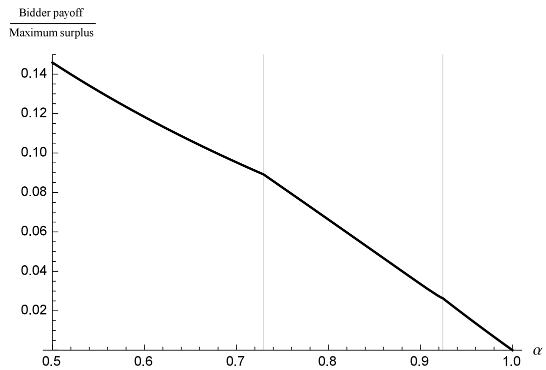
Figure 5: **Learning probabilities in the first-price auction under I^2.** Primitive values: $\alpha = \beta = 0.6$ and $k = 0.07$. In panel (a), $k < \overline{k}_0$ for $\alpha < 0.73$; $k \in [\overline{k}_0, \overline{k}_1)$ for $\alpha \in [0.73, 0.92)$; $k \geq \overline{k}_1$ for $\alpha \in [0.92, 1)$. In panel (b), $k < \overline{k}_0$ for $\beta < 0.67$; $k \in [\overline{k}_0, \overline{k})$ for $\beta \in [0.67, 1]$. The solid and the dashed lines respectively represent π_1 and π_0.

their learning probability as α increases (even in the range of α where $\pi_0 = 0$) unless α is too high, as depicted in Figure 5(a). This is because a weak bidder who is uninformed of the rival's signal tends to bid less aggressively as α increases—which implies that his rival is more likely to be weak as well—, causing a strong bidder to shade his bid more upon learning that his rival is weak.

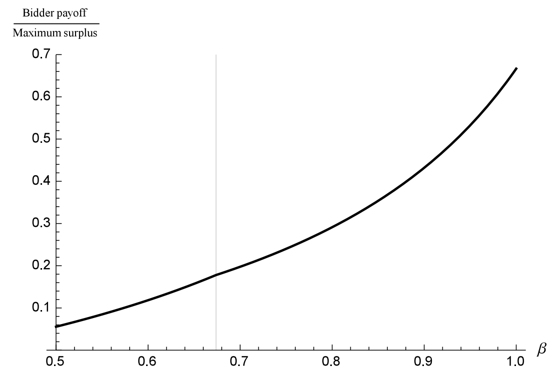
The benefit from the strategic advantage depends also on the degree of value interdependence: a higher β—i.e., lower interdependence—increases the value discrepancy between strong and weak bidders, which enables strong bidders to shade their bids more and thereby draw more benefit from learning that their rival is weak. This explains why the learning probability of strong bidders, π_1, is increasing in β for any $k < \overline{k}_1$. In contrast, π_0 is negatively affected by higher β.[18] This follows from the fact that the lower interdependence makes learning the other's signal less valuable for one's value estimation—that is, it reduces the informational advantage—while a weak bidder derives the benefit of learning mostly from the informational advantage. With π_0 being lower due to higher β, weak bidders are less likely to bid competitively when facing strong bidders, which will reinforce the learning incentive of strong bidders. Figure 5(b) depicts the changes in learning probabilities according to the value interdependence.

The equilibrium payoff in Part (iv) of Theorem 2 follows from the fact that each bidder obtains a positive payoff only when his own signal is high and his rival's signal is low, which

[18]It can be shown that $\pi_0 = 0$ when β is sufficiently high.



(a) Normalized bidder payoff according to α



(b) Normalized bidder payoff according to β

Figure 6: **Bidder's normalized payoff in the first-price auction under I^2.** Primitive values: $\alpha = \beta = 0.6$ and $k = 0.07$.

yields the (ex-ante) payoff $\frac{1}{2}(1-\alpha)\Gamma_{10} = \frac{1}{2}(1-\alpha)(v_{10} - \overline{b}_{10})$ since, by bidding $\overline{b}_{10}$, he wins against the rival with low signal (irrespective of whether the latter is learning). The fact that this payoff is increasing in k means that a lower learning cost (i.e., lower k) is harmful to bidders' payoff, which is intuitive since a lower cost induces both strong and weak bidders to learn with higher probability, thereby intensifying the bidding competition between them.

Note also that the increase in signal correlation (i.e., higher α) reduces bidders' payoff, though it induces less learning by weak bidders. This is because a higher signal correlation in itself has the effect of intensifying the bidding competition. For instance, if signals are perfectly correlated, then the entire rent for bidders will be competed away. In contrast, a higher β increases bidders' payoff through its contrary effects on weak and strong bidders' learning incentives that facilitate the bid shading by strong bidders. The effect of α and β in Part (*iv*) should be taken with some caution since the maximum surplus, $\frac{1}{2}\alpha + (1-\alpha)\beta$, also varies with those parameters. However, their effect on the normalized payoff, which is defined as the bidders' payoff divided by the maximum surplus, remains qualitatively the same as in Part (*iv*) of Theorem 2, as depicted in Figure 6.

Remark 1. So far we have assumed that bidders learn their rival's signal perfectly whenever they pay the learning cost. The model can be easily extended to the case of imperfect learning. To do so, assume that when each bidder i decides to learn the rival's signal, he learns s_j, $j \neq i$, with probability $q \in (0, 1]$ but learns nothing with the remaining probability.[19] Note that q measures the precision of learning, and that $q = 1$ in our original setup I^2.

[19] One can consider an alternative extension in which each bidder observes another signal that is *imperfectly correlated* to the rival's signal. This model is beyond the scope of our analysis, however.

Consider the case in which only strong bidders decide to learn with a positive probability. The analysis is modified only in so far as the indifference condition (6) changes to

$$\Gamma_1 = q\big((1-\alpha)\Gamma_{10} + \alpha\Gamma_{11}\big) + (1-q)\Gamma_1 - k, \tag{10}$$

where the left hand side is the payoff from no learning and the right hand side is the payoff a bidder i with high prior signal expects from deciding to learn s_j. This expression follows from the fact that the learning succeeds with probability q, in which case the payoff of the bidder with high prior signal is equal to Γ_{10} and Γ_{11} with probability $1-\alpha$ and α, respectively, while, if the learning fails with probability $1-q$, then his payoff equals Γ_1. Rearranging the terms in (10), we have

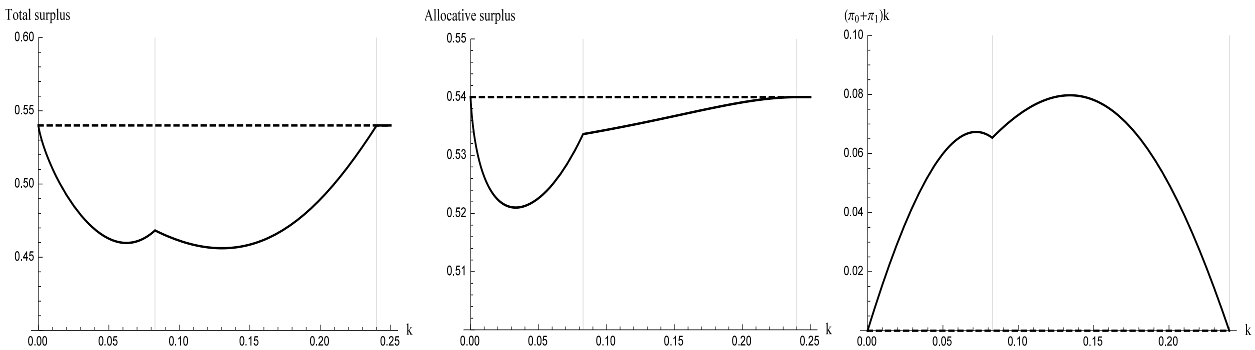
$$\Gamma_1 = (1-\alpha)\Gamma_{10} + \alpha\Gamma_{11} - \frac{k}{q}.$$

Comparing this with (6) reveals that the bidder must pay a higher cost $k/q > k$ to have the same information that he would have obtained if the learning was perfect. It is also straightforward to see that all other equilibrium conditions remain unchanged, except that π_1 is replaced by $q\pi_1$. As a consequence of these observations and Theorem 2, the *effective* learning probability, $q\pi_1$, is increasing in the learning precision q.[20] An analogous analysis applies to the case in which both strong and weak bidders decide to learn with positive probabilities.

5 Comparison of the Two Auction Formats

Using the equilibrium characterization obtained so far under I^2, we compare the performance of the first-price and second-price auctions in terms of total surplus, bidders' payoff and the seller's revenue. To do so, let T_{FPA} and T_{SPA} denote the total surplus in the equilibrium of the first-price and second-price auctions, respectively. Likewise let B_{FPA} and B_{SPA} denote the bidders' equilibrium payoff, and let R_{FPA} and R_{SPA} denote the seller's equilibrium revenue. Note that for $k \geq k_1$, there is no learning in the equilibrium of both auctions, which leads to the outcomes in which $T_{FPA} = T_{SPA}$, $B_{FPA} = B_{SPA}$, and $R_{FPA} = R_{SPA}$. Henceforth, we focus on the case where $k < \overline{k}_1$.

[20] However, our numerical analysis shows that the probability of learning decision, π_1, is changing non-monotonically in q.



(a) Total surplus (b) Allocative surplus (c) Learning costs

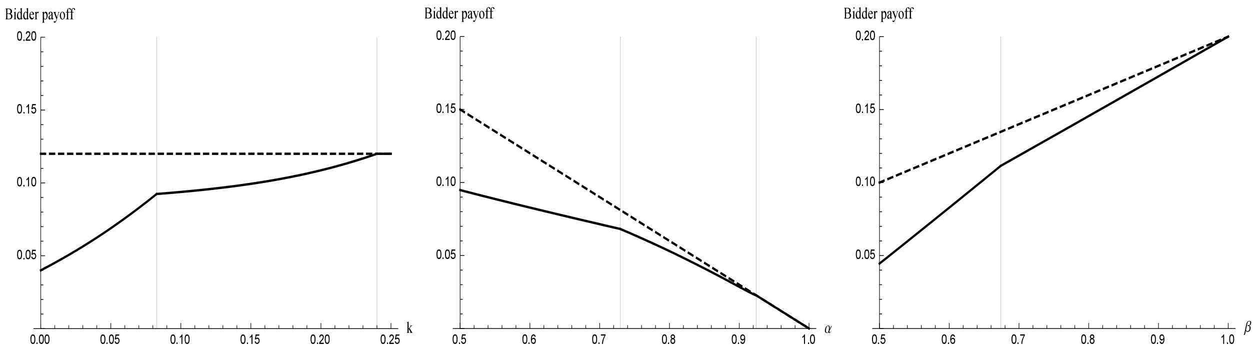
Figure 7: **Comparison of the total surplus** Primitive values: $\alpha = \beta = 0.6$. The solid and the dashed lines represent the first-price and the second-price auctions, respectively.

□ **Total surplus.** Recall that the total surplus—or simply referred to as surplus—is the same as the allocative surplus minus the costs of information acquisition. Recall also from Theorem 1 that the second-price auction achieves the highest possible surplus for two reasons: (i) the allocation is efficient; and (ii) the learning cost is not incurred. The dashed lines in Figure 7 depicts total surplus and the allocative surplus in the second-price auction. The first-price auction, however, fails both (i) and (ii). In particular, (i) fails since strong bidders often lose to weak bidders, as can be seen from the fact that the support E_{10} overlaps with E_0 or E_{01}. The solid lines in Figure 7 depict the total surplus, the allocative surplus, and the learning costs of the first-price auction.

Proposition 7. *For any $k < \overline{k}_1$, $T_{FPA} < T_{SPA}$.*

□ **Bidders' payoff.** In both auction formats, each bidder obtains a positive payoff only when his (prior) signal is high and his rival's signal is low. The resulting equilibrium payoff for each bidder is $\frac{1}{2}(1-\alpha)(v_{10} - v_{00})$ in the second-price auction and $\frac{1}{2}(1-\alpha)(v_{10} - \overline{b}_{10})$ in the first-price auction, as shown by Theorem 1 and Theorem 2, respectively. Given that $\overline{b}_{10} > v_{00} = 0$, the bidders' payoff is higher in the second-price auction than in the first-price auction.

In addition, Figure 8 reveals that the difference in the bidders' payoff between the two auctions becomes larger as k, α or β becomes smaller. Recall that with smaller k, α or β, weak bidders learn their rival's signal with a higher probability. The direct effect of this learning on the bidding strategy of weak bidders themselves is ambiguous: they will bid more or less aggressively as their rival turns out to be strong or weak, respectively. However,



(a) According to k (b) According to α (c) According to β

Figure 8: **Comparison of the bidders' payoff.** Primitive values: $\alpha = \beta = 0.6$ and $k = 0.07$. The solid and the dashed lines represent B_{FPA} and B_{SPA}, respectively.

it has an indirect effect of making strong bidders bid more aggressively, since they expect their weak rival to be informed of their high signal and thus bid more aggressively (with higher probability). It thus decreases the payoff of strong bidder facing a weak rival, which causes the bidders' equilibrium payoff in the first-price auction to decrease as well, since each bidder obtains a positive payoff only when the bidder himself is strong while his rival is weak. Indeed, the following proposition shows that the payoff difference between the two auctions is widening as k, α or β becomes smaller.

Proposition 8. *For any $k < \bar{k}_1$, $B_{SPA} > B_{FPA}$ and $B_{SPA} - B_{FPA}$ is decreasing in k, α and β.*

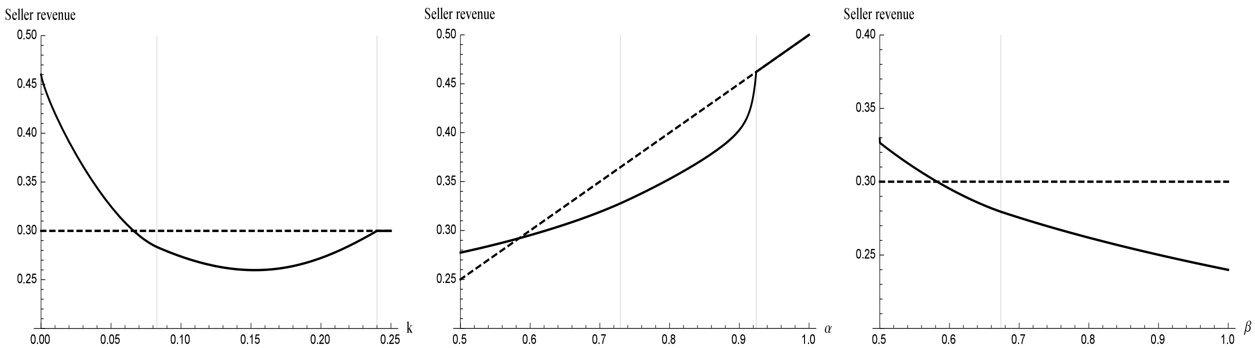
Proof. See Appendix C.1. ■

□ **Seller's revenue.** Note that the seller's revenue is equal to the total surplus minus the sum of two bidders' payoffs. As both the total surplus and the bidders' payoff are higher in the second-price auction than in the first-price auction, the revenue ranking between the two auctions can go either way. Indeed, the following proposition shows that the seller's revenue in the first-price auction is higher than that in the second-price auction when either k is small or both α and β are small, while the ranking is reversed when β is sufficiently large.

Proposition 9. *For any $k < \bar{k}_1$, the following results hold true:*

(i) $R_{FPA} > R_{SPA}$ if k is close to 0, while R_{FPA} and $R_{FPA} - R_{SPA}$ are maximized at $k = 0$[21]*;*

[21] Since we assume $k > 0$, this should be understood as a limit result with k converging to zero.



(a) According to k (b) According to α (c) According to β

Figure 9: **Comparison of the seller's revenue.** Primitive values: $\alpha = \beta = 0.6$ and $k = 0.07$. The solid and the dashed lines represent R_{FPA} and R_{SPA}, respectively.

(ii) $R_{FPA} > R_{SPA}$ *if* α *and* β *are close to* $\frac{1}{2}$*;*

(iii) $R_{FPA} < R_{SPA}$ *if* β *is close to 1.*

Proof. See Appendix C.2. ■

To understand Parts (i) and (ii), recall from Part (iii) of Theorem 2 that the weak bidder's learning probability is decreasing in the parameter values (k, α, and β). Recall also that the weak bidder's learning has a positive effect on the strong bidder's bidding strategy and thus on the seller's revenue. With the weak bidder's learning probability being sufficiently high due to small parameter values, this effect is so magnified as to make the first-price auction revenue-superior to the second-price auction. Moreover, according to Part (i), the seller's revenue from the first-price auction is maximized at zero learning cost (with other parameters fixed). In this case, strong bidders learn their rival's signal at zero cost and outbid weak rival with probability of one, which implies the total surplus achieves its first-best.[22] On the other hand, the bidders' payoff is minimized at $k = 0$ according to Part (iv) of Theorem 2. So the seller's revenue, which equals the total surplus minus bidders' payoffs, is maximized at $k = 0$.

In contrast, with β close to 1 (i.e, values being almost private), weak bidders never learn and then make very low bids since their value is low (close to 0). This induces strong bidders to learn with high probability as long as the learning cost is not too high (i.e., $k \leq \overline{k}_1$). Upon learning that their rival is weak, strong bidders can win the object at very low price, which is detrimental to the seller, reversing the revenue ranking in Part (iii). In fact, our numerical analysis, as in Figure 9(c), shows that there is a threshold level of value interdependence

[22]See Figure 7(a) for an illustration of this result.

such that the first-price auction is revenue-superior to second-price auction if and only if the value interdependence is larger than that level. This result is consistent with the finding by Fang and Morris (2006) that in the private values case, the first-price auction is revenue-inferior to second-price auction when bidders observe signals correlated with their rival's value, although the signals are given exogenously unlike our model.

6 Concluding Remarks

This paper investigates the problem of endogenous information acquisition in interdependent value auctions. We characterize the symmetric equilibrium in both first-price and second-price auctions and analyze bidders' learning and bidding behavior through two channels—informational and strategic advantages. We show that under I^2, the total surplus and the bidder payoff are higher in the second-price auction, but the ranking of the seller revenue between the two auction formats depends on the magnitude of learning cost as well as the degrees of signal correlation and value interdependent. These findings are different from those under I^1, which is the case studied in the previous auction literature in that the total surplus and the seller revenue are higher in the second-price auction, while the bidder payoff is higher in the first-price auction.

Appendix

We introduce a few notations at first. For any $t, t' \in \overline{\Omega}$, let $p(t'|t)$ be the probability with which each bidder of type t believes his rival to be of type t', given the equilibrium learning strategy, and $\Omega_t := \{t' \in \Omega \mid p(t'|t) > 0\}$ be the set of all rival types that a bidder of type t faces with positive probabilities.

In what follows, we provide proofs for the second-price auction and then those for the first-price auction. All of omitted proofs are provided in the Supplementary Appendix.

A Proofs for Section 4.1

We first provide a couple of lemmas to prove Theorem 1.

Lemma 1. *Under I^n, $n = 1, 2$, if $t \in \{U, 0, 1\} \cap \Omega$, then $\underline{b}_t \geq \underline{v}(t) := \min_{t' \in \Omega_t} v(t, t')$ and $\overline{b}_t \leq \overline{v}(t) := \max_{t' \in \Omega_t} v(t, t')$ in any symmetric equilibrium.*

Lemma 2. *In any symmetric equilibrium under I^2, the following results hold:*
(i) If $t = mm \in \Omega$, where $m = 0, 1$, then $E_t = \{v_t\}$;
(ii) If $\pi_1 > 0$, then $\pi_0 > 0$ and $\bar{b}_{01} > v_{01}$, while $E_t \cap (v_{01}, \bar{b}_{01}) = \emptyset$ for any $t \in \Omega_{01}$;
(iii) If $\pi_0 > 0$ and $1 \in \Omega$, then $E_1 \cap [\bar{b}_{01}, v_{11}) = \emptyset$;
(iv) If $\pi_1 = 0$, then $E_0 = \{v_{00}\}$.

A.1 Proof of Theorem 1

Proof of Part (i). To show $\pi_1 = 0$, suppose for a contradiction that $\pi_1 > 0$. Let us first consider the case that $\pi_1 \in (0, 1)$ so $1 \in \Omega$. In this case, $\pi_0 > 0$ by Part (ii) of Lemma 2, and $\underline{b}_1 \geq \underline{v}(1) = v_{10}$ by Lemma 1. Also, $E_1 \cap (v_{01}, v_{11}) = \emptyset$ from Parts (ii) and (iii) of Lemma 2. Hence, it must be the case that $E_1 \subset \{v_{01}, v_{11}\}$. If $v_{01} \in E_1$ so that type $t = 1$ puts a mass at v_{01}, then the same type can profitably deviate to bid $v_{01} + \varepsilon$ for sufficiently small $\varepsilon > 0$. Assume thus that $E_1 = \{v_{11}\} = E_{11}$, where the second equality follows from Part (i) of Lemma 2. This means that each bidder i with $s_i = 1$ can never earn a positive payoff if $s_j = 1$, which implies that it is also optimal for him to bid v_{10} irrespective of s_j. Then, he can do better by not learning s_j and bidding v_{10}, since it saves the information acquisition cost k. A similar contradiction can be established in the case $\pi_1 = 1$.

We now show $\pi_0 = 0$. Consider a bidder i with $s_i = 0$ and suppose he learns s_j. If $s_j = 0$, then he obtains zero payoff in the bidding stage, clearly. If $s_j = 1$, then the rival must be of type $t = 1$, given the fact that $\pi_1 = 0$. Since $\underline{b}_1 = \bar{b}_1 = v_1 \geq v_{01}$ by Lemma 1, bidder i can never earn a positive payoff. So, bidding v_{00} without learning s_j is better for bidder i than learning s_j, since it saves the information acquisition cost. ■

Proof of Parts (ii) and (iii). The argument to show that each bidder of type $t = m \in \{0, 1\}$ must bid v_{mm} in symmetric equilibrium is similar to the proof of Part (ii) of Proposition 1 (see the Supplementary Appendix) and hence omitted. Part (iii) immediately follows from Part (i). ■

B Proofs for Section 4.2

To analyze the first-price auction, observe first that for any $t \in \overline{\Omega}$ and $b \geq 0$,

$$\Gamma_t(b) = \sum_{t' \in \Omega_t} p(t'|t) \left[H_{t'}(b_-) + \frac{H_{t'}(b) - H_{t'}(b_-)}{2} \right] (v(t, t') - b),$$

where $H_{t'}(b_-) := \lim_{b' \nearrow b} H_{t'}(b')$. The expression in the square bracket is due to the assumption that any bid tie is broken randomly. Note that the above payoff does not take into account the learning cost. Note also that $\Gamma_t = \Gamma_t(b)$ for $b \in E_t$.[23]

Lemma 3. *Call a subset $\Omega' \subset \Omega$ a* ***component*** *of Ω if $\Omega_t \subset \Omega'$ for any $t \in \Omega'$, i.e. types in Ω' face each other and no others. Then, for any component Ω' of Ω, there exists at least one type $t \in \Omega'$ with $\Gamma_t = 0$.*

Lemma 4. *Define for any $t \in \Omega$, $L_t := \{t' \,|\, \underline{b}_t \geq \bar{b}_{t'}\}$. Consider any type t deviating to bid $b \in E_{t'} \backslash E_t$ with $t' \in \Omega_t \backslash \{t\}$ such that no type puts a mass at b and there is only one type $t'' \in \Omega_t \cap \Omega_{t'}$ with $b \in E_{t''}$. Then, $\Gamma_t(b)$ is nonincreasing at such b if*

$$\frac{p(L_{t'}|t)}{p(t''|t)} \geq \frac{p(L_{t'}|t')}{p(t''|t')} \quad \textit{and} \quad v(t,t'') \leq v(t',t''). \tag{B.1}$$

Also, $\Gamma_t(b)$ is nondecreasing at such b if

$$\frac{p(L_{t'}|t)}{p(t''|t)} \leq \frac{p(L_{t'}|t')}{p(t''|t')} \quad \textit{and} \quad v(t,t'') \geq v(t',t''). \tag{B.2}$$

B.1 Proof of Proposition 4

Proof of Part (i). Suppose that $\pi_0 = \pi_1 = 0$ in equilibrium. The existing literature, for instance Campbell and Levin (2000), shows that in this case, there is a unique equilibrium bidding strategy in which each type-0 bidder bids v_{00} for sure, while each type-1 bidder randomizes his bid over interval $[v_{00}, \bar{b}_1]$ with $\bar{b}_1 = \alpha v_{11} + (1-\alpha)v_{00}$, following the distribution

$$H_1(b) = \frac{(1-\alpha)(b - v_{00})}{\alpha(v_{11} - b)}. \tag{B.3}$$

The equilibrium payoffs for type-0 and type-1 are respectively equal to 0 and $(1-\alpha)(v_{10}-v_{00})$.

We prove the first statement by showing that no bidder has an incentive to learn his rival's signal if and only if $k \geq \bar{k}_1$. If bidder i with $s_i = 1$ deviates to learn s_j, then the maximum payoff from this deviation, exclusive of the learning cost, is given as

$$\tilde{\Gamma} = (1-\alpha)(v_{10} - v_{00}) + \max_{b \in [v_{00}, \bar{b}_1]} \alpha H_1(b)(v_{11} - b), \tag{B.4}$$

[23] To be precise, $\Gamma_t = \Gamma_t(b)$ for some $b \in int(E_t)$ or a mass point b of the distribution H_t. This is because some bid in E_t, for instance $\underline{b}_t$, can be suboptimal for type t (though $\underline{b}_t \in E_t$), in particular if there is some other type who puts a mass at $\underline{b}_t$.

where the first term is the payoff from bidding $v_{00}+\varepsilon$ (for an arbitrary small $\varepsilon > 0$) after learning $s_j = 0$ while the second term is the payoff from bidding the optimal $b \in [v_{00}, \bar{b}_1]$ after learning $s_j = 1$. By substituting (B.3) into (B.4), we obtain

$$\tilde{\Gamma} = (1-\alpha)(v_{10} - v_{00}) + \max_{b \in [v_{00}, \bar{b}_1]} (1-\alpha)(b - v_{00}) = (1-\alpha)(v_{10} - v_{00}) + (1-\alpha)(\bar{b}_1 - v_{00}).$$

Thus, bidder i with $s_i = 1$ has no incentive to deviate and learn s_j if and only if

$$\tilde{\Gamma} - k \leq (1-\alpha)(v_{10} - v_{00}) \;\Leftrightarrow\; k \geq \alpha(1-\alpha)(v_{11} - v_{00}) = \bar{k}_1.$$

Similarly, each bidder i with $s_i = 0$ has no incentive to deviate if $k \geq \bar{k}_1$. ■

Proof of Part (ii). Suppose $\pi_1 = 1$ for a contradiction. Then, the singleton set $\{11\}$ is a component, so that $\Gamma_{11} = 0$ by Lemma 3. Thus, the payoff for each bidder i with s_i from learning s_j equals $\alpha\Gamma_{11} + (1-\alpha)\Gamma_{10} - k = (1-\alpha)\Gamma_{10} - k$. However, if he bids some $b \in E_{10}$ without learning, then the resulting payoff will be at least $(1-\alpha)\Gamma_{10} > (1-\alpha)\Gamma_{10} - k$, a contradiction. Next, suppose $\pi_0 = 1$ for a contradiction. Then, the singleton set $\{00\}$ is a component, so $\Gamma_{00} = 0$ by Lemma 3. We argue that $\Gamma_{01} = 0$, which will establish the desired contradiction since it means that bidder i with $s_i = 0$, after learning s_j would earn zero payoff in the bidding stage, so learning only entails the cost k. If $\Gamma_{01} > 0$ to the contrary, then we must have $\bar{b}_{01} < v_{01}$, which in turn implies $\Gamma_1 > 0$ since the type-1 bidder can get a positive payoff by bidding some $b \in (\bar{b}_{01}, v_{01})$. By the above observations, we must also have $\Gamma_{11}, \Gamma_{10} > 0$. In sum, $\Gamma_t > 0$ for all $t \in \Omega' = \{1, 01, 10, 11\}$, which cannot hold true, however, since Ω' contains a component if $\pi_0 = 1$. ■

Proof of Part (iii). Suppose for a contradiction that $\pi_0 > 0 = \pi_1$. We then have $\Omega_{01} = \{1\}$, implying that $\Gamma_{01} = H_1(\bar{b}_{01})(v_{01} - \bar{b}_{01})$. Thus, the payoff of each bidder i with $s_i = 0$ from learning s_j is

$$0 \leq \Gamma_0 = \alpha\Gamma_{00} + (1-\alpha)\Gamma_{01} - k = (1-\alpha)\Gamma_{01} - k = (1-\alpha)H_1(\bar{b}_{01})(v_{01} - \bar{b}_{01}) - k, \quad \text{(B.5)}$$

where the first equality holds since $\pi_0 \in (0,1)$ means that bidder i with $s_i = 0$ is indifferent between learning and not learning, while the second equality holds since $\Gamma_{00} = 0$. Next, we must have $\underline{b}_1 \leq \underline{b}_{01}$, since otherwise $\Gamma_{01} = 0$. Thus, $\Gamma_1 = (1-\alpha)(1-\pi_0)H_0(\underline{b}_1)(v_{10} - \underline{b}_1)$. Consider now bidder i with $s_i = 1$ deviating to learn s_j. If he bids $\underline{b}_1$ after learning $s_j = 0$

and $\bar{b}_{01}$ after learning $s_j = 1$, then the resulting payoff, exclusive of the learning cost, is

$$(1-\alpha)H_0(\underline{b}_1)(v_{10} - \underline{b}_1) + \alpha H_1(\bar{b}_{01})(v_{11} - \bar{b}_{01}) = \Gamma_1 + \alpha H_1(\bar{b}_{01})(v_{11} - \bar{b}_{01}).$$

So, the net gain from the deviation is at least

$$\left[\Gamma_1 + \alpha H_1(\bar{b}_{01})(v_{11} - \bar{b}_{01}) - k\right] - \Gamma_1 = \alpha H_1(\bar{b}_{01})(v_{11} - \bar{b}_{01}) - k > 0,$$

where the inequality follows from (B.5) and the facts that $\alpha > 1 - \alpha$ and $v_{11} > v_{01}$. This means that bidder i with $s_i = 1$ has a strict incentive to learn s_j, a contradiction. ■

B.2 Proof of Proposition 5

We first provide some characterizations of symmetric equilibrium with $\pi_1 > 0$:

Lemma 5. *In any symmetric equilibrium with $\pi_1 > 0$ (whether or not $\pi_0 = 0$),*
(i) $\bigcup_{t\in\Omega} E_t$ is a connected interval while no type $t \neq 00$ puts a mass at any $b \in \left(\bigcup_{t\in\Omega} E_t\right) \setminus \{v_{00}\}$;
(ii) $E_{m0} \cap E_{m1} = \emptyset$ for $m = 0, 1$;
(iii) $\Gamma_t > 0 = \Gamma_0 = \Gamma_{00}$ for all $t \neq 0, 00$;
(iv) $E_t \subseteq E^t := \bigcup_{t'\in\Omega_t} E_{t'}$ for any $t \in \Omega$;
(v) $\underline{b}_{00} = \bar{b}_{00} = \underline{b}_0 = v_{00}$;
(vi) $v_{00} < \underline{b}_1 < \underline{b}_{11}$;
(vii) $\underline{b}_0 = v_{00} < \bar{b}_0$;
(viii) $\underline{b}_{10} = v_{00} < \bar{b}_{10} = \underline{b}_1$ while $E_{10} = [\underline{b}_{10}, \bar{b}_{10}]$;
(ix) $\Gamma_1 = (1-\alpha)\Gamma_{10}$ and $k = \alpha\Gamma_{11}$.

Lemma 6. *In any symmetric equilibrium with $\pi_1 > 0 = \pi_0$,*
(i) $\bar{b}_0 = \bar{b}_{10} = \underline{b}_1$ while $E_0 = E_{10} = [v_{00}, \bar{b}_0]$;
(ii) $\bar{b}_1 = \underline{b}_{11}$ while $E_1 = [\underline{b}_1, \bar{b}_1]$ and $E_{11} = [\underline{b}_{11}, \bar{b}_{11}]$.

Proof of Parts (i) and (ii). Lemma 5 and Lemma 6 together imply that the supports of the equilibrium bids distributions must look like those in Figure 3. Given this, one can write the equilibrium conditions as follows:

$$0 = \Gamma_0(\bar{b}_0) = \alpha(v_{00} - \bar{b}_0) + (1-\alpha)\pi_1(v_{01} - \bar{b}_0) \tag{B.6}$$

$$(1-\alpha)(v_{10} - \bar{b}_0) = \Gamma_1(\bar{b}_0) = \Gamma_1(\bar{b}_1) = (1-\alpha)(v_{10} - \bar{b}_1) + \alpha(1-\pi_1)(v_{11} - \bar{b}_1) \tag{B.7}$$

$$(1-\pi_1)(v_{11}-\bar{b}_1) = \Gamma_{11}(\bar{b}_1) = \Gamma_{11}(\bar{b}_{11}) = v_{11} - \bar{b}_{11} \tag{B.8}$$

$$k = \alpha\Gamma_{11}(\bar{b}_{11}) = \alpha(v_{11} - \bar{b}_{11}), \tag{B.9}$$

where the first equalities of (B.6) and (B.9) hold due to Parts (*iv*) and (*ix*) of Lemma 5, respectively. From (B.9), $\bar{b}_{11} = v_{11} - \frac{k}{\alpha}$. Substituting this into (B.8) yields $\bar{b}_1 = v_{11} - \frac{k}{(1-\pi_1)\alpha}$, which can then be substituted into (B.7) to yield $\bar{b}_0 = v_{11} - \frac{k}{(1-\pi_1)\alpha} - \frac{k}{1-\alpha}$. We thus obtain (4). To obtain (3), rearrange (B.6) to get

$$\bar{b}_0 = \frac{\alpha v_{00} + (1-\alpha)\pi_1 v_{01}}{\alpha + (1-\alpha)\pi_1} = \frac{(1-\alpha)\pi_1 v_{01}}{\alpha + (1-\alpha)\pi_1}. \tag{B.10}$$

Equating this with $\bar{b}_0$ in (4) yields (3). It is straightforward to check that the RHS of (3) is increasing in π_1 so there exists a unique solution (if any) that solves (3). ■

Proof of Part (iii). We show that there is some $\bar{k}_0 < \bar{k}_1$ such that if $k \notin [\bar{k}_0, \bar{k}_1)$, there is no equilibrium with $\pi_1 > 0 = \pi_0$. First, one can easily check that for $k = \bar{k}_1 = \alpha(1-\alpha)$, $\pi_1 = 0$ is the (unique) solution of (3). Thus, there is no positive solution to (3) if $k \geq \bar{k}_1$, since the RHS of (3) is increasing in π_1. Next, we show that if $k < \bar{k}_0$, then each bidder i with $s_i = 0$ can profitably deviate to learn s_j. To see the payoff from this deviation, after learning $s_j = 0$, the bidder i of type $t_i = 00$ can only obtain zero payoff (by bidding v_{00}). After learning $s_j = 1$ (with probability $1-\alpha$), the bidder i of type $t_i = 01$ can bid $\bar{b}_0$ to obtain $(v_{01} - \bar{b}_0)$. Thus, the deviation payoff is at least $(1-\alpha)\pi_1(v_{01} - \bar{b}_0)$, which is equal to $\alpha(\bar{b}_0 - v_{00})$ by (B.6). This payoff is decreasing in k since $\bar{b}_0$ is decreasing in k.[24] This implies that the deviation is profitable for $k < \bar{k}_0$, given the definition of $\bar{k}_0$ in (5). ■

B.3 Proof of Proposition 6

Let us first provide some characterizations of symmetric equilibrium with $\pi_1, \pi_0 > 0$.

Lemma 7. *In any symmetric equilibrium with $\pi_1, \pi_0 > 0$,*
(i) $\bar{b}_0 = \underline{b}_{01}$ *while* $E_0 = [v_{00}, \bar{b}_0]$ *and* $E_{01} = [\underline{b}_{01}, \bar{b}_{01}]$;
(ii) $\bar{b}_1 = \underline{b}_{11}$ *while* $E_{11} = [\underline{b}_{11}, \bar{b}_{11}]$;
(iii) $\bar{b}_0 \leq \underline{b}_1$;
(iv) $\bar{b}_{01} \in [\underline{b}_1, \bar{b}_1]$ *while* $E_{01} = [\underline{b}_{01}, \bar{b}_{01}]$;
(v) $(1-\alpha)\Gamma_{01} = k$.

[24] To see it, rewrite (B.6) to get $\bar{b}_0 = \frac{(1-\alpha)\pi_1 v_{01}}{\alpha+(1-\alpha)\pi_1}$, which is decreasing in k since π_1 is decreasing in k.

Lemma 8. *Suppose that $H_{01}(\bar{b}_{10}) = 1$ (that is, $\bar{b}_{10} = \bar{b}_{01}$). If $k = \bar{k}_0$, then the solution π_1 of (7) coincides with that of (3) while π_0 defined in (8) is equal to zero. Moreover, $\bar{b}_0 = \bar{b}_{01}$ in (9), which is the same as $\bar{b}_0$ in (4) at $k = \bar{k}_0$.*

Lemma 9. *If $k \geq \bar{k}_0$, then there is no symmetric equilibrium with $\pi_1, \pi_0 > 0$.*

Lemma 10. *If $k < \bar{k}_0$, then $\bar{b}_{01} = \bar{b}_{10}$ in any symmetric equilibrium with $\pi_1, \pi_0 > 0$.*

Proof of Parts (i) and (ii). By Lemma 5, Lemma 7, and Lemma 10, the supports of the equilibrium bid distributions must look like those in Figure 4. Using this, we can write the equilibrium conditions as follows:

$$0 = \Gamma_0(\bar{b}_0) = \alpha(v_{00} - \bar{b}_0) + (1-\alpha)\pi_1 H_{10}(\bar{b}_0)(v_{01} - \bar{b}_0) \tag{B.11}$$

$$(1-\alpha)(v_{10} - \bar{b}_{10}) = \Gamma_1(\bar{b}_{10}) = \Gamma_1(\bar{b}_1) = (1-\alpha)(v_{10} - \bar{b}_1) + \alpha(1-\pi_1)(v_{11} - \bar{b}_1) \tag{B.12}$$

$$\pi_1 H_{10}(\underline{b}_{01})(v_{01} - \underline{b}_{01}) = \Gamma_{01}(\underline{b}_{01}) = \Gamma_{01}(\bar{b}_{01}) = \pi_1(v_{01} - \bar{b}_{01}) \tag{B.13}$$

$$(v_{10} - \bar{b}_{10}) = \Gamma_{10}(\bar{b}_{10}) = \Gamma_{10}(\bar{b}_0) = (1-\pi_0)(v_{10} - \bar{b}_0) \tag{B.14}$$

$$k = (1-\alpha)\Gamma_{01}(\bar{b}_{01}) = (1-\alpha)\pi_1(v_{01} - \bar{b}_{01}) \tag{B.15}$$

$$k = \alpha\Gamma_{11}(\bar{b}_1) = \alpha(1-\pi_1)(v_{11} - \bar{b}_1) \tag{B.16}$$

$$k = \alpha\Gamma_{11}(\bar{b}_{11}) = \alpha(v_{11} - \bar{b}_{11}), \tag{B.17}$$

where the first equalities in (B.15) to (B.17) hold due to Part (ix) of Lemma 5 and Part (v) of Lemma 7. Observe that $\bar{b}_{01}$, $\bar{b}_1$ and $\bar{b}_{11}$ in (9) are directly obtained by rearranging (B.15), (B.16), and (B.17), respectively. Next, rearranging (B.11) yields

$$\bar{b}_0 = \frac{\alpha v_{00} + (1-\alpha)\pi_1 H_{10}(\bar{b}_0) v_{01}}{\alpha + (1-\alpha)\pi_1 H_{10}(\bar{b}_0)}. \tag{B.18}$$

Note that

$$H_{10}(\bar{b}_0) = H_{10}(\underline{b}_{01}) = \frac{v_{01} - \bar{b}_{01}}{v_{01} - \bar{b}_0} = \frac{k}{(1-\alpha)\pi_1(v_{01} - \bar{b}_0)}, \tag{B.19}$$

where the first equality follows from $\bar{b}_0 = \underline{b}_{10}$, the second from (B.13), and the third from substituting the expression for $\bar{b}_{01}$ in (9). We obtain the expression of $\bar{b}_0$ in (9) by substituting (B.19) into (B.18) and then solving for $\bar{b}_0$.

Let us now obtain π_0 and π_1. For π_0, rearrange (B.14) to get

$$\pi_0 = 1 - \frac{v_{10} - \bar{b}_{10}}{v_{10} - \bar{b}_0} = \frac{\bar{b}_{10} - \bar{b}_0}{v_{10} - \bar{b}_0}. \tag{B.20}$$

Substituting this equation into the expressions for $\bar{b}_{10} = \bar{b}_{01}$ and $\bar{b}_0$ in (9) yields (8). To show that π_1 is obtained by solving (7), substitute $\bar{b}_{10} = v_{01} - \frac{k}{(1-\alpha)\pi_1}$ into (B.12) to get

$$\begin{aligned}(1-\alpha)\left(v_{10} - v_{01} + \frac{k}{(1-\alpha)\pi_1}\right) &= (1-\alpha)(v_{10} - \bar{b}_1) + \alpha(1-\pi_1)(v_{11} - \bar{b}_1) \\ &= (1-\alpha)\left(v_{10} - v_{11} + \frac{k}{\alpha(1-\pi_1)}\right) + k,\end{aligned}$$

where the second equality holds since $\bar{b}_1 = v_{11} - \frac{k}{\alpha(1-\pi_1)}$. Then, (7) is obtained by rearranging the leftmost and rightmost terms of the above equation. The RHS of (7) increases from $-\infty$ to ∞ as π_1 increases from 0 to 1 while the LHS is constant, and hence there is a unique solution $\pi_1 \in (0,1)$ to (7).

Lastly, to show $\pi_1 > \pi_0$, observe first that

$$\pi_1 - \pi_0 = \pi_1 - \frac{\bar{b}_{01} - \bar{b}_0}{v_{10} - \bar{b}_0} = \frac{v_{10} - \bar{b}_{01} - (1-\pi_1)(v_{10} - \bar{b}_0)}{v_{10} - \bar{b}_0}, \tag{B.21}$$

where the first equality follows from (B.20). Next, we use $v_{11} - v_{01} = v_{10}$ and $\bar{b}_0 = \frac{k}{\alpha}$ to rewrite (7) as

$$(1-\pi_1)(v_{10} - \bar{b}_0) = \frac{k(1-\pi_1)}{1-\alpha} - \frac{k(1-\pi_1)}{(1-\alpha)\pi_1} + \frac{k\pi_1}{\alpha}$$

Substituting this and $\bar{b}_{01} = v_{01} - \frac{k}{(1-\alpha)\pi_1}$ into the numerator of the last term in (B.21),

$$\begin{aligned}v_{10} - \bar{b}_{01} - (1-\pi_1)(v_{10} - \bar{b}_0) &= v_{10} - v_{01} + \frac{k}{(1-\alpha)\pi_1} - \frac{k(1-\pi_1)}{1-\alpha} + \frac{k(1-\pi_1)}{(1-\alpha)\pi_1} - \frac{k\pi_1}{\alpha} \\ &= v_{10} - v_{01} + \frac{k}{(1-\alpha)\pi_1} - \frac{k}{1-\alpha} + \frac{k\pi_1}{1-\alpha} + \frac{k(1-\pi_1)}{(1-\alpha)\pi_1} - \frac{k\pi_1}{\alpha} \\ &= v_{10} - v_{01} + \frac{2k}{1-\alpha}\left(\frac{1}{\pi_1} - 1\right) + \pi_1\left(\frac{k}{1-\alpha} - \frac{k}{\alpha}\right) > 0,\end{aligned}$$

where the inequality holds since $v_{10} > v_{01}$, $\pi_1 < 1$ and $\alpha > \frac{1}{2}$. We thus have that $\pi_1 > \pi_0$. ■

Proof of Part (iii). The result follows directly from Lemma 9. ■

B.4 Proof of Theorem 2

Proof of Part (i). By Part (*iii*) of Proposition 4, there does not exist an equilibrium with $\pi_1 = 0 < \pi_0$. Then, Parts (*iv*) of Proposition 5 and Proposition 6 together imply that bidders are learning with positive probability only if $k < \overline{k}_1$. Thus, we must have $\pi_1 = \pi_0 = 0$ if $k \geq \overline{k}_1$, in which case the uniqueness (and existence) of equilibrium follows from Proposition 4. ■

Proof of Part (ii). By Parts (*ii*) and (*iii*) of Proposition 4 and Part (*iii*) of Proposition 6, there is no equilibrium where $\pi_1 = \pi_0 = 0$ or $\pi_1 = 0 < \pi_0$ or $\pi_1, \pi_0 > 0$ if $k \in [\overline{k}_0, \overline{k}_1)$. We must thus have $\pi_1 > 0 = \pi_0$, in which case π_1 is given by (3).

We now show that π_1 is decreasing in k and increasing in β. It is immediate that π_1 is decreasing in k from the fact that the LHS of (3) is decreasing in k while the RHS is increasing in k and π_1. To show π_1 is increasing in β, rewrite (3) as

$$\frac{v_{11}}{k} = \frac{1}{1-\alpha} + \frac{1}{\alpha(1-\pi_1)} + \frac{v_{01}}{k}\left(1 - \frac{\alpha}{\alpha + (1-\alpha)\pi_1}\right).$$

With $v_{01} = 1 - \beta$, the RHS of this equation is decreasing in β, which implies that π_1 is increasing in β since the RHS is increasing in π_1 while the LHS is constant.

Next, while the uniqueness of equilibrium follows from Proposition 5, it remains to show that no bidder has a profitable deviation from the equilibrium bidding or learning strategy. For no profitable deviation from the equilibrium bidding strategy, we need to prove that no bidder type $t \in \Omega$ has an incentive to deviate to place a bid in $E_{t'}$ with $t' \in \Omega_t$. As with the proof of Proposition 2 (see the Supplementary Appendix), this proof follows directly from applying Lemma 4, and hence is omitted. For no profitable deviation from the equilibrium learning strategy, it suffices to show that each bidder i with $s_i = 0$ has no incentive to deviate to learn s_j. To do so, note that after learning $s_j = 0$, it is optimal for $t_i = 00$ to bid v_{00} and obtain zero payoff, since $00 \in \Omega$. Let $\Gamma^*_{01}(k)$ denote the payoff of $t_i = 01$, as a function of k, from bidding optimally after learning $s_j = 1$. Then, the best payoff that $t_i = 0$ can expect from learning s_j is given by $(1-\alpha)\Gamma^*_{01}(k) - k$.

Claim 1. *$\Gamma^*_{01}(k)$ is decreasing in k.*

Claim 2. $(1-\alpha)\Gamma^*_{01}(\overline{k}_0) = \overline{k}_0$.

Claim 1 and Claim 2 together imply that $(1-\alpha)\Gamma^*_{01}(k) - k \leq 0$ if $k \geq \overline{k}_0$, which means that the deviation is unprofitable. ■

Proof of Part (iii). By Parts (*ii*) and (*iii*) of Proposition 4 and Part (*iv*) of Proposition 5, there is no equilibrium in which $\pi_1 = \pi_0 = 0$ or $\pi_1 > 0 = \pi_0$ or $\pi_1 = 0 < \pi_0$, if $k < k_0$. We must thus have $\pi_1, \pi_0 > 0$, in which case π_1 and π_0 are given by (7) and (8), respectively.

We now prove the effects of k on π_1 and π_0. The fact that π_1 is decreasing in k is immediate from the fact that the RHS of (7) is increasing in π_1 while the LHS is increasing in k. For the effect of k on π_0, note that $\bar{b}_0$ and $\bar{b}_{10}$ in (9) are increasing and decreasing in k, respectively. Given this, the middle expression of (B.20) is decreasing in k, and so is π_0. The effects fo α and β on π_1 and π_0 follow from the next claim.

Claim 3. *For any $k < \bar{k}_0$, $\partial\pi_1/\partial\alpha, \partial\pi_1/\partial\beta > 0$, while $\partial\pi_0/\partial\alpha, \partial\pi_0/\partial\beta < 0$.*

Lastly, while the uniqueness of equilibrium follows from Proposition 6, we need to show that no type $t \in \Omega$ has an incentive to deviate to bid some $b \in E_{t'}$ where $t' \in \Omega_t \backslash \{t\}$.[25] We only analyze the cases in which this statement does not follow directly from Lemma 4.

Claim 4. *$\Gamma_0(b)$ is decreasing in $b \in E_{01}$, $\Gamma_1(b)$ is constant for $b \in E_{10}$, and $\Gamma_{01}(b)$ is increasing in $b \in E_0$.*

Consider first type $t = 0$—for whom $\Omega_t = \{0, 1, 10, 00\}$—deviating to bid some $b \in E_{10} \backslash E_0 = E_{01}$. Since $\Gamma_0(b)$ is decreasing in $b \in E_{01}$, this deviation is unprofitable. Next, consider $t = 1$—for whom $\Omega_t = \{1, 0, 01, 11\}$—deviating to bid some $b \in E_{10} = E_0 \cup E_{10}$. Since the deviation payoff $\Gamma_1(b)$ is constant across the interval E_{10}, we have $\Gamma_1(b) = \Gamma_1(\bar{b}_{10}) = \Gamma_1(\underline{b}_1) = \Gamma_1$ for all $b \in E_{10}$, so such deviation is unprofitable. Lastly, consider type $t = 01$—for whom $\Omega_t = \{1, 10\}$—deviating to bid $b \in E_0 = E_{10} \backslash E_{01}$. Since $\Gamma_{01}(b)$ is increasing in $b \in E_0$, we have $\Gamma_{01}(b) \le \Gamma_{01}(\bar{b}_0) = \Gamma_{01}(\underline{b}_{01}) = \Gamma_{01}$ for all $b \in E_0$, as desired. ■

Proof of Part (iv). We begin with observing the following claims.

Claim 5. *$\frac{\partial(1-\alpha)(v_{10}-\bar{b}_0)}{\partial\alpha} < 0$ for any $k \in [\bar{k}_0, \bar{k}_1)$.*

Claim 6. *For any $k < \bar{k}_1$, $\bar{b}_{10}$ is decreasing in k, α and β.*

Consider first the case $k \in [\bar{k}_0, \bar{k}_1)$. The (ex-ante) equilibrium payoff for each bidder equals

$$\tfrac{1}{2}\Gamma_0 + \tfrac{1}{2}\left[(1-\pi_U)\Gamma_1 + \pi_U(\alpha\Gamma_{11} + (1-\alpha)\Gamma_{10} - k)\right] = \tfrac{1}{2}(1-\alpha)\Gamma_{10} = \tfrac{1}{2}(1-\alpha)(v_{10} - \bar{b}_0),$$

[25] Clearly, there is no profitable deviation from the equilibrium learning strategy, since each bidder is indifferent between learning and not learning irrespective of his signal.

where the first equality holds since $\Gamma_0 = 0$, $\Gamma_1 = (1-\alpha)\Gamma_{10}$, and $\alpha\Gamma_{11} = k$. Note that since $\bar{b}_0 = \bar{b}_{10}$ and $\bar{b}_{10}$ is decreasing in k and β by Claim 6, the equilibrium payoff, $\frac{1}{2}(1-\alpha)(v_{10}-\bar{b}_0)$, is increasing in k and β. The comparative statics regarding α follows from Claim 5.

Consider next the case $k < \bar{k}_0$. The (ex-ante) equilibrium payoff for each bidder equals

$$\begin{aligned}&\tfrac{1}{2}\left[(1-\pi_0)\Gamma_0 + \pi_0(\alpha\Gamma_{00} + (1-\alpha)\Gamma_{01} - k)\right] + \tfrac{1}{2}\left[(1-\pi_1)\Gamma_1 + \pi_1(\alpha\Gamma_{11} + (1-\alpha)\Gamma_{10} - k)\right] \\ &= \tfrac{1}{2}\pi_0\left((1-\alpha)\Gamma_{01} - k\right) + \tfrac{1}{2}\left[(1-\alpha)\Gamma_{10} + \pi_1(\alpha\Gamma_{11} - k)\right] = \tfrac{1}{2}(1-\alpha)\Gamma_{10} = \tfrac{1}{2}(1-\alpha)(v_{10} - \bar{b}_{10}),\end{aligned}$$

where the first equality follows from $\Gamma_0 = \Gamma_{00} = 0$ and $\Gamma_1 = (1-\alpha)\Gamma_{10}$, and the second equality from $\alpha\Gamma_{11} = k = (1-\alpha)\Gamma_{01}$. To see how this changes in k and β, note that $\bar{b}_{10} = \bar{b}_{01}$ is decreasing in β and k by Claim 6. Thus, the equilibrium payoff is increasing in k and β. To see the effect of α, write the ex-ante payoff as

$$\tfrac{1}{2}(1-\alpha)\Gamma_{10} = \tfrac{1}{2}(1-\alpha)(v_{10}-\bar{b}_0) = \tfrac{1}{2}(1-\alpha)\left(v_{10} - v_{01} + \frac{k}{(1-\alpha)\pi_1}\right) = \tfrac{1}{2}(1-\alpha)(2\beta-1) + \frac{k}{2\pi_1},$$

which is decreasing in α since $2\beta - 1 \geq 0$ and π_1 is increasing in α. ∎

C Proofs for Section 5

C.1 Proof of Proposition 8

The difference of bidders' payoff between the two auctions is $\frac{1}{2}(1-\alpha)\bar{b}_{10}$. Since $\bar{b}_{10}$ is decreasing in k and β by Claim 6, so is $\frac{1}{2}(1-\alpha)\bar{b}_{10}$. Next, note that $\frac{\partial(1-\alpha)\bar{b}_{10}}{\partial\alpha} = -\bar{b}_{10} + (1-\alpha)\frac{\partial\bar{b}_{10}}{\partial\alpha} < 0$, where the inequality follows from Claim 6.

C.2 Proof of Proposition 9

Proof of Part (i). We first prove that as $k \to 0$, the total surplus in the first-price auction, T_{FPA}, approaches the first-best level $\frac{1}{2}\alpha + (1-\alpha)\beta$. Since the learning cost vanishes as $k \to 0$, we only need to show that the allocative surplus approaches the first-best level, which holds true if the winning probability of type $t = 10$ against the rival of type $t = 0$ or $t = 01$ approaches 1 as $k \to 0$. To show this, it suffice to prove that for any fixed small $\varepsilon > 0$, there is sufficiently small $\bar{k}(<\bar{k}_0)$ such that for $k < \bar{k}$, $H_{01}(b') > 1-\varepsilon = 1 - H_{10}(b')$ for some $b' \in int(E_{01})$, since it will imply that the winning probability of type $t = 10$ against type $t = 0$ or $t = 01$ is at least $(1-\pi_0) + \pi_0(1-\varepsilon)^2$, which becomes arbitrarily close to 1 by making ε sufficiently small. With k close to 0 and thus smaller than $\bar{k}_0$, the bidding distributions of type $t = 01$ and $t = 10$ on E_{01} are given as

$$H_{01}(b) = \frac{(1-\pi_0)}{\pi_0}\frac{(b-\bar{b}_0)}{v_{10}-b} \quad \text{and} \quad H_{10}(b) = \frac{v_{01}-\bar{b}_{01}}{v_{01}-b}. \tag{C.1}$$

Observe also that as $k \to 0$, we have $\pi_1 \to 1$, $\pi_0 \to \frac{v_{01}}{v_{10}}$, $\bar{b}_0 \to v_{00} = 0$, and $\bar{b}_{01} = \bar{b}_{10} \to v_{01}$. Now let b' be defined such that $H_{10}(b') = \varepsilon$. By (C.1), we have $b' = \frac{\bar{b}_{01}-(1-\varepsilon)v_{01}}{\varepsilon}$, which converges to v_{01} as $k \to 0$ since $\bar{b}_{01} \to v_{01}$ as $k \to 0$. Given this and (C.1), we have $H_{01}(b') = \frac{(1-\pi_0)}{\pi_0}\frac{(b'-\bar{b}_0)}{v_{10}-b'} \to \frac{v_{10}-v_{01}}{v_{01}}\frac{v_{01}}{v_{10}-v_{01}} = 1$ as $k \to 0$ since $\pi_0 \to \frac{v_{01}}{v_{10}}$ and $\bar{b}_0 \to v_{00} = 0$ as $k \to 0$. Thus, one can find sufficiently small k such that $H_{01}(b') > 1-\varepsilon$, as desired.

To prove that $R_{FPA} > R_{SPA}$ at $k \simeq 0$, recall from Part (*iv*) of Theorem 2 that each bidder's payoff in the first-price auction is $\frac{1}{2}(1-\alpha)(v_{10}-\bar{b}_{10}) = \frac{1}{2}(1-\alpha)\left(2\beta - 1 + \frac{k}{(1-\alpha)\pi_1}\right)$, where the equality follows from $\bar{b}_{10}$ in (9). Thus, $B_{FPA} = (1-\alpha)(2\beta - 1 + \frac{k}{(1-\alpha)\pi_1})$, which converges to $(1-\alpha)(2\beta-1)$ as $k \to 0$. Therefore, at $k \simeq 0$,

$$R_{FPA} = T_{FPA} - B_{FPA} \simeq \tfrac{1}{2}\alpha + (1-\alpha)\beta - (1-\alpha)(2\beta-1) = \tfrac{1}{2}\alpha + (1-\alpha)(1-\beta).$$

In the second-price auction, according to Theorem 1, a positive payment is made only when both bidders have high signal and equals $v_{11} = 1$, which means that $R_{SPA} = \frac{1}{2}\alpha < R_{FPA} \simeq \frac{1}{2}\alpha + (1-\alpha)(1-\beta)$ at $k \simeq 0$, as desired.

Next, by Theorem 2, the bidders' payoff decreases as k decreases. Thus, it is minimized as $k \to 0$. Combining this with the above finding that the total surplus approaches the first-best level as $k \to 0$ implies that the seller's revenue is maximized as $k \to 0$. ■

Proof of Part (ii). We show that at $\alpha = \beta = \frac{1}{2}$, $R_{FPA} > R_{SPA}$, from which the desired result will follow since the seller's revenue as well as the equilibrium strategy is continuous at $\alpha = \beta = \frac{1}{2}$. So let $\alpha = \beta = \frac{1}{2}$, and note that $R_{SPA} = \frac{1}{2}\alpha = \frac{1}{4}$ while the allocative surplus in the first-price auction is equal to $\frac{1}{2}$.[26] Consider the case of $k \in [\bar{k}_0, \bar{k}_1)$ in the first-price auction in which

$$(1-\alpha)(v_{10} - \bar{b}_0) = (1-\alpha)\left(v_{10} - \frac{(1-\alpha)\pi_1 v_{01}}{\alpha + (1-\alpha)\pi_1}\right) = \frac{1}{4(1+\pi_1)}, \tag{C.2}$$

[26] With $\beta = \frac{1}{2}$, the values are common across bidders and equal to $\frac{1}{2}$, which means that the allocative surplus is $\frac{1}{2}$ irrespective of the allocation, as long as someone obtains the object.

where the first equality follows from $\bar{b}_0$ in (4) and some rearrangement, and the second equality holds since $\alpha = \beta = \frac{1}{2}$. Hence, $R_{FPA} = \frac{1}{2} - \pi_1 k - \frac{1}{4(1+\pi_1)}$, where $\pi_1 k$ is the learning cost. We thus have

$$R_{FPA} - R_{SPA} = \frac{1}{2} - \pi_1 k - \frac{1}{4(1+\pi_1)} - \frac{1}{2} = \pi_1 \left(\frac{1}{4(1+\pi_1) - k} \right) = \frac{\pi_1^3}{8 + 4\pi_1(1-\pi_1)} > 0,$$

where the last equality holds since

$$k = \frac{(1-\pi_1)(2+\pi_1)}{4(2-\pi_1)(1+\pi_1)}, \tag{C.3}$$

which follows from (3) and the fact that $\alpha = \beta = \frac{1}{2}$.

Before turning to the case of $k < \bar{k}_0$, we show that $\bar{k}_0 = \frac{1}{10}$ at $\alpha = \beta = \frac{1}{2}$. To see this, recall that from (5), $\bar{k}_0$ is the unique solution to

$$k = (1-\alpha)(v_{01} - \bar{b}_0)\pi_1 = \frac{\pi_1}{4(1+\pi_1)},$$

where the second equality follows from the fact that $v_{10} = v_{01} = \frac{1}{2}$ (since $\beta = \frac{1}{2}$) and (C.2). Equating this with (C.3), we have $\pi_1 = \frac{2}{3}$ and $\bar{k}_0 = \frac{1}{10}$. Next, for $k < \bar{k}_0$,

$$B_{FPA} = (1-\alpha)(v_{10} - \bar{b}_{01}) = (1-\alpha)\left(v_{10} - v_{01} + \frac{k}{(1-\alpha)\pi_1} \right) = \frac{k}{\pi_1},$$

where the last equality holds since $\alpha = \beta = \frac{1}{2}$. Thus,

$$\begin{aligned} R_{FPA} - R_{SPA} = T_{FPA} - B_{FPA} - R_{SPA} &= \frac{1}{2} - (\pi_0 + \pi_1)k - \frac{k}{\pi_1} - \frac{1}{4} \\ &= \frac{1}{4} - \left(1 - \frac{4k}{(1-4k)\pi_1} + \pi_1\right) k - \frac{k}{\pi_1} = \frac{2 - \pi_1 - 2\pi_1^3 + 2\pi_1^4}{4(1-2\pi_1)(1-3\pi_1+\pi_1^2)}, \end{aligned}$$

where the second equality follows from (8) and the fact that $\alpha = \beta = \frac{1}{2}$, and the last equality holds since $k = \frac{(1-\pi_1)\pi_1}{4(3\pi_1 - \pi_1^2 - 1)}$ from substituting $\alpha = \beta = \frac{1}{2}$ into (7). One can show that the numerator of the RHS of the last equality attains its minimum value 0.019 at $\pi_1 \approx 0.637$, and the denominator is strictly positive for any $\pi_1 > \frac{1}{2}$, which holds true since the fact that $k = \frac{(1-\pi_1)\pi_1}{4(3\pi_1 - \pi_1^2 - 1)} < \bar{k}_0 = \frac{1}{10}$ implies $\pi_1 > \frac{2}{3}$. ∎

Proof of Part (iii). With $\beta \simeq 1$, we have $v_{01} \simeq 0$, so the expression in (8) becomes negative, meaning that we must have $\pi_0 = 0$ in the equilibrium. Thus, from (3), we obtain

$\pi_1 \simeq \frac{\alpha(1-\alpha)-k}{\alpha(1-\alpha-k)} > 0$. Substituting this into $\overline{b}_0 = \overline{b}_{10}$ in (4), we have $\overline{b}_0 = \overline{b}_{10} \simeq 0$ and thus

$$B_{FPA} = 2 \times \tfrac{1}{2}(1-\alpha)(v_{10} - \overline{b}_{10}) \simeq (1-\alpha) \tag{C.4}$$

since $v_{10} \simeq 1$ with $\beta \simeq 1$. Also, $T_{FPA} \leq \frac{1}{2}\alpha + (1-\alpha) - \pi_1 k$. Hence,

$$R_{FPA} = T_{FPA} - B_{FPA} \leq \tfrac{1}{2}\alpha + (1-\alpha) - \pi_1 k - B_{FPA} \simeq \tfrac{1}{2}\alpha - \pi_1 k < \tfrac{1}{2}\alpha = R_{SPA}.$$

where the approximate equality follows from (C.4) and the strictly inequality holds since $k, \pi_1 > 0$. ■

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