Declining Real Interest Rates: The Role of Energy Prices in Energy Importers

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Abstract

This study investigates whether the upward trend in real energy prices since the 1990s can account for the decreasing trend in real interest rates in energy-importing countries. By estimating dynamic panel data models, I first provide empirical evidence that a rise in real energy prices leads to falls in real interest rates and aggregate consumption in energy-importing OECD member countries, whereas this effect is much weaker in energy-exporting OECD member countries than in energy-importing ones. Using a life-cycle model, I show that the increasing trend in real energy prices decreased the equilibrium real interest rate by about one percentage point between 1990 and 2018. The mechanism for this effect is as follows: As a result of the increases in the real energy price, energy consumption falls, dampening the consumption of manufactured goods because of the complementarity in consumption between energy and manufactured goods. Accordingly, aggregate consumption declines, putting downward pressure on the real interest rate. Moreover, the increased real energy price decreases energy input in production, raising the capital/energy ratio and inducing a lower real interest rate via a fall in the marginal product of capital. Through a simulation, I also demonstrate that increases in the real energy price had a greater influence on the declining real interest rate in energy-importing countries during 1990-2018, compared to population aging.

JEL Classification: D15, E43, J11, Q43

Keywords: energy prices; real interest rate; energy importers; life expectancy; population growth

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1 Introduction

The real interest rates in many developed economies have been declining steadily since the 1990s, remaining in negative territory even after the global financial crisis (Figure 1).[1] Meanwhile, real energy prices have been trending upward overall. Since energy is both consumed by households and used as an input in production, fluctuations in real energy prices influence household consumption as well as the marginal product of capital.[2] Thus, it is natural to expect that the upward trend in real energy prices would have influenced the falling trend in real interest rates since the 1990s. Surprisingly, however, increases in real energy prices have yet to be considered in the literature related to declining real interest rates. Most studies have focused mainly on demographics and fiscal factors. Therefore, this study examines the impact of the upward trend in real energy prices on the declining real interest rates in energy-importing countries, and shows that increases in real energy prices have played a substantial role in decreasing real interest rates.[3] Accordingly, this paper adds to the literature by demonstrating that the upward trend in real energy prices since the 1990s may be contributing to the decline in real interest rates in developed economies during the same period.

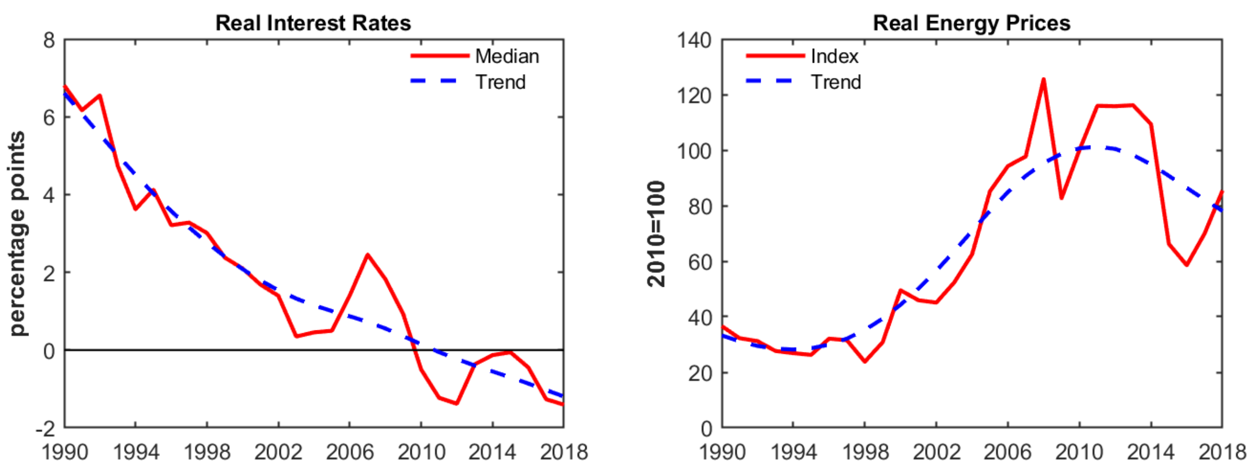

Figure 1: Real interest rates and real energy prices

Notes: "Median" in the left panel denotes the median ex-post real interest rate among OECD member countries except for Turkey, whose nominal short-term interest rate data are not available either from the OECD data or the Federal Reserve Economic Data (FRED) of the St. Louis Fed. Ex-post real interest rates are defined as the nominal short-term interest rates minus realized CPI inflation. The source of the data on nominal short-term interest rates and CPI inflation is the OECD, except for Japan, whose nominal short-term interest rate data were sourced from the FRED. The real energy price index data are from the World Bank. The trends of the median real interest rates and real energy price index are obtained from the Hodrick-Prescott filter.

[1] This phenomenon has led to the resuscitation of the term "secular stagnation" (initially coined by Hansen 1939) in recent years, by scholars such as Gordon (2012), Krugman (2013), and Summers (2013).

[2] Since changes in energy prices affect both firms and households, energy prices have a more widespread effect on the economy compared to the prices of other goods. Hence, macroeconomists generally regard changes in energy prices as an important source of economic fluctuation (Blanchard and Galí 2010).

[3] This paper does not consider energy exporters, since a rise in real energy prices in such countries does not lead to a fall in real interest rates, unlike the case with energy importers, as shown in section 2.

As an illustration, consider the case of an energy-importing country, and suppose that energy is included in the consumption basket of households and that firms use energy as an input in production, along with capital and labor. A rise in real energy prices will dampen energy consumption. Since energy and manufactured goods are complementary in consumption, the dampened energy consumption will put downward pressure on the consumption of manufactured goods.[4] Aggregate consumption will fall, meaning that household savings will increase. Moreover, as real energy prices increase, energy input in production will reduce. The decreased energy input in production will cause a higher capital/energy ratio, leading to a fall in the marginal product of capital. Since the decreases in both aggregate consumption and the marginal product of capital will put downward pressure on real interest rates, it is expected that an increase in real energy prices would push real interest rates downward in energy-importing countries.

However, the case of energy-exporting countries is different. Unlike energy importers, a rise in real energy prices in the former will induce an increase in energy export income. Thus, the increased income will partly or fully offset the negative effects of the increased real energy prices on aggregate consumption. Consequently, the aggregate consumption in energy-exporting countries will fall by less than that in energy-importing countries or may even increase. Moreover, since energy is the main product for energy exporters, the increase in real energy prices will not cause a fall in the marginal product of capital but rather, lead to an increase. Therefore, for energy-exporting countries, an increase in real energy prices will not lead to a fall in real interest rates, or even if it does, will cause a far smaller fall compared to the fall in energy-importing counties. This supposition is consistent with the empirical evidence in section 2. For this reason, I focus on energy importers (especially energy-importing OECD member countries).[5]

The primary aim of this study is to show that the upward trend in real energy prices induces a declining trend in real interest rates in energy-importing countries. Furthermore, this study investigates whether the increasing trend in real energy prices has greater explanatory power for the decrease in real interest rates in energy-importing countries, compared to demographic transition factors.

This paper begins by providing empirical evidence that a rise in real energy prices has negative influences on real interest rates and consumption in energy-importing OECD member countries.[6] Specifically, I construct a panel dataset of OECD member countries from

[4] Many studies such as Cooper (2003), Hughes et al. (2006), and Bodenstein et al. (2011) have shown that energy and non-energy in consumption are complements.

[5] Moreover, since most OECD member countries are net energy importers, considering only energy-importing OECD member countries may not matter significantly.

[6] It should be noted that real energy prices affect energy consumption and the capital/energy ratio in production and hence the marginal product of capital and aggregate consumption, resulting in changes in real interest rates, as already explained. Therefore, it is more desirable to show how energy consumption and

1970–2018 and estimate dynamic panel data models using the system generalized method of moments (GMM). The estimation results show that an increase in real energy prices brings about falls in real interest rates and aggregate consumption in energy-importing OECD member countries, and that this effect is much smaller in energy-exporting OECD member countries compared to energy-importing ones. In fact, according to the estimation results, real interest rates and aggregate consumption in energy-exporting OECD member countries do not fall in response to a rise in real energy prices.

I then construct a life-cycle model by extending the life-cycle model of Gertler (1999) to include energy in consumption and production.[7] Specifically, I consider a representative energy-importing OECD member country; energy is used as an input in production and consumed by households in the model. By calibrating the model to capture the observed trend of real energy prices, I analyze the effect of increasing real energy prices on the equilibrium real interest rate. The model simulation results show that the upward trend in the real energy price substantially decreases the real interest rate through the aforementioned mechanism, that is, by decreasing consumption and increasing the capital/energy ratio. Specifically, the model predicts that increases in real energy prices decreased real interest rates in energy-importing OECD member countries by about one percentage point between 1990 and 2018.

In addition, I compare the effects of demographic transition (i.e., population aging) and increasing real energy prices on real interest rates for energy importers via a model simulation. To do so, I calibrate the model to capture not only the observed transitions in life expectancy and population growth (and thus, the dependency ratio) in energy-importing OECD member countries but also the observed trend of real energy prices between 1990 and 2018. The results reveal that the negative effect of the increasing trend in real energy prices on the real interest rate in energy-importing OECD member countries is about two times greater than that of population aging.

This study is relevant to the literature on the recent decline in real interest rates in developed economies. Many studies such as Carvalho et al. (2016) and Fujita and Fujiwara (2016) argue that population aging is the primary reason for declining real interest rates.[8] For example, according to Carvalho et al. (2016), longer life expectancy leads to an increase in the retirement period for a given retirement age; hence, individuals have stronger incentives

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the capital/energy ratio, marginal product of capital, aggregate consumption and real interest rates respond to changes in real energy prices. However, since data for energy consumption, the capital/energy ratio, and the marginal product of capital are not available, I consider only aggregate consumption and real interest rates in empirical evidence.

[7] Since one of the main aims of this paper is to compare the effects of real energy prices and demographic transition, I chose the life-cycle model, in which both energy and demographics can be considered.

[8] Ikeda and Saito (2014), Kara and von Thadden (2016), Favero and Galasso (2016), Gagnon et al. (2016), Lisack et al. (2017), Aksoy et al. (2019), and Ferrero et al. (2019), among others, also argue that population aging has contributed to the decreasing trend in real interest rates.

to save throughout the life cycle, which pushes down real interest rates. Moreover, a drop in population growth increases the capital/labor ratio; consequently, the marginal product of capital falls, putting downward pressure on real interest rates. The decreased population growth, however, also brings about a rise in the dependency ratio (the ratio of people aged 65 years and older to people aged 15–64 years). Since the marginal propensity to save of retired people is generally lower, an increase in the dependency ratio tends to raise real interest rates. Nevertheless, the overall effect of population aging on real interest rates is negative, according to the simulation results of their life-cycle model. Some papers such as Rachel and Summers (2019) and Kobayashi and Ueda (2021) emphasize fiscal factors in the falling trend of real interest rates. Specifically, Kobayashi and Ueda (2021) show that the decline in real interest rates in Japan can be explained by the increasing fear of large-scale taxation on capital and the misallocations of capital in future debt. However, the role of energy prices in declining real interest rates has yet to be explored in the relevant literature.

The remainder of this paper proceeds as follows. Section 2 provides empirical evidence that a rise in real energy prices leads to falls in real interest rates and aggregate consumption in energy-importing OECD member countries, and that it brings about greater decreases in real interest rates and aggregate consumption in energy-importing countries than in energy-exporting ones. Section 3 describes the life-cycle model. Section 4 presents the simulation results of the life-cycle model. Model calibration and further experiments—introduction of social security into the model, comparison with population aging, and sensitivity analysis—are also included in this section. Section 5 presents the conclusion.

2 Empirical Evidence

This section provides empirical evidence that real interest rates and aggregate consumption in energy-importing OECD member countries decline in response to a rise in real energy prices, and that these falls are greater than that of energy-exporting OECD member countries.[9] Specifically, I construct an annual panel dataset for OECD member countries from 1970–

[9] Thirty one OECD member countries are considered in this section. There are five energy-exporting OECD member countries with positive net energy exports, according to the OECD Trade in Value Added dataset for 2005-2015: Australia, Canada, Denmark, Mexico, and Norway. Energy exports and imports are defined as exports and imports of mining and quarrying industries. Several OECD member countries such as Chile, Hungary, Lithuania, Latvia, and Turkey are not included in the sample countries since their data are not available.

2018, and estimate the following two dynamic panel data models.[10]

$$
\begin{aligned}
R_{i,t} &= \vartheta_i + \alpha R_{i,t-1} + \alpha_1 EP_t + \alpha_2 (EP_t \times D_{i,t}) + \beta' X_{i,t} + \varepsilon_{i,t}, \\
C_{i,t} &= \zeta_i + \gamma C_{i,t-1} + \gamma_1 EP_t + \gamma_2 (EP_t \times D_{i,t}) + \delta' Z_{i,t} + \varsigma_{i,t},
\end{aligned}
$$

where R and C are the real interest rate and aggregate consumption, respectively, ϑ_i and ζ_i are a country fixed effect, EP is the logarithm (log) of the real energy price index, and D is a dummy variable whose value is 1 if the country is an energy exporter.[11] X and Z are a vector of other control variables. X consists of the log of real GDP, the log of the core consumer price index (CPI), and the log of the exchange rate between the national currency and the US dollar.[12] X also includes a global financial crisis dummy that takes the value of 1 for the period 2008–2011. Z includes real interest rates and all variables in X. The subscripts i and t denote countries and years, respectively.

The above dynamic panel data models are estimated using the system GMM developed by Arellano and Bover (1995) and Blundell and Bond (1998). Real interest rates and aggregate consumption in a country affect its macroeconomic variables such as GDP, core CPI, and exchange rates, while real interest rates and aggregate consumption in small open economies do not influence real energy prices, a global variable. Furthermore, all OECD member countries, with the exception of the US, can be regarded as small open economies.[13] Therefore, in choosing an instrument set, the variables GDP, core CPI, and exchange rate in the first dynamic panel data model, and these plus real interest rates in the second model are treated as endogenous; whereas real energy prices and the interaction term between real energy price

[10] Due to data availability, the sample start date for each country differs. Specifically, the sample start date for Australia is 1972; it is 1974 for New Zealand and Switzerland, 1977 for Belgium and Spain, 1979 for Italy and Norway, 1980 for Japan, 1982 for the Netherlands and Sweden, 1984 for Ireland, 1986 for the UK, 1987 for Denmark and Finland, 1988 for Iceland and Portugal, 1991 for Korea, 1992 for Israel and Poland, 1995 for the Czech Republic and Greece, 1997 for Mexico, 1998 for Estonia, 1999 for Luxembourg and the Slovak Republic, and 2002 for Slovenia. The sample start date for the remaining countries is 1970.

[11] The real interest rates are ex-post real short-term interest rates, calculated using the nominal short-term interest rates minus realized CPI inflation. The nominal short-term interest rate, whose source is the OECD, is either the three-month interbank rate or the three-month Treasury bill rate, except in the case of Japan. The nominal short-term interest rate for Japan is the three-month rate for certificates of deposit, and the source of this data is the FRED. The source of the real energy price index is the World Bank

[12] The source of consumption, real GDP, and core CPI data is the OECD and that of exchange rates is the FRED, except for the US. In the case of the US exchange rate, I use the nominal effective exchange rate (neer) from the BIS. Since an increase in the exchange rate between the national currency and the US dollar implies a depreciation of the currency, while an increase in the US neer implies an appreciation of the US dollar, the log inverse of US neer is entered in the equation. Note that the core CPI is used instead of overall CPI. This is because overall CPI includes energy prices, and hence using core CPI, excluding food and energy prices, seems to be more reasonable.

[13] US real interest rates and consumption influence real energy prices since the US is regarded as a large economy. Hence, in the case of the US, real energy prices should be treated as endogenous. Considering this fact, I provide estimation results excluding the US in Online Appendix A. Even if the US is excluded in the sample countries, estimation results are very similar to those using all sample countries.

and the dummy variable for energy exporters ($EP_t \times D_{i,t}$) are treated as exogenous.[14]

The purpose of this analysis is to provide empirical evidence that a rise in real energy prices leads to falls in real interest rates and aggregate consumption in energy-importing OECD member countries, and that the rise brings about smaller decreases in real interest rates and aggregate consumption in energy-exporting OECD member countries than in energy-importing ones. Hence, this study is interested in the signs and significance of α_1 and γ_1, the coefficients of the real energy prices, and those of the coefficients of the interaction terms α_2 and γ_2. According to the hypothesis, α_1 and γ_1 should be negative and statistically significant, which implies that an increase in real energy prices triggers falls in real interest rates and aggregate consumption. Moreover, if α_2 and γ_2 are positive and statistically significant, it suggests that the negative influence of an increase in real energy prices on real interest rates and aggregate consumption in energy-exporting OECD member countries is weaker than that in energy-importing ones.

The second column in Table 1 presents the estimation results of the first dynamic panel data model. First, the estimated α_1 (the coefficient of real energy prices) in the second row of the second column is negative and statistically significant at the 1% level, implying that an increase in real energy prices induces a decrease in real interest rates in energy-importing OECD member countries. Furthermore, the estimated coefficient of the interaction term α_2 in the third row of the column is positive and statistically significant at the 10% level, showing that the negative impact of a rise in real energy prices on real interest rates in energy-exporting OECD member countries tends to be far smaller than that in energy-importing ones. Indeed, the estimated effect of an increase in real energy prices on real interest rates in energy-exporting OECD member countries is the sum of the estimated coefficients of real energy prices and the interaction term (i.e., estimated $\alpha_1+\alpha_2$). Since the estimated $\alpha_1+\alpha_2$ is positive, a rise in real energy prices tends to increase real interest rates in energy-exporting OECD member countries.

The third column in Table 1 presents the estimation results of the second dynamic panel data model. The estimated coefficient of real energy prices γ_1 in the second row of the column is negative and statistically significant at the 1% level. This implies that if real energy prices increase, aggregate consumption in energy-importing OECD member countries decreases. Moreover, the estimated γ_2 (coefficient of the interaction term in the third row of the column) is positive and statistically significant at the 5% level, implying that the negative influence of an increase in real energy prices on aggregate consumption in energy-exporting OECD member countries is much smaller than that in energy-importing ones. In particular, the estimated effect of a rise in real energy prices on consumption in energy-exporting OECD

[14] I reduce lags for instruments and collapse the instrument set, as recommended by Roodman (2009), to limit instrument proliferation which can weaken the Hansen test.

member countries is the estimated $\gamma_1+\gamma_2$. Considering that the estimated $\alpha_1+\alpha_2$ is positive, an increase in real energy prices tends to increase consumption in energy-exporting OECD member countries.

Table 1: Estimation results

	Dependent variable:	
Explanatory variables:	Real interest rate	Consumption
Lagged dependent variable	0.7644***	0.5706***
	(0.0583)	(0.1608)
Real energy price (α_1 and γ_1)	**-0.0087*** **	**-0.0200*** **
	(0.0033)	**(0.0071)**
Real energy price×Exporter dummy (α_2 and γ_2)	**0.0128* **	**0.0516** **
	(0.0073)	**(0.0226)**
GDP	0.0030	0.4110***
	(0.0021)	(0.1524)
Core CPI	-0.0006	0.0107
	(0.0079)	(0.0151)
Exchange rate	-0.0102**	-0.0177
	(0.0052)	(0.0176)
Global financial crisis dummy	-1.3235***	-0.5202*
	(0.2262)	(0.3107)
Real interest rate		0.0967
		(0.1071)
Number of countries	31	31
Number of instruments	31	28
AB test for AR(2) (p-values)	0.301	0.198
Hansen test (p-values)	0.315	0.333

Notes: Clustered standard errors are reported in parentheses. ***, ** and * denote significance at the 1%, 5%, and 10% level, respectively. AB test means the Arellano-Bond test.

In summary, the estimation results clearly show that an increase in real energy prices tends to decrease real interest rates and aggregate consumption in energy-importing OECD member countries, and that this effect is much weaker in energy-exporting OECD member countries. These results provide empirical support for the aforementioned hypothesis that the upward trend in real energy prices since the 1990s has played a substantial role in the declining trend in real interest rates in energy-importing countries during the same period.

3 The Life-cycle Model

In this section, I describe the life-cycle model for a representative energy-importing OECD member country. The model is an extended version of the life-cycle model of Gertler (1999). Specifically, I incorporate energy into Gertler's (1999) model. Hence, in the model, households consume energy as well as manufactured goods, with firms using energy as an input in production, along with labor and capital. As in previous studies (Atkeson and Kehoe (1999), Kim (2020), etc.), energy needs to be imported from abroad at an exogenous world price. There is no aggregate uncertainty in the model, and thus, agents have perfect foresight. Nonetheless, they can be surprised by initial unexpected shocks.

3.1 Life-cycle Structure and Households

There are two types of households, workers and retirees. In period t, N_t^w workers and N_t^r retirees exist.[15] Between periods $t-1$ and t, a typical worker retires with a probability $1-\omega_t$, and a typical retiree survives with a probability γ_t. In period t, $(1-\omega_t+n_t)N_{t-1}^w$ new workers are born, and thus, the law of motion for the labor force is

$$N_t^w = (1-\omega_t+n_t)N_{t-1}^w + \omega_t N_{t-1}^w = (1+n_t)N_{t-1}^w. \tag{1}$$

Hence, n_t is the growth rate of workers between periods $t-1$ and t. The number of retirees evolves according to

$$N_t^r = (1-\omega_t)N_{t-1}^w + \gamma_t N_{t-1}^r. \tag{2}$$

From Equations (1) and (2), we can obtain the law of motion for the dependency ratio ($\psi_t \equiv N_t^r/N_t^w$):

$$(1+n_t)\psi_t = (1-\omega_t) + \gamma_t \psi_{t-1}. \tag{3}$$

It is assumed that workers inelastically supply one unit of labor, whereas retirees do not work.[16] In addition, as in Yaari (1965) and Blanchard (1985), there is a perfect annuities market. That is, a mutual fund exists. The fund receives wealth from retirees and invests the proceeds. Each surviving retiree receives a return proportionate to their contribution, while retirees who die receive nothing. Due to the existence of the perfect annuities market, the uncertainty of death that retirees face can be eliminated.

To address the uncertainty of retirement that workers face, I assume risk neutrality in

[15] The superscripts w and r denote workers and retirees, respectively, throughout this paper.

[16] According to Carvalho et al. (2016), although the demographic trends in developed economies would require individuals to supply more hours, the data shows a decreasing trend for the variable. Thus, they view the assumption of inelastic labor supply may not matter greatly for the purposes of their paper.

preferences, as in Gertler (1999). Specifically, the preferences are a class of the recursive non-expected utility function. Denoting V_t^z, $z \in \{w, r\}$, the value of utility is

$$V_t^z = \left[C_t^{z^\rho} + \beta_{t+1}^z \left\{ \mathbb{E}_t \left(V_{t+1} \mid z \right) \right\}^\rho \right]^{\frac{1}{\rho}}, \tag{4}$$

where C_t^z is a consumption index, and $\mathbb{E}_t$ denotes the expectations operator. Since retirees face a positive death probability, the subjective discount factors of workers and retirees are different:

$$\beta_t^w = \beta, \;\; \beta_t^r = \beta \gamma_t.$$

Hence, workers and retirees have a different $\mathbb{E}_t \left(V_{t+1} \mid z \right)$:

$$\mathbb{E}_t \left(V_{t+1} \mid w \right) = \omega_{t+1} V_{t+1}^w + (1 - \omega_{t+1}) V_{t+1}^r, \;\; \mathbb{E}_t \left(V_{t+1} \mid r \right) = V_{t+1}^r.$$

The consumption index C_t^z, which is identical for workers and retirees, is defined by the consumption of manufactured goods $C_{M,t}^z$ and energy consumption $C_{E,t}^z$, using the constant elasticity of substitution (CES) aggregator.

$$C_t^z = \left\{ (1 - \chi)^{\frac{1}{\eta}} C_{M,t}^{z \frac{\eta - 1}{\eta}} + \chi^{\frac{1}{\eta}} C_{E,t}^{z \frac{\eta - 1}{\eta}} \right\}^{\frac{\eta}{\eta - 1}}. \tag{5}$$

3.1.1 Retirees

The representative retiree, born and retired in periods j and τ respectively, maximizes the following utility by choosing consumption $C_t^r(j, \tau)$, capital $K_t^r(j, \tau)$, and government bond $B_t^r(j, \tau)$, for $t \geq \tau$.

$$V_t^r(j, \tau) = \left[\left\{ C_t^r(j, \tau) \right\}^\rho + \beta \gamma_{t+1} \left\{ V_{t+1}^r(j, \tau) \right\}^\rho \right]^{\frac{1}{\rho}}, \tag{6}$$

The corresponding budget constraint is:

$$C_t^r(j, \tau) + K_t^r(j, \tau) + B_t^r(j, \tau) = \frac{1}{\gamma_t} \left[\left\{ R_t^k + (1 - \delta) \right\} K_{t-1}^r(j, \tau) + R_{t-1} B_{t-1}^r(j, \tau) \right]. \tag{7}$$

In addition, the initial asset holdings of a retiree are equal to his asset holdings in the last period as a worker.

$$K_{\tau-1}^r(j, \tau) = K_{\tau-1}^w(j), \;\; B_{\tau-1}^r(j, \tau) = B_{\tau-1}^w(j).$$

Since there is no aggregate uncertainty in the model, asset returns must be equal:

$$R_t = R^K_{t+1} + (1 - \delta).$$

Let $A^r_t(j,\tau) \equiv K^r_t(j,\tau) + B^r_t(j,\tau)$ be the total assets for a retiree. We can then rewrite the budget constraint more compactly:

$$C^r_t(j,\tau) + A^r_t(j,\tau) = \frac{R_{t-1}A^r_{t-1}(j,\tau)}{\gamma_t}. \tag{8}$$

Then, the retiree's consumption can be written as:[17]

$$C^r_t(j,\tau) = \xi^r_t \left(\frac{R_{t-1}A^r_{t-1}(j,\tau)}{\gamma_t} \right), \tag{9}$$

where ξ^r_t refers to retirees' marginal propensity to consume (MPC). The MPC evolves according to the following law of motion:

$$\xi^r_t = 1 - \gamma_{t+1}\beta^\sigma R_t^{\sigma-1} \frac{\xi^r_t}{\xi^r_{t+1}}. \tag{10}$$

3.1.2 Workers

The representative worker, born in period $j \leq t$, also chooses consumption and assets to maximize the following utility:

$$V^w_t(j) = \left[\{C^w_t(j)\}^\rho + \beta \left\{ \omega_{t+1}V^w_{t+1}(j) + (1-\omega_{t+1})V^r_{t+1}(j,t+1) \right\}^\rho \right]^{\frac{1}{\rho}}, \tag{11}$$

subject to

$$C^w_t(j) + A^w_t(j) = R_{t-1}A^w_{t-1}(j) + W_t - T^w_t, \tag{12}$$

where $A^w_t(j) \equiv K^w_t(j) + B^w_t(j)$, W_t is the real wage, T^w_t denotes a lump-sum tax paid by each worker, and $A^w_t(j) = 0$ for $t = j$.

Online Appendix C explains how to solve the worker's optimization problem. Workers' consumption is defined as:

$$C^w_t(j) = \xi^w_t \left(R_{t-1}A^w_{t-1}(j) + H^w_t \right), \tag{13}$$

where ξ^w_t and H^w_t denote the MPC of workers and the human wealth of workers, respectively.

[17] Online Appendix B explains how to solve the retiree's problem.

H_t^w is defined as the present discounted value of current and future after-tax labor income:

$$H_t^w = W_t - T_t^w + \sum_{l=1}^{\infty} \frac{W_{t+l} - T_{t+l}^w}{\prod_{s=1}^{l} \left(\Omega_{t+s} R_{t+s-1}/\omega_{t+s}\right)} = W_t - T_t^w + \frac{\omega_{t+1} H_{t+1}^w}{\Omega_{t+1} R_t}. \tag{14}$$

The MPC of workers ξ_t^w evolves according to the following equation:

$$\xi_t^w = 1 - \gamma_{t+1} \beta^\sigma \Omega_{t+1}^{\sigma-1} R_t^{\sigma-1} \frac{\xi_t^w}{\xi_{t+1}^w}. \tag{15}$$

Finally, the adjustment term Ω_t is defined as:

$$\Omega_t = \omega_t + (1 - \omega_t) \varepsilon_t^{\frac{1}{1-\sigma}}, \tag{16}$$

with $\varepsilon_t = \xi_t^r / \xi_t^w$. Workers' equations are more complicated compared to those of retirees, due to the term Ω_t. To explain, workers consider the possibility that they may retire in the next period. That is, when a worker retires, he/she faces a different MPC, which is captured by the term ε_t.

3.1.3 Aggregation

Since the MPC, human wealth for workers, adjustment factor, interest rates, and wages are the same for all retirees or for all workers, we can easily aggregate the equations for the households' problems. We define an aggregate variable S_t^z, $z \in \{w, r\}$, as $\int_0^{N^z} S_t^z(i) di$.

Aggregate consumption for retirees and workers is:

$$C_t^r = \xi_t^r \left(R_{t-1} A_{t-1}^r\right), \tag{17}$$

$$C_t^w = \xi_t^w \left(R_{t-1} A_{t-1}^w + H_t\right). \tag{18}$$

In Equation (17), the survival probability γ_t is absent. This is because only surviving retirees receive a return of R_{t-1}/γ_t. The law of motion for the aggregate human wealth for workers is:

$$H_t = W_t N_t^w - T_t + \frac{\omega_{t+1} H_{t+1}}{(1 + n_{t+1}) \Omega_{t+1} R_t}. \tag{19}$$

Hence, the aggregate consumption C_t is:

$$C_t = \xi_t^r \left(\lambda_{t-1} R_{t-1} A_{t-1}\right) + \xi_t^w \left((1 - \lambda_{t-1}) R_{t-1} A_{t-1} + H_t\right), \tag{20}$$

with $\lambda_t = A_t^r / A_t$. Aggregate assets for retirees are the sum of the savings of surviving retirees and just-retired workers:

$$A_t^r = R_{t-1}A_{t-1}^r - C_t^r + (1-\omega_{t+1})\left\{R_{t-1}A_{t-1}^w + W_tN_t^w - T_t - C_t^w\right\}. \tag{21}$$

Aggregate assets for workers are the savings of workers remaining in the labor force:

$$A_t^w = \omega_{t+1}\left\{R_{t-1}A_{t-1}^w + W_tN_t^w - T_t - C_t^w\right\}. \tag{22}$$

From the above equations, we can find the law of motion for λ_t:

$$(\lambda_t - (1-\omega_{t+1}))\,A_t = \omega_{t+1}\,(1-\xi_t^r)\,\lambda_{t-1}R_{t-1}A_{t-1}. \tag{23}$$

Finally, aggregate assets are the sum of aggregate capital and government bonds:

$$A_t = K_t + B_t. \tag{24}$$

3.2 Firms

The representative firm produces manufactured goods using capital, labor, and energy as inputs. Its production function is a nested CES with constant returns to scale, as in Kim and Loungani (1992) and Kim (2021):

$$Y_t = (X_tN_t^w)^\alpha\left\{(1-\phi)K_{t-1}^{-\nu} + \phi E_t^{-\nu}\right\}^{-\frac{(1-\alpha)}{\nu}}, \tag{25}$$

where E_t denotes energy inputs, and α is the labor share. ϕ determines the importance of energy and $\nu = \frac{1-\varsigma}{\varsigma}$, where ς denotes the elasticity of substitution between capital and energy. X_t is the total factor productivity, which grows exogenously at a rate x_t:

$$X_t = (1+x_t)X_{t-1}. \tag{26}$$

The first-order conditions are as follows:

$$W_tN_t^w = \alpha p_{M,t}Y_t, \tag{27}$$

$$R_t^K = (1-\alpha)(1-\phi)K_{t-1}^{-\nu-1}\frac{p_{M,t}Y_t}{(1-\phi)K_{t-1}^{-\nu} + \phi E_t^{-\nu}}, \tag{28}$$

$$p_{E,t} = \phi(1-\alpha)E_t^{-\nu-1}\frac{p_{M,t}Y_t}{(1-\phi)K_{t-1}^{-\nu} + \phi E_t^{-\nu}}, \tag{29}$$

where $p_{M,t}$ and $p_{E,t}$ are the real prices of manufactured goods and energy, respectively.

3.3 Fiscal Policy

In order to finance government spending G_t, the government levies taxes T_t and issues government bonds B_t; hence, its budget constraint is:

$$B_t = R_{t-1}B_{t-1} + G_t - T_t. \tag{30}$$

We assume the ratios of government debt and spending to GDP are fixed as in Gertler (1999).

$$B_t = b_y p_{M,t} Y_t, \tag{31}$$

$$G_t = g_y p_{M,t} Y_t. \tag{32}$$

3.4 Equilibrium

Given the demographic processes (n_t, ω_t, and γ_t), the productivity growth rate (x_t), and the real energy price ($p_{E,t}$), a competitive equilibrium is a sequence of $\{C_t, C_t^r, C_t^w, C_{E,t}, C_{M,t}, A_t,$ $A_t^r, A_t^w, \lambda_t, H_t, Y_t, I_t, K_t, E_t, T_t, G_t, B_t, \xi_t^r, \xi_t^w, \Omega_t, \varepsilon_t, R_t, R_t^K, W_t, p_{M,t}, \psi_t\}$, such that:

i. Taking prices as given, households maximize their utility subject to the budget constraint.

ii. Firms maximize their profits subject to the production function.

iii. The government chooses its debt and taxes to satisfy its budget constraint.

iv. The labor, capital, and goods markets are clear, and therefore, the aggregate resource constraint of the economy is $p_{M,t}Y_t = C_t + p_{M,t}I_t + G_t + p_{E,t}E_t$. The law of motion of capital is given by $K_t = (1-\delta)K_{t-1} + p_{M,t}I_t$.

Furthermore, since all quantity variables in this economy grow at a rate of $(1+x_t)(1+n_t)$ in the steady state, these variables are detrended by $N_t^w X_t$. Specifically, the detrended variables are expressed as lower case letters, that is, $z_t = \frac{Z_t}{N_t^w X_t}$. In addition, hereafter, I assume a constant probability of retirement for simplicity, that is, $\omega_t = \omega$ for all t. All the detrended equilibrium conditions are listed in Online Appendix D.

4 Quantitative Analysis

This section first provides the model calibration, and then studies whether the upward trend in real energy prices causes a downward trend in real interest rates for energy importers. Subsequently, I add social security to the model. Since many studies argue that demographic transition (i.e., population aging) has played an important role in the declining real interest rates in developed economies since the 1990s, I also compare its effects on real interest rates with those of the increasing trend in real energy prices by simulating the model. Finally, this section provides several sensitivity analyses since the energy-related parameters are newly introduced in the model compared to standard models.

4.1 Calibration

The parameter values in the model are presented in Table 2. One period in the model corresponds to one year. I assume that individuals are born at age 20 and retire at age 65, as is typical in the literature, such as Ferrero (2010) and Carvalho et al. (2016). Hence, the probability that a typical worker remains in the labor force ω is 0.9778. Since demographic transition and productivity are not of interest in the baseline model, I assume that the demographic parameters and productivity growth rate are constant.[18] In the representative energy-importing OECD member country, the values of the population growth rate n and dependency ratio ψ are set to 0.46% and 22.4% respectively, which matches the average population growth rate and dependency ratio for energy-importing OECD member countries from 1990–2018 based on World Bank data. From Equation (3), we can obtain the value of 0.9055 for the survival probability γ, which is consistent with an n of 0.46% and ψ of 22.4%. It is assumed that productivity growth rate x is 1%.

Regarding the energy-related parameters, the elasticity of substitution between energy and manufactured goods in consumption η is set to 0.3, borrowing from Natal (2012). The weight of energy in consumption χ is assumed to be 0.058, which matches the average ratio of consumption of energy goods and services to total consumption expenditure in the US from 1990–2018 using National Income and Product Accounts (NIPA) data. Following Kim and Loungani (1992), we assume that the elasticity of substitution between capital and energy inputs in production $1/(1+\nu)$ is 0.5882, and the steady state capital/energy ratio K/E is assumed to be 50. Accordingly, the parameter related to the importance of energy in production ϕ is 0.0079, which is determined by ν, K/E, β, and δ. Since these parameters are new compared to standard models, due to the introduction of energy, I assess whether the main results of this paper are sensitive to changes in the values of these parameters later in this section.

For some parameters, I assign standard values from the literature. The elasticity of intertemporal substitution σ, the labor share α, and the depreciation rate δ are set to 0.5, 0.67, and 0.1, respectively. The discount factor β is set so that the real interest rate in initial steady state is 6.19%, which is the same as the median ex-post short-term real interest rate of energy-importing OECD member countries in 1990. Based on the 1990–2018 data from

[18] In order to compare the effects of population aging and the upward trend in real energy prices, demographic transition is also considered later in this section.

the IMF and OECD, I assume the ratios of government spending and debt to GDP to be 19% and 39%, respectively.

Table 2: Parameter values

Parameter	Value	Definition
ω	0.9778	Probability of remaining in the labor force
n	0.0046	Population growth rate
ψ	0.224	Dependency ratio
γ	0.9055	Survival probability
x	0.01	Productivity growth rate
η	0.3	Elasticity of substitution between energy and manufactured goods in consumption
χ	0.058	Weight of energy in consumption
$1/(1+\nu)$	0.5882	Elasticity of substitution between capital and energy in production
ϕ	0.0079	Parameter related to importance of energy in production
σ	0.5	Elasticity of intertemporal substitution
α	0.67	Labor share
δ	0.1	Depreciation rate
β	0.9342	Discount factor
g_y	0.19	Ratio of government spending to output
b_y	0.39	Ratio of government debt to output

4.2 Effects of the Real Energy Price on the Real Interest Rate

I now present the main results of this paper. Specifically, I analyze whether the increasing trend in real energy prices leads to a decreasing trend in the equilibrium real interest rate in the model. To do so, I assume that the real energy price increases from 1 in the period 0 to 2.5 in the period 28, with equal increments in each period. This assumption is consistent with the observed real energy price index, which was approximately 2.5 in 2018 when the real energy price index data is normalized to 1 in 1990. Therefore, one can associate the period 0 in the model with 1990 and the period 28 with 2018.

Figure 2 presents the main results. In response to the increasing trend in the real energy price (top-left panel), energy consumption falls (top-right panel). The decreased energy consumption dampens the consumption of manufactured goods because there is complementarity between energy and manufactured goods in consumption (i.e., $\eta < 1$). Since energy and manufactured goods consumption decreases, aggregate consumption falls (middle-left panel), putting downward pressure on the real interest rate.

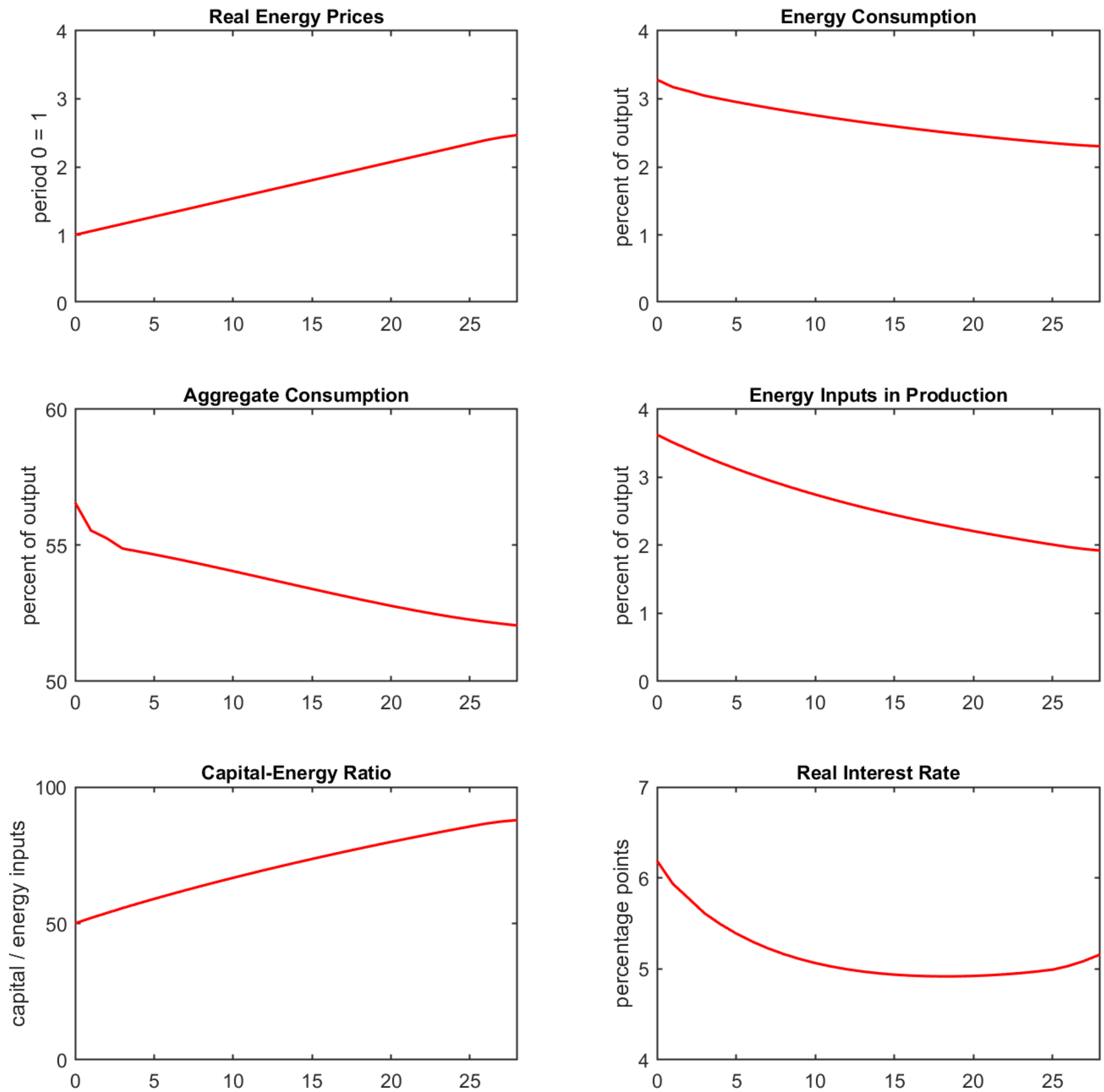

Figure 2: Simulated variables in the model

Furthermore, the increased real energy price also decreases energy input in production (middle-right panel). Accordingly, the capital/energy ratio increases (bottom-left panel), which also pushes down the real interest rate. As a consequence, the real interest rate declines from 6.19% in the period 0 to around 5.16% in the period 28 (bottom-right panel). In other words, according to the simulation results, the upward trend in real energy prices explains about 14% of the observed decline in real interest rates in energy-importing OECD member countries.

The model simulation shows, as expected, that the upward trend in real energy prices since the 1990s has played a substantial role in the declining trend in real interest rates in energy-importing OECD member countries during the same period.

4.3 Further Experiments

Several more experiments are carried out in this section. First, social security is added to the model so that retirees can receive security benefits. Then, I compare the effects of population aging on real interest rates with those of the increasing trend in real energy prices. The final experiment evaluates whether the main results of this paper are sensitive to the parameter values related to energy in the model.

4.3.1 Adding Social Security

Until now, for simplicity, it was assumed that retirees finance consumption through savings when they were workers in the model, that is, social security is absent. Although this assumption seems reasonable for some countries like the US, it is not for others, such as European countries, which have a robust public pension system. Given that a representative energy-importing OECD member country could be a European country, it is reasonable to test whether adding social security to the model changes our main results.

In order to introduce social security to the model, several modifications are needed. Specifically, we need to add social security benefits E_t^r to the budget constraint of retirees:

$$C_t^r(j,\tau) + A_t^r(j,\tau) = R_{t-1}A_{t-1}^r(j,\tau)/\gamma_t + E_t^r.$$

Accordingly, retirees' consumption is:

$$C_t^r(j,\tau) = \xi_t^r \left(\frac{R_{t-1}}{\gamma_t} A_{t-1}^r(j,\tau) + S_t^r \right),$$

where S_t^r is the present discounted value of E_t^r, that is, $S_t^r = E_t^r + \gamma_{t+1}S_{t+1}^r/R_t$.

The budget constraint of workers is the same as before. Their consumption function, however, needs to be altered since workers now take their social security benefits after retirement into consideration when they consume:

$$C_t^w(j) = \xi_t^w \left(R_{t-1}A_{t-1}^w(j) + H_t(j) + S_t^w \right),$$

where S_t^w denotes the present discounted value of social security benefits that workers can expect after retirement. That is, $S_t^w = (\omega/\Omega_{t+1})(S_{t+1}^w/R_t) + (1-\omega/\Omega_{t+1})(S_{t+1}^r/R_t)$. Due to the existence of social security payments, the government's budget constraint also changes to:

$$B_t = R_{t-1}B_{t-1} + G_t + E_t - T_t,$$

with $E_t = N_t^r E_t^r$. As in Gertler (1999), the government fixes the ratio of social security payments to GDP:

$$E_t = e_y p_{M,t} Y_t.$$

Following Gertler (1999), the ratio of social security to GDP e_y is assumed to be 2%.

Figure 3 presents the simulated equilibrium real interest rate in two cases (the baseline model, in which social security is omitted, and the model with social security) in response to the same changes in real energy price as in the previous section. The red solid line is the simulated real interest rate in the model with social security, and the blue dashed line is that in the baseline model. The real interest rate responses in the two models are very similar. The real interest rate in the model with social security, however, declines by a slightly less margin than in the baseline model. The reason for this is quite straightforward. Workers in the model with social security have less incentive to save than those in the baseline model since retirees receive social security benefits. In short, even though social security is added to the model, the main results of this paper hold. That is, as in the baseline model in which social security is absent, the upward trend in the real energy price leads to a fall in the real interest rate even in the model with social security.

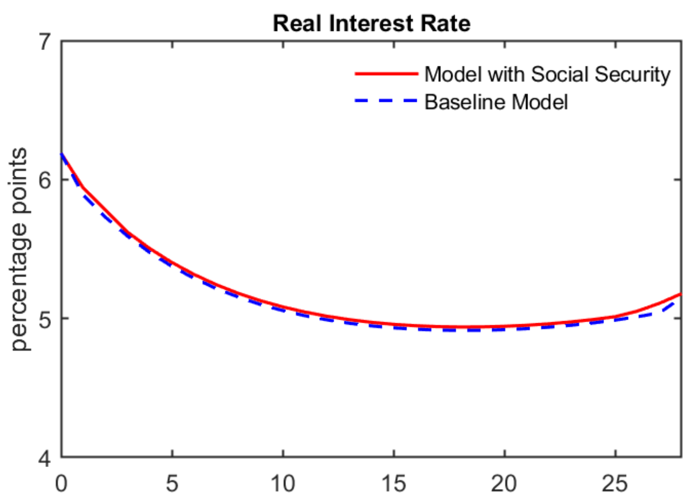

Figure 3: Simulated real interest rates in the baseline model and in the model with social security

4.3.2 Comparison with Population Aging

According to some studies, such as Carvalho et al. (2016), the demographic transition (i.e., population aging) in advanced economies is an important factor explaining their decreasing trend in real interest rates since the 1990s. Hence, it is necessary to evaluate whether the effects of population aging on the downward trend in real interest rates are greater than those of the upward trend in real energy prices. For this exercise, I simulate the model without social security and assume that the population growth rate decreases from 0.56% in the period 0 to 0.50% in the period 28, and that the survival probability increases from 0.8862 to 0.9266, which is similar to World Bank data.[19] The implied dependency ratio rises from 18.6% in the period 0 to 28.3% in the period 28, which exactly matches the data.[20] Similarly, one can associate the periods 0 and 28 in the model with 1990 and 2018, respectively.[21]

Figure 4 shows the simulated real interest rate in response to the assumed transitions in population growth rate and survival probability and the implied changes in the dependency ratio, holding the real energy price constant. Consistent with other studies such as Carvalho et al. (2016), the real interest rate declines in response to population aging. Carvalho et al. (2016) explain well why population aging leads to a fall in the real interest rate. Specifically, when the survival probability increases (i.e., life expectancy rises), individuals save more to prepare for a longer retirement period, which depresses the real interest rate. A fall in population growth leads to a higher capital/labor ratio, putting downward pressure on the real interest rate.[22]

More importantly, our simulation results show that the equilibrium real interest rate falls by approximately 0.5 percentage points between 1990 and 2018 due to population aging (red solid line, bottom-right panel of Figure 4) when the real energy price is constant, and that the upward trend in real energy prices decreases the rate by about one percentage point during the same period (blue dashed line, bottom-right panel of Figure 4) when the demographic variables are constant, as shown in the previous section. Therefore, according to our simulation results, the increasing trend in real energy prices since the 1990s has played a more important role in the decreasing trend in real interest rates in energy-importing OECD member countries during the same period, compared to population aging. Specifically, the negative effect of the upward trend in real energy prices on real interest rates in energy-importing OECD member countries is approximately two times greater than that of population aging.

[19] According to the World Bank, the average population growth rate in energy-importing OECD member countries was 0.56% in 1990 and 0.50% in 2018. Moreover, the average life expectancy in energy-importing OECD member countries was 74.2 years in 1990 and 80.8 years in 2018. In the model, the survival probabilities of 0.8862 and 0.9266 correspond to the life expectancy of 73.8 and 78.6, respectively.

[20] Based on World Bank data, the average dependency ratio in energy-importing OECD member countries increased from 18.6% in 1990 to 28.3% in 2018.

[21] Note that this is different from Carvalho et al. (2016), in which the demographic transition from 1990 to 2100, using the UN's population prospects, is considered.

[22] A decrease in population growth also pushes down the real interest rate, since it stimulates a greater dependency ratio and since retirees save less compared to workers. However, this effect is relatively small, according to Carvalho et al. (2016).

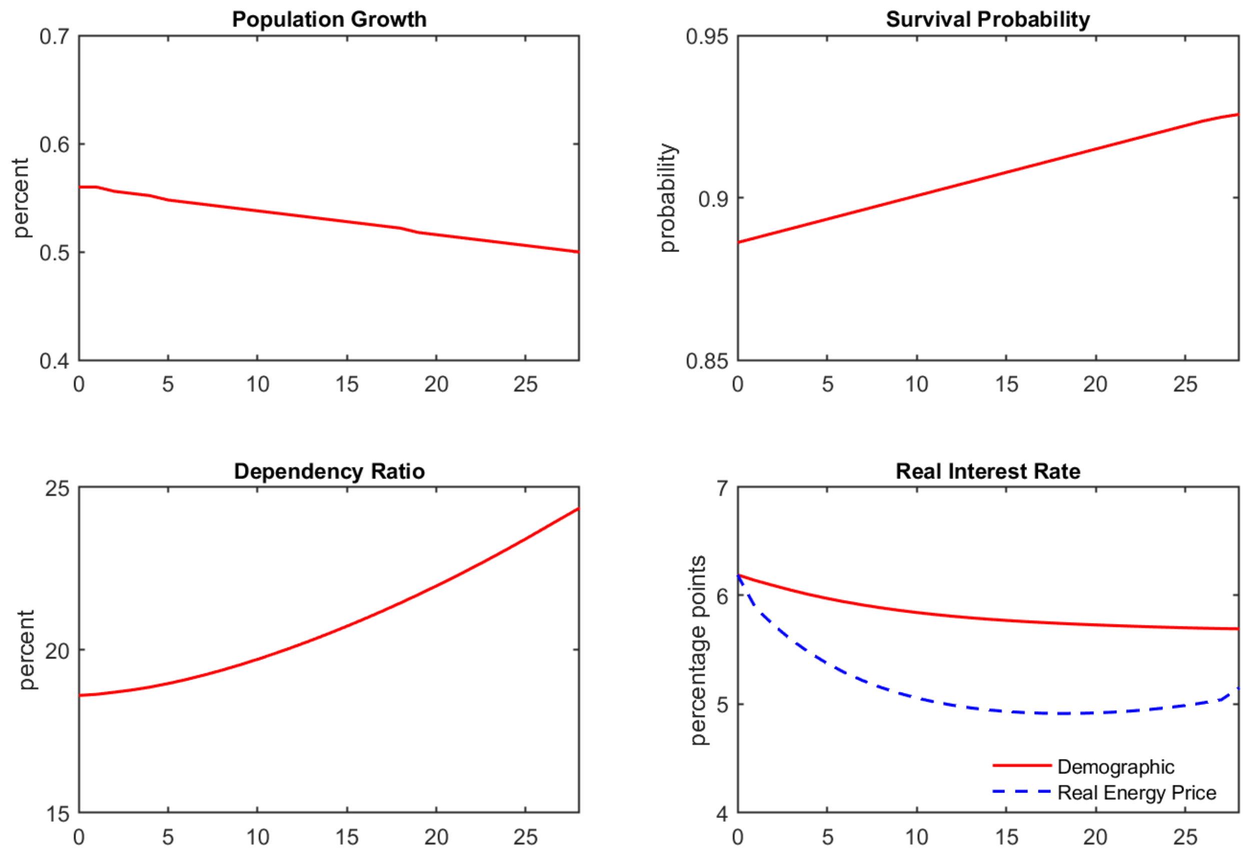

Figure 4: Simulated variables in the model in response to population aging

Note: "Demographic" denotes the simulation results in which only the demographic transition is considered (holding the real energy price constant), and "Real Energy Price" denotes the simulation results in which only the upward trend in real energy prices is considered (holding the demographic variables constant).

4.3.3 Sensitivity Analysis

Since energy is added to the model of Gertler (1999), the parameters for the elasticity of substitution between energy and manufactured goods in consumption (η), the weight of energy in the consumption index (χ), and the elasticity of substitution between energy and capital in production ($1/(1+\nu)$) are all new parameters compared to existing models.[23] Hence, we need to assess whether our model's main result—an upward trend in real energy prices leads to a decreasing trend in real interest rates—is sensitive to changes in the values of these three parameters. For this sensitivity analysis, I consider two more values for each parameter. Specifically, in addition to the baseline values, I consider 0.1 and 0.5 for the value of η, 0.03 and 0.09 for the value of χ, and 0.25 and 1 for the value of $1/(1+\nu)$. The simulation results are shown in Figure 5.

[23] In fact, the parameter related to the importance of energy in production (ϕ) is a new one as well. However, since its value is determined by the steady state real rental rate (R^K), the steady state ratio of capital to energy in production (K/E) and ν, I do not assess the sensitivity of the result to changes in its value.

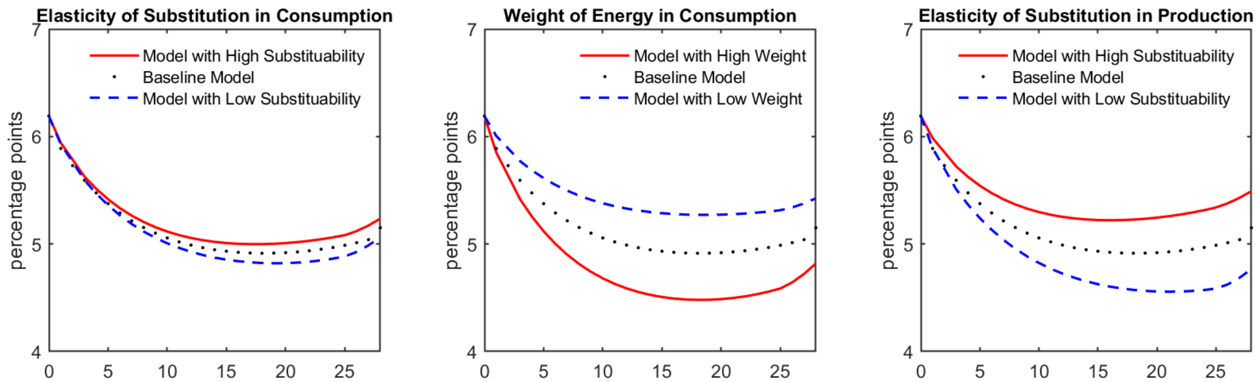

Figure 5: Simulated real interest rate in the model with different parameter values

Note: "Elasticity of Substitution in Consumption" denotes the elasticity of substitution between energy and manufactured goods in consumption (η), "Weight of Energy in Consumption" denotes the parameter related to the weight of energy in the consumption index (χ), and "Elasticity of Substitution in Production" denotes the elasticity of substitution between energy and capital in production ($1/(1+\nu)$).

Elasticity of Substitution between Energy and Manufactured Goods in Consumption (η) The first panel of Figure 5 presents the equilibrium paths of the real interest rate in the model with high, low, and baseline values of η in response to the same changes in the real energy price, as in the previous section. Even if the value of η is different from our baseline value, the main results of this study hold. That is, the increasing trend in the real energy price brings about a decline in the real interest rate.

Nonetheless, when the elasticity of substitution between energy and manufactured goodss in consumption is low, there is a greater decline in the real interest rate. The lower the elasticity, the stronger the complementarity (i.e., lower substitutability) between energy and manufactured goods. Hence, if elasticity is low, the decreased energy consumption—owing to the rise in real energy prices—dampens consumption of manufactured goods even more. This leads to a greater decrease in aggregate consumption. Accordingly, the real interest rate falls by a greater value when the elasticity is small, compared to when the value of elasticity is large.

Weight of Energy in the Consumption Index (χ) Now, let us consider different values for the parameter related to the weight of energy in the consumption index (χ). The equilibrium paths of the real interest rate in the model with different values of χ in response to the increasing trend in the real energy price are provided in the middle panel of Figure 5.

Although we assume different values of χ from our baseline, our main results hold. However, in the case of a high value of χ, the real interest rate decreases more. Since increases in the real energy price decrease energy consumption, and since a large χ indicates a greater weight of energy in aggregate consumption, aggregate consumption falls more when χ is high than when it is low.

Elasticity of Substitution between Energy and Capital in Production ($1/(1+\nu)$) The last panel of Figure 5 presents the equilibrium paths of the real interest rate in the model with different values of the elasticity of substitution between energy and capital in production ($1/(1+\nu)$).

When $1/(1+\nu)$ is low (i.e., ν is high),), the real interest rate declines more in response to increases in the real energy price. A low $1/(1+\nu)$ indicates stronger complementarity (i.e., weaker substitutability) between capital and energy in production. As the real energy price rises, energy inputs fall and it is harder for energy inputs to be substituted by capital inputs when $1/(1+\nu)$ is low. Accordingly, output falls more than in the case when $1/(1+\nu)$ is high; thus, the marginal product of capital reduces more.

5 Conclusion

This study explores whether the upward trend in real energy prices since the 1990s accounts for the declining trend in real interest rates in energy-importing OECD member countries during the same period.

By estimating dynamic panel data models, I first show empirically that an increase in real energy prices brings about falls in real interest rates and aggregate consumption in energy-importing OECD member countries. Moreover, I also provide empirical evidence that a rise in real energy prices has much stronger effects on real interest rates and aggregate consumption in energy-importing OECD member countries than in energy-exporting ones.

I then construct a life-cycle model, extending the model of Gertler (1999). In the model, households consume energy, while firms use energy to produce goods. By simulating the model, I show that the increasing trend in the real energy price decreased the equilibrium real interest rate by about one percentage point between 1990 and 2018. This is because an increase in the real energy price leads to a fall in energy consumption, and thus consumption of manufactured goods falls as well, thanks to the complementarity between energy and manufactured goods in consumption. Accordingly, aggregate consumption reduces, which has a dampening effect on the real interest rate. In addition, the increase in real energy price causes a decrease in energy inputs in production, leading to a rise in the capital/energy ratio. Hence, the marginal product of capital declines, putting downward pressure on the real interest rate.

I also show that the main results hold even if social security is introduced in the model, and that the main results of this paper are insensitive to the values of the parameters related to energy in the model.

Using the model, I further compare the effects of the upward trend in real energy prices and population aging on real interest rates in energy-importing OECD member countries.

The model simulation reveals that the upward trend in real energy prices since the 1990s has played a far more important role in the declining trend in real interest rates in energy-importing OECD member countries than population aging has.

Finally, it should be noted that monetary and fiscal policies are not considered in this paper, though they may have played an important role in the decline in real interest rates in advanced countries. Although considering them and comparing them with the increasing trend in real energy prices and population aging would be interesting, I leave this for future research.

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Online Appendix (not for publication)

A. Estimation Results (excluding the U.S.)

Explanatory variables:	Dependent variable: Real interest rate	Dependent variable: Consumption
Lagged dependent variable	0.7564***	0.5471***
	(0.0583)	(0.1586)
Real energy price (α_1 and γ_1)	**-0.0078****	**-0.0198*****
	(0.0033)	**(0.0075)**
Real energy price×Exporter dummy (α_2 and γ_2)	**0.0122***	**0.0528***
	(0.0072)	**(0.0274)**
GDP	0.0032	0.4351***
	(0.0020)	(0.1517)
Core CPI	-0.0011	0.0023
	(0.0069)	(0.0128)
Exchange rate	-0.0108***	-0.0115
	(0.0042)	(0.0186)
Global financial crisis dummy	-1.2191***	-0.5183
	(0.2511)	(0.3413)
Real interest rate		0.0608
		(0.0959)
Number of countries	30	30
Number of instruments	27	28
AB test for AR(2) (p-values)	0.328	0.211
Hansen test (p-values)	0.272	0.297

Notes: Clustered standard errors are reported in parentheses. ***, ** and * denote significance at the 1%, 5%, and 10% level, respectively. AB test means the Arellano-Bond test.

B. Retirees' Problem

The first-order conditions are

$$
\begin{aligned}
\left(C_t^r(j,\tau)\right)^{\rho-1} &= \beta\gamma_{t+1}\left(V_{t+1}^r(j,\tau)\right)^{\rho-1}\frac{\partial V_{t+1}^r(j,\tau)}{\partial K_t^r(j,\tau)},\\
\left(C_t^r(j,\tau)\right)^{\rho-1} &= \beta\gamma_{t+1}\left(V_{t+1}^r(j,\tau)\right)^{\rho-1}\frac{\partial V_{t+1}^r(j,\tau)}{\partial B_t^r(j,\tau)}.
\end{aligned}
$$

We can obtain the last terms in the above equations by using the envelope conditions:

$$
\begin{aligned}
\frac{\partial V_t^r(j,\tau)}{\partial K_{t-1}^r(j,\tau)} &= \left(V_t^r(j,\tau)\right)^{1-\rho}\left(C_t^r(j,\tau)\right)^{\rho-1}\frac{R_t^K+1-\delta}{\gamma_t},\\
\frac{\partial V_t^r(j,\tau)}{\partial B_{t-1}^r(j,\tau)} &= \left(V_t^r(j,\tau)\right)^{1-\rho}\left(C_t^r(j,\tau)\right)^{\rho-1}\frac{R_{t-1}}{\gamma_t}.
\end{aligned}
$$

Combining above equations gives the following equations:

$$
\begin{aligned}
\left(C_t^r(j,\tau)\right)^{\rho-1} &= \beta\left(C_{t+1}^r(j,\tau)\right)^{\rho-1}\left(R_{t+1}^K+1-\delta\right),\\
\left(C_t^r(j,\tau)\right)^{\rho-1} &= \beta\left(C_{t+1}^r(j,\tau)\right)^{\rho-1}R_t.
\end{aligned}
$$

Rearranging yields

$$
1 = \beta R_t\left[\frac{C_{t+1}^r(j,\tau)}{C_t^r(j,\tau)}\right]^{-\frac{1}{\sigma}} = \beta\left(R_{t+1}^K+1-\delta\right)\left[\frac{C_{t+1}^r(j,\tau)}{C_t^r(j,\tau)}\right]^{-\frac{1}{\sigma}}, \tag{B.1}
$$

where, $\sigma = \frac{1}{1-\rho}$. Conjecture retirees consumption as:

$$
C_t^r(j,\tau) = \xi_t^r\left(\frac{R_{t-1}A_{t-1}^r(j,\tau)}{\gamma_t}\right). \tag{B.2}
$$

Combining Equations (B.1) and (B.2) gives

$$
\xi_{t+1}^r\left(\frac{R_tA_t^r(j,\tau)}{\gamma_{t+1}}\right) = \left(\beta R_t\right)^{\sigma}\xi_t^r\left(\frac{R_{t-1}A_{t-1}^r(j,\tau)}{\gamma_t}\right). \tag{B.3}
$$

Using Equation (B.2), we can rewrite the budget constrain for retirees as:

$$
A_t^r(j,\tau) = (1-\xi_t^r)\frac{R_{t-1}A_{t-1}^r(j,\tau)}{\gamma_t}. \tag{B.4}
$$

By combining Equations (B.3) and (B.4), we can obtain the law of motion for ξ_t^r:

$$
\frac{1}{\xi_t^r} = 1+\gamma_{t+1}\beta^{\sigma}(R_t)^{\sigma-1}\frac{1}{\xi_{t+1}^r}.
$$

C. Workers' Problem

The first-order condition is

$$\begin{aligned}(C_t^w(j))^{\rho-1} = \beta \left[\omega_{t+1} V_{t+1}^w(j) + (1-\omega_{t+1}) V_{t+1}^r(j,t+1)\right]^{\rho-1} \\ \left[\omega_{t+1} \frac{\partial V_{t+1}^w(j)}{\partial A_t^w(j)} + (1-\omega_{t+1}) \frac{\partial V_{t+1}^w(j,t+1)}{\partial A_t^w(j)}\right].\end{aligned}$$

The envelope conditions are

$$\begin{aligned}\frac{\partial V_t^w(j)}{\partial A_{t-1}^w(j)} &= \left(V_t^w(j)\right)^{1-\rho} \left(C_t^w(j)\right)^{\rho-1} R_{t-1}, \\ \frac{\partial V_t^r(j,\tau)}{\partial A_{t-1}^w(j,\tau)} &= \frac{\partial V_t^r(j,t)}{\partial A_{t-1}^r(j,t)} \frac{\partial A_{t-1}^r(j,t)}{\partial A_{t-1}^w(j)} = \frac{\partial V_t^r(j,t)}{\partial A_{t-1}^r(j,t)}.\end{aligned}$$

Combining the envelop conditions and the first-order condition gives

$$\begin{aligned}&(C_t^w(j))^{\rho-1} = \beta R_t \left[\omega_{t+1} V_{t+1}^w(j) + (1-\omega_{t+1}) V_{t+1}^r(j,t+1)\right]^{\rho-1} \\ &\left[\omega_{t+1} \left(V_{t+1}^w(j)\right)^{1-\rho} \left(C_{t+1}^w(j)\right)^{\rho-1} + (1-\omega_{t+1}) \left(V_{t+1}^r(j,t+1)\right)^{1-\rho} \left(C_{t+1}^r(j,t+1)\right)^{\rho-1}\right].\end{aligned} \tag{C.1}$$

Now, conjecture value functions for workers and retirees:

$$V_t^w(j) = \xi_t^{w^{-\frac{1}{\rho}}} C_t^w(j), \tag{C.2}$$

$$V_t^r(j,\tau) = \xi_t^{r^{-\frac{1}{\rho}}} C_t^r(j,\tau). \tag{C.3}$$

Using Equations (C.1), (C.2) and (C.3), we can obtain

$$\begin{aligned}(C_t^w(j))^{\rho-1} = \beta R_t \left[\omega_{t+1} \xi_{t+1}^{w^{-\frac{1}{\rho}}} C_{t+1}^w(j) + (1-\omega_{t+1}) \xi_{t+1}^{r^{-\frac{1}{\rho}}} C_{t+1}^r(j,t+1)\right]^{\rho-1} \\ \left[\omega_{t+1} \left(\xi_{t+1}^{w^{-\frac{1}{\rho}}}\right)^{1-\rho} + (1-\omega_{t+1}) \left(\xi_{t+1}^{r^{-\frac{1}{\rho}}}\right)^{1-\rho}\right].\end{aligned}$$

Substituting the definition of the adjustment term Ω_t into the above equation yields the consumption Euler equation:

$$\omega_{t+1} C_{t+1}^w(j) + (1-\omega_{t+1}) \varepsilon_{t+1}^{-\frac{1}{\rho}} C_{t+1}^r(j,t+1) = \left(\beta \Omega_{t+1} R_t\right)^{\sigma} C_t^w(j), \tag{C.4}$$

where $\varepsilon_t \equiv \frac{\xi_t^r}{\xi_t^w}$. The guess for workers consumption and consumption of a retiree born at j who just retired are

$$C_t^w(j) = \xi_t^w \left(R_{t-1}A_{t-1}^w(j) + H_t^w\right), \tag{C.5}$$

$$C_t^r(j) = \xi_t^r R_{t-1}A_{t-1}^w(j). \tag{C.6}$$

Using Equations (C.3), (C.4) and (C.5),

$$\omega_{t+1}\left(R_tA_t^w(j) + H_{t+1}^w\right) + (1-\omega_{t+1})R_tA_t^w(j)\varepsilon_{t+1}^{\frac{\rho-1}{\rho}} = (\beta R_t\Omega_{t+1})^\sigma \frac{\xi_t^w}{\xi_{t+1}^w}\left(R_{t-1}A_{t-1}^w(j) + H_t^w\right). \tag{C.7}$$

Combining Equation (C.5), the definition of human wealth and the budget constraint gives

$$A_t^w(j) + \frac{\omega_{t+1}H_{t+1}^w}{\Omega_{t+1}R_t} = (1-\xi_t^w)\left(R_{t-1}A_{t-1}^w(j) + H_t^w\right). \tag{C.8}$$

Substituting Equation (C.8) into (C.7) gives the following law of motion for ξ_t^w:

$$\frac{1}{\xi_t^w} = 1 + \beta^\sigma(\Omega_{t+1}R_t)^{\sigma-1}\frac{1}{\xi_{t+1}^w}. \tag{C.9}$$

D. Detrended Equilibrium Conditions

$$(1+n_t)\psi_t = (1-\omega_t) + \gamma_t\psi_{t-1}, \tag{D.1}$$

$$\xi_t^r = 1 - \gamma_{t+1}\beta^\sigma(R_t)^{\sigma-1}\frac{\xi_t^r}{\xi_{t+1}^r}, \tag{D.2}$$

$$\xi_t^w = 1 - \beta^\sigma(\Omega_{t+1}R_t)^{\sigma-1}\frac{\xi_t^w}{\xi_{t+1}^w}, \tag{D.3}$$

$$\varepsilon_t = \frac{\xi_t^r}{\xi_t^w}, \tag{D.4}$$

$$\Omega_t = \omega_t + (1-\omega_t)(\varepsilon_t)^{\frac{1}{1-\sigma}}, \tag{D.5}$$

$$c_t = \xi_t^w\left((1-\lambda_{t-1})R_{t-1}a_{t-1} + h_t\right) + \xi_t^r\left(\lambda_{t-1}R_{t-1}a_{t-1}\right), \tag{D.6}$$

$$c_t = \left\{(1-\chi)^{\frac{1}{\eta}}c_{M,t}^{\frac{\eta-1}{\eta}} + \chi^{\frac{1}{\eta}}c_{E,t}^{\frac{\eta-1}{\eta}}\right\}^{\frac{\eta}{\eta-1}}, \tag{D.7}$$

$$c_{E,t} = \chi p_{E,t}^{-\eta}c_t, \tag{D.8}$$

$$c_{M,t} = (1-\chi)p_{M,t}^{-\eta}c_t, \tag{D.9}$$

$$c_t^r = \xi_t^r R_{t-1}a_{t-1}^r, \tag{D.10}$$

$$c_t^w = \xi_t^w \left(R_{t-1} a_{t-1}^w + h_t \right), \tag{D.11}$$

$$h_t = w_t - t_t + h_{t+1} \frac{\omega_{t+1}(1 + x_{t+1})}{\Omega_{t+1} R_t}, \tag{D.12}$$

$$(1 + x_{t+1})(1 + n_{t+1}) \left[\lambda_t - (1 - \omega_{t+1}) \right] a_t = \omega_{t+1}(1 - \xi_t^r) \lambda_{t-1} R_{t-1} a_{t-1}, \tag{D.13}$$

$$R_t = R_{t+1}^K + (1 - \delta), \tag{D.14}$$

$$y_t = \left\{ (1 - \phi) k_{t-1}^{-\nu} + \phi e_t^{-\nu} \right\}^{\frac{-(1-\alpha)}{\nu}}, \tag{D.15}$$

$$w_t = \alpha p_{M,t} y_t, \tag{D.16}$$

$$R_t^K = (1 - \alpha)(1 - \phi) k_{t-1}^{-\nu-1} \frac{p_{M,t} y_t}{(1 - \phi) k_{t-1}^{-\nu} + \phi e_t^{-\nu}}, \tag{D.17}$$

$$p_{E,t} = \phi (1 - \alpha) e_t^{-\nu-1} \frac{p_{M,t} y_t}{(1 - \phi) k_{t-1}^{-\nu} + \phi e_t^{-\nu}}, \tag{D.18}$$

$$a_t = k_t + b_t, \tag{D.19}$$

$$a_t^r = \lambda_t a_t, \tag{D.20}$$

$$a_t^w = (1 - \lambda_t) a_t, \tag{D.21}$$

$$(1 + x_{t+1})(1 + n_{t+1}) k_t = (1 - \delta) k_{t-1} + p_{M,t} i_t. \tag{D.22}$$

$$(1 + x_{t+1})(1 + n_{t+1}) b_t = R_{t-1} b_{t-1} + g_t - t_t, \tag{D.23}$$

$$(1 + x_{t+1})(1 + n_{t+1}) b_t = b_{y,t} p_{M,t} y_t, \tag{D.24}$$

$$g_t = g_{y,t} p_{M,t} y_t, \tag{D.25}$$

$$p_{M,t} y_t = c_t + p_{M,t} i_t + g_t + p_{E,t} e_t. \tag{D.26}$$