A Triple Regime Stochastic Volatility Model with Threshold and Leverage Effects*

Heejoon Han[†] Chang Sik Kim[‡] Eunhee Lee[§]

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Abstract

This paper considers a new stochastic volatility model, in which both sign and magnitude of stock return play roles in explaining a more detailed relationship between stock return and volatility. The model allows for both threshold and leverage effects and accommodates three regimes (extreme negative return, mid-range including moderate negative and positive returns, and large positive return) to better capture the time varying aspect of the leverage effect. Applications of the model suggest strong evidence of time varying leverage effect. The comparison of the deviance information criterion reveals a good fit of our model.

JEL classification: C22, C50, G12

Keywords; Stochastic volatility model, leverage effect, threshold effect, multiple regime, MCMC, Gibbs sampling.

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[†]Department of Economics, Sungkyunkwan University. E-mail: heejoonhan@skku.edu.

[‡]Department of Economics, Sungkyunkwan University. E-mail: skimcs@skku.edu.

[§]Corresponding author. Institute of Life Science and Natural Resources, Department of Food and Resource Economics, Korea University. E-mail: leeeunhee@korea.ac.kr.

1 Introduction

It is well known that the relationship between return and volatility in equity markets is asymmetric; a negative return is associated with higher volatility than a positive return. In the literature of autoregressive conditional heteroskedasticity (ARCH)-type volatility models, various models have been proposed to accommodate this stylized fact. Examples include Engle and Ng (1993), Glosten et al. (1993), Nelson (1991), and Pagan and Schwert (1990). There also has been active research of stochastic volatility (SV) models addressing an asymmetric relationship between return and volatility. SV models specify volatility as a separate random process and therefore can have advantages over the ARCH-type models for modeling the dynamics of return series (Kim et al. 1998). Moreover, Poon and Granger (2003) reported that SV models in general outperform ARCH-type models in out-of-sample volatility forecasting. With the rapid development in estimation methods of SV models, these models recently become more popular than they used to be.

In the framework of SV models, one common approach to accommodate the asymmetric relationship between return and volatility is to adopt a correlation coefficient between two innovations in lagged return and volatility process (Harvey and Shephard 1996, Yu 2004). If the correlation is negative, a negative lagged return will be associated with higher subsequent volatility. This asymmetry based on the correlation coefficient is typically referred to as the *leverage effect* in the stochastic volatility literature. The other approach to explain asymmetric relationship between return and volatility is adopting the threshold effect considered by So et al. (2002), who defined two regimes based on the sign of stock returns and let the parameters in the SV model have different values in each regime. Recently, researchers tried to accommodate both threshold and leverage effects in the SV model (Smith 2009, Wu and Zhou 2014[1], Xu 2010).

The leverage effect was assumed to be constant. However, recent studies have found evidence against a constant parameter for the leverage effect. Empirical data have shown that the leverage parameter characterizing the correlation between innovations to return and innovations to variance vary with time. For example, Daouk and Ng (2011) reported evidence of stronger leverage effect when prices decreases. Using the daily United States stock index return series from 1926 to 2010, Christensen et al. (2015) found a negative leverage effect throughout, but a significant increase in magnitude during financial crises. Moreover, nonparametric or semiparametric modeling for time varying leverage effect has

[1] Wu and Zhou (2014) designated their model a triple-threshold leverage SV model, not because they actually consider three different regimes in the model, but they allow the state dependent leverage effects, that is, two regime-specific correlation coefficients, and each regime is still determined by only the sign of return.

been extensively studied. Examples are Ait-Sahalia et al. (2013), Bandi and Reno (2012), Linton et al. (2016), Wang and Mykland (2014) and Yu(2012).

In this paper, we focus on the idea that the relationship between return and volatility depends on the magnitude of return as well as its sign. One important common feature of most existing SV models is that the relationship between return and volatility is determined only by the sign of return, regardless of its magnitude. For example, in most prior studies, both moderate negative return and extreme negative return have the same relationship with volatility. However, it is not realistic and it is more natural to expect that investors behave differently when stock prices drop (or rise) below (or above) a certain level and, consequently, the relationship between return and volatility would be different. We propose a new stochastic volatility model, in which both sign and magnitude of return play roles in explaining more detailed relationship between return and volatility. Specifically, we accommodate three regimes in the model (extreme negative return, mid-range including moderate negative and positive returns, and large positive return) to better capture the time varying aspect of the leverage effect instead of the usual two regimes depending only on the sign. We let the parameter for the leverage effect have a different value for each regime, expecting that the behavior of investors would be different in each regime.

We applied our model on two stock return series from 03 January 2006 to 30 June 2015: the return series of the S&P 500 Index and the stock return of Microsoft Corporation (MSFT). We utilized the Markov chain Monte Carlo (MCMC) method to implement a practical Bayesian estimation approach for our model. Chib and Greenberg (1996) and Chib (2001) have provided extensive reviews on the method. This method has been successfully applied to estimate basic and extended stochastic volatility models (e.g. Chib et al. 2002, Jacquier et al. 1994, Kim et al. 1998). The MCMC method is a simulation technique that generates a sample from the target distribution. The simulation is conducted by specifying the transition density of an irreducible aperiodic Markov chain whose limiting invariant distribution is the target posterior distribution. Then, the Markov chain is iterated a large number of times in a computer-generated Monte Carlo simulation and the draws generated from the simulation can be used to summarize the posterior distribution.

We report evidence that the relationship between return and volatility depends on the magnitude of return as well as its sign. In both the S&P 500 Index and the MSFT cases, the estimated leverage effect differd in each of the three regimes. In both cases, when the stock price dropped (or rose) beyond a certain level, the leverage effect either disappeared or became much weaker. In regime 3 (with large positive return), the leverage effect disappeared in both cases. In regime 1 (with extreme negative return), the leverage effect weakly appeared for the index while it disappeared for the MSFT. Second, when stock return was moderately negative or positive (regime 2), the conventional leverage effect appeared in both the index and the individual firm's stock. In this case, the leverage effect was much stronger in the index than the individual firm. Third, comparison of the deviance information criterion for various SV models showed that our model fit the data well compared to various existing SV models.

The rest of the paper is organized as follows. Section 2 introduces the model and explains the estimation method. Sections 3 provides the main results and Section 4 concludes the paper. Appendix contains tables and figures.

2 The Model and Estimation Method

2.1 The Model

We denote by r_t a demeaned stock return series and let θ be the the vector of unknown parameters that will be specified in the next subsection. We define a sequence of random variables s_t^j by

$$s_t^1 = \begin{cases} 1 & \text{if } r_t < \tau_1 \\ 0 & \text{otherwise} \end{cases} \qquad \text{(Regime 1)}$$

$$s_t^2 (= 1 - s_t^1 - s_t^3) = \begin{cases} 1 & \text{if } \tau_1 \leq r_t < \tau_2 \\ 0 & \text{otherwise} \end{cases} \qquad \text{(Regime 2)}$$

$$s_t^2 = \begin{cases} 1 & \text{if } r_t \geq \tau_2 \\ 0 & \text{otherwise} \end{cases} \qquad \text{(Regime 3).}$$

We let $s_t = (s_t^1, s_t^2, s_t^3)'$ for $t = 1, ..., n$. The triple regime stochastic volatility model with threshold and leverage effects (TRSV model) is defined as:

$$r_t = \sqrt{h_t} u_t$$

$$\log h_{t+1} - \mu_{s_t} = \beta_{s_t}(\log h_t - \mu_{s_t}) + \varepsilon_t \ , \ \varepsilon_t = \sigma v_{t+1}$$

where

$$\left(\begin{array}{c} u_t \\ v_{t+1} \end{array} \middle| s_t, \theta \right) \sim N \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho_{s_t} \\ \rho_{s_t} & 1 \end{pmatrix} \right)$$

and

$$\begin{aligned}
\mu_{s_t} =&\mu_1 s_t^1 + \mu_2 s_t^2 + \mu_3 s_t^3 \\
\beta_{s_t} =&\beta_1 s_t^1 + \beta_2 s_t^2 + \beta_3 s_t^3 \\
\rho_{s_t} =&\rho_1 s_t^1 + \rho_2 s_t^2 + \rho_3 s_t^3.
\end{aligned}$$

Therefore, we may rewrite

$$\begin{aligned}
\log h_{t+1} - \mu_1 &= \beta_1(\log h_t - \mu_1) + \varepsilon_t \ , \ cor(u_t, v_{t+1}) = \rho_1 \ \text{ if } r_t < \tau_1 \\
\log h_{t+1} - \mu_2 &= \beta_2(\log h_t - \mu_2) + \varepsilon_t \ , \ cor(u_t, v_{t+1}) = \rho_2 \ \text{ if } \tau_1 \le r_t < \tau_2 \\
\log h_{t+1} - \mu_3 &= \beta_3(\log h_t - \mu_3) + \varepsilon_t \ , \ cor(u_t, v_{t+1}) = \rho_3 \ \text{ if } r_t \ge \tau_2
\end{aligned}$$

Our TRSV model is a triple-regime model, in which each regime is determined by return. It is important to note that both the sign of return and the magnitude of return determine the regime in the model. When the return is negative with a large magnitude, it belongs to regime 1. When return is moderately negative or positive, it is in regime 2. When return is positive with a large magnitude, it belongs to regime 3. In each regime, the leverage effect represented by the correlation coefficient ρ_{s_t} takes a different value. The model also allows for the threshold effect, which means that μ_{s_t} and β_{s_t} have different values in each regime.

The reason why we introduce a triple-regime model rather than the traditional two-regime models is that the empirical results about the relationship between volatility and returns for the periods of large negative return or financial crisis has been found to be mixed. For example, Christensen et al. (2015) recently found that the risk–return trade-off is significantly positive only during financial crises, and is insignificant during non-crisis periods. This can be explained by the fact that a given increase in the debt/equity ratio leads to increased risk during crisis, and a increase in risk increases the discount rate more during financial crisis than during normal periods following the volatility feedback interpretation. The authors also found that the magnitude of leverage effect changes drastically during financial crises using the daily U.S. stock index return series from 1926 through 2010. In this paper, we show that time-variant leverage effect can be better explained by our triple-regime model, since the strength of the leverage effect can be drastically changing for the periods of extremely negative lagged return or financial crisis period.

Our model is related with recent studies on nonparametric or semiparametric modeling for time varying leverage effect (Ait-Sahalia et al. 2013, Bandi and Reno 2012, Linton et al. 2016, Wang and Mykland 2014 and Yu 2012). Yu (2012) and Bandi and Reno (2012) found strong evidence for time varying aspects for asymmetric relationships between lagged return and volatility. Linton et al. (2016) suggested a way of testing the leverage hypothesis nonparametrically using the concept of first order distributional dominance; the authors found that investors consider not just the level of volatility but the entire conditional distribution of volatility. In fact, Bandi and Reno (2012) and Patton and Sheppard (2015) considered the current level of volatility as the main driving force or the strength of the time varying leverage effect, whereas Yu (2012) assumed that the driving factor for the time varying leverage was the lagged return.

Yu (2012) proposed a SV model that can allow for multiple regimes. Even if his empirical applications support two-regime models instead of three-regime models, he also considered a three-regime model. However, our model is different from his model in two main aspects. First, we estimate τ_i in our model while it is predetermined in Yu (2012). In his three-regime model, τ_1 and τ_2 are chosen so that each regime has a nearly equal split of observations (34.5%, 31% and 34.5% of returns, respectively). However, it is expected that investors' behavior would be different if stock prices dropped (or rose) below (or above) a certain level, it would be more desirable to estimate τ_1 and τ_2 to accommodate the effect of such a behavior in the model. Second, we allow for the threshold effect in the volatility level parameter μ_{s_t} and volatility persistence parameter β_{s_t} while it is not allowed in his model.

Danielsson (1994) and Asai and McAleer (2006) also considered SV models that incorporate both the sign and magnitude of return. However, their models are based on an EGARCH type representation and do not focus on correlation coefficient between two innovations. Therefore, their models do not provide the detailed features of leverage effect that depend on both the sign and magnitude of return series as our model does.

2.2 Bayesian Estimation Method

Estimating SV-type models is quite challenging since these models do not have closed form likelihood functions due to their latent structure of the conditional variance. Therefore, maximum likelihood estimation can not be directly used. Several estimation methods have been proposed in the literature including quasi-maximum likelihood method (QML) (Harvey et al. 1994), the simulated maximum likelihood method (Danielsson 1994, Durbin and Koopman 1997, and Sandmann and Koopman (1998)), the efficient method of moments (Gallant and Tauchen 1996 and Andersen et al. 1999), the simulated method of moments (Duffie and Singleton 1993) and the generalized method of moments (GMM; Melino and Turnbull 1990, Andersen and Sørensen 1996, and Sørensen 2000). In addition to these methods, the Bayesian Markov chain Monte Carlo (MCMC) method has been used to estimate the parameters of SV models. Compared with other estimation methods, the Bayesian method is explicitly suitable and has been proven to perform well and provide relatively accurate results. Moreover, Andersen et al. (1999) showed that MCMC is one of the most efficient method. The first Bayesian approach was provided by Jacquier et al (1994) where the posterior distribution of the unknown parameters was sampled by MCMC method and they also show that, in SV framework, the MCMC method is superior to both QML and GMM. Recently, Kim et al (1998) and Chib et al (2002) developed an alternative, more efficient, MCMC algorithm for SV models.

In this study, therefore, we use the Bayesian approach to estimate our model. We define the vector of observed samples $R = (r_1, r_2. \cdots, r_n)'$ with n sample size. We let $\theta = (\mu, \beta, \sigma^2, \rho, \tau, h_1)'$ be the the vector of unknown parameters with $\mu = (\mu_1, \mu_2, \mu_3), \beta = (\beta_1, \beta_2, \beta_3), \rho = (\rho_1, \rho_2, \rho_3)$, $\tau = (\tau_1, \tau_2)$, and $H = (h_1, h_2, \cdots, h_n)'$ and $S = (s_1, s_2, \cdots, s_n)'$ and be the vectors of the latent variables. Following Yu (2005), our model can be rewritten as

$$\log h_{t+1} | \log h_t, \theta, s_t \sim N(\mu_{s_t} + \beta_{s_t}(\log h_t - \mu_{s_t}), \sigma^2)$$
$$r_t | \log h_{t+1}, \log h_t, \theta, s_t \sim N(\frac{\rho_{s_t}}{\sigma}\sqrt{h_t}(\log h_{t+1} - \mu_{s_t}), h_t(1 - \rho^2_{s_t}))$$

By Bayes' theorem, we can construct the joint posterior distribution of the unobservables given the data in terms of the prior distribution $p(\theta)$, and the likelihood function as follows:

$$p(\theta, H|R) \propto p(R, H|\theta)p(\theta) \tag{1}$$

where

$$\begin{aligned} p(R, H|\theta, S) &\propto p(\log h_1|\theta) \prod_{t=1}^{n-1} p(r_t, \log h_{t+1}| \log h_t, \theta, s_t) p(r_n| \log h_n, \theta) \\ &= p(\log h_1|\theta) \prod_{t=1}^{n-1} p(r_t| \log h_{t+1}, \theta, s_t) p(\log h_{t+1}| \log h_t, \theta, s_t) p(r_n| \log h_n, \theta) \\ p(\theta) =& p(\mu_1)p(\mu_2)p(\mu_3)p(\beta_1)p(\beta_2)p(\beta_3)p(\sigma^2)p(\rho_1)p(\rho_2)p(\rho_3)p(\tau_1)p(\tau_2)p(\log h_1) \end{aligned}$$

Regarding the prior distribution of θ, we follow the literature, namely, all variables of θ are assumed to be independent. For parameters β, and σ^2, we follow exactly the prior specification of Kim et al (1998); $\sigma^2 \sim Inverse - Gamma(2.5, 0.025)$, which has a mean of 0.167 and a standard deviation of 0.024. For β, Kim et al (1998) specified a beta distribution with parameters 20 and 1.5 implying a mean of 0.86 and a standard deviation of 0.11. Regarding the parameter μ, we take a slightly informative prior such as $\mu_j \sim N(-10, 4)$ for all j. The correlation parameter ρ_j for all j is assumed to be uniformly distributed with support between -1 and 1, and hence is completely flat. Therefore, the prior distributions with different regimes are not informative. For the threshold level parameters τ, to ensure that each regime has enough observations, we assume that the threshold has a uniform prior, $U[\overline{\tau_1}, \overline{\tau_2}]$, where the lower and upper bounds correspond to selected quantiles of r_t. We suggest that each regime must contain at least 10% of the sample r_t and τ is constrained to satisfy, $\tau_1 < \tau_2$. When the stock return of Microsoft Corporation is used, we impose $\tau_1 < \overline{\tau_m}$ and $\tau_2 > \overline{\tau_m}$ to speed up the convergence. $\overline{\tau_m}$ denotes 50% quantile of the sample r_t.

For the usual Bayesian procedure, we implemented a MCMC method to sample latent variables and unknown parameters from the joint posterior density $p(\theta, H|R)$ in (1). The MCMC algorithm repeatedly samples from the posterior distributions, which generates a Markov chain over (θ, H), until converging to the equilibrium/stationary posterior distribution, $p(\theta, H|R)$. For our MCMC procedure, we used the Gibbs sampler and the Metropolis-Hastigns (MH) algorithm within the Gibbs sampler. These methods have had a widespread influence on the theory and practice of Bayesian inference. For instance, Chib and Greenberg (1995) provided a detailed account of the Metropolis-Hastings algorithm.

Let $\omega = (\theta, H, S)$ and ω_{-h_t} denotes ω excluding h_t. The Gibbs sampler, employed to generate a Markov chain whose stationary distribution is the joint posterior distribution (1), works as follows in the first step. Given the initialization (θ^0, H^0), we draw from each of the following distributions:

1. (a) Sample h_1 from $p(\log h_1|\omega_{-h_1}, r_t) \propto p(r_1, \log h_2|\log h_2, \theta, s_1)p(\log h_1)^2$.
(b) Sample h_t from $p(\log h_t \mid \omega_{-h_t}, r_t) \propto p(r_t, \log h_{t+1}|\log h_t, \theta, s_t)p(r_{t-1}, \log h_t|\log h_{t-1}, \theta, s_{t-1})$ for $t = 2, ..., n-1$,
(c) Sample h_n from $p(\log h_n \mid \omega_{-h_n}, r_t) \propto p(r_n|\log h_n, \theta, s_n)p(r_{n-1}, \log h_n|\log h_{n-1}, \theta, s_{n-1})$
2. Sample (ρ_1, ρ_2, ρ_3) from $p(\rho_0, \rho_1, \rho_2|w_{-\rho}, R) \propto \prod_t p(r_t|logh_{t+1}, \theta_{-\rho}, s_t)p(\rho_1, \rho_2, \rho_3)$
3. Sample σ from $p(\sigma|w_{-\sigma}, R) \propto \prod_t p(r_t, logh_{t+1}|logh_t, \theta_{-\sigma}, s_t)p(\sigma)$
4. Sample μ_j from $p(\mu_j, |w_{-\mu_j}, R) \propto \prod_t p(r_t, logh_{t+1}|logh_t, \theta_{-\mu_j}, s_t)p(\mu_j)$ $j = 1, 2, 3$
5. Sample β_j from $p(\beta_j, |w_{-\beta_j}, R) \propto \prod_t p(r_t, logh_{t+1}|logh_t, \theta_{-\beta_j}, s_t)p(\beta_j)$ $j = 1, 2, 3$
6. Sample τ_i from $p(\tau_i|w_{-\tau_i}, R) \propto \prod_t p(r_t, logh_{t+1}|logh_t, \theta_{-\tau_i}, s_t)p(\tau_i)$ $i = 1, 2$
7. Sample s_t, $t = 1, ..., n$.
8. Go to 1.

The random walk chain MH algorithm is applied to sample the parameters θ and a common and convenient choice of density for the increment random variable is the normal. The

[2] $p(\log h_1) \sim N(\bar{\mu}, \frac{\bar{\sigma^2}}{1-\bar{\rho^2}})$ where $\bar{\mu}, \bar{\sigma^2}$ and $\bar{\rho^2}$ are obtained from the results of SV model with leverage effect (SVL).

scale parameter for increment random variable determines the precise form of the candidate generating density. A suitable value of the scale parameter with good convergence properties can be selected by having an acceptance probability of 20% to 60%.

The most difficult part of this Gibbs sampler is to sample h_t from $p(h_t \mid \omega_{-h_t}, r_t)$. For sampling h_t, we use the grid-based chain suggested by Tierney (1994). Ritter and Tanner (1992) proposed the griddy Gibbs sampler for Gibbs sampling in problems where the conditional distributions cannot be sampled directly. This method was also described in Tierney (1994). Using this algorithm in its pure form may require quite a fine grid and thus a very large number of posterior density evaluations are necessarily required to control the error in the approximation. To deal with this problem, Tierney (1994) proposed this algorithm in a Metropolis chain to ensure the equilibrium distribution is exactly the target distribution even a coarse grid.

3 Main Results

3.1 Data and Benchmark Models

We considered two daily stock return series from 03 January 2006 to 30 June 2015. One is the S&P 500 Index return series and the other is the MSFT. The sample size was 2,388 in each case. Figure 1 shows the graphs of both return series. Each return series is demeaned by subtracting its sample mean.

For each return series, we estimate the following five models. We let

$$r_t = \sqrt{h_t} u_t, \qquad u_t \sim N(0,1).$$

1. basic SV model (SV_0):

$$\log h_{t+1} - \mu = \beta(\log h_t - \mu) + \sigma v_{t+1},$$

   where $v_t \sim N(0,1)$.

2. SV model with leverage effect (SVL):

$$\log h_{t+1} - \mu = \beta(\log h_t - \mu) + \sigma v_{t+1},$$

   where $cor(u_t, v_{t+1}) = \rho$.

3. SV model with threshold effect (SVT):

$$logh_{t+1} - \mu_{s_t} = \beta_{s_t}(logh_t - \mu_{s_t}) + \sigma v_{t+1}$$

where

$$s_t = \begin{cases} 0 & \text{if } r_t < 0 \\ 1 & \text{if } r_t \geq 0 \end{cases} . \quad (2)$$

4. SV model with threshold and (constant) leverage effects (SVTL):

$$logh_{t+1} - \mu_{s_t} = \beta_{s_t}(logh_t - \mu_{s_t}) + \sigma v_{t+1},$$

where $cor(u_t, v_{t+1}) = \rho$ and s_t is defined as in (2).

5. Triple regime SV model with threshold and leverage effects (TRSV) defined in Section 2.

The basic SV model does not allow for any asymmetric relationship between return and volatility. The SVL model introduced by Harvey and Shephard (1996) allows for the leverage effect by incorporating the correlation between lagged return and volatility. So et al. (2002) proposed the SVT model, which accommodates the threshold effect in the model. In the SVT model, each regime is determined by the sign of lagged return. Smith (2009) introduced the SVTL model by combining these two models. It should be noted that the correlation coefficient ρ in the SVTL model is constant and does not depend on any one regime. While these three models (SVL, SVT and SVTL) explain the asymmetric relationship between return and volatility, it is only the sign of lagged return that determines the asymmetric relationship in these models. On the other hand, our TRSV model incorporates both sign and magnitude of lagged return in determining each regime. As explained in Section 2, the TRSV model allows three regimes depending on sign and magnitude of lagged return.

All of these SV models are estimated using the Bayesian method described in Section 2.2. Regarding the prior distribution for the benchmark SV models, we use the same distributions as the TRSV model, which are specified in Section 2.2.

3.2 Results for the S&P 500 Index

We first consider the S&P 500 Index return series. For the basic SV model, after burn-in period of 30,000 iterations and a follow-up period of 70,000, we collected every 10th iteration. For SVL model, total 200,000 iterations were drawn. We chose a burn-period of 11,000 iterations and stored every 10th iteration. For the rest models, we iterated total 300,000 and stored every 20th iteration because posterior correlations among the parameters are rather higher and convergence of Gibbs samplers is quite slow. In SVT and SVTL models, first 20,000 samples and 10,000 samples were discarded, respectively. For TRSV model, the results were reported after burn-in period of 170,000.

Table 1 provides the estimation result of the basic SV model. It reports the posterior mean, posterior standard deviations, 5% quantile and 95% quantile of all the parameters. The convergence diagnostics by Geweke (1992) is also provided in the table. The autoregressive coefficient β represents volatility persistence and it is estimated to be very close to unity ($\hat{\beta} = 0.984$). This is not surprising because the sample period contains the financial crisis in 2008. During the crisis period, volatility is much higher than the rest period and such a persistency makes the logarithm of volatility be estimated to be a near unit root process.

Table 2 presents the estimation result of the SVL model. The model includes the correlation coefficient ρ that exhibits the relationship between return and future volatility. It is estimated to be -0.775, which indicates that return and volatility has a negative relationship and the relationship is relatively strong. The negative value of $\hat{\rho}$ confirms what is already known in the literature. Examples include Harvey and Shephard (1996) and Yu (2004). The autoregressive coefficient β is estimated to be 0.971, which is a little lower but still similar to that in the basic SV model.

Table 3 reports the estimation result of the SVT model. In this model, the parameters μ and β take different values depending on whether return is negative or not. The level of volatility is represented by the value of μ. When return is negative, μ_0 is estimated to be -5.200. When return is non-negative, μ_1 is estimated to be -13.904. This implies that the level of volatility is much higher when return is negative. While $\hat{\mu}_0$ and $\hat{\mu}_1$ are estimated to be quite different from each other, the volatility persistence parameter β is estimated to be similar in both regimes ($\hat{\beta}_0 = 0.965$ and $\hat{\beta}_1 = 0.971$).

Table 4 provides the estimation result of the SVTL model. Compared to the SVT model, the model now includes a (constant) correlation coefficient ρ. The correlation coefficient ρ is estimated to be -0.735, which is similar to that in the SVL model. Compared to the SVT model in Table 3, the estimates of β_0 and β_1 are similar while the estimates of μ_0 and μ_1 are quite different. This shows that incorporating the leverage effect does not substantially change the volatility persistence estimates while it affect substantially the volatility level estimates. Compared to the SVT model, in particular the difference between $\hat{\mu}_0$ and $\hat{\mu}_1$ becomes much smaller.

The estimation result of our TRSV model is provided in Table 5. The convergence diagnostics by Geweke (1992) in Table 5 show that the Markov chains converged well. Figures

2-4 give the trace of the MCMC iterates after burn-in period, the autocorrelations of the draw sequences and the estimated posterior densities of all parameters. From the trace and the autocorrelation plots, we observed the high speed of convergence. The autocorrelations of the iterates decayed very quickly in all parameters. Figure 5 provides the plot of the posterior mean of MCMC iterates for h_t.

First, for our TRSV model, the parameter τ_1 is estimated to be -0.0103, which is the 0.14 quantile of the demeaned return series. When return is lower than its 14 percent quantile, it belongs to regime 1. The parameter τ_2 is estimated to be 0.0111, which is the 0.88 quantile of the return series. Hence, when return is between its 14 percent quantile and 88 percent quantile, it belongs to regime 2. When return is larger than its 88 percent quantile, it is in regime 3. Therefore, regime 1 contains 14% extreme negative returns, regime 2 includes 74% moderate negative/positive returns and regime 3 has 12% extreme positive returns.

Second, it is interesting to compare the estimates of the correlation coefficient ρ. Depending on each regime, ρ is estimated to be quite different. In regime 2, it is estimated to be -0.787, which is somewhat similar to the values in SVL and SVTL models. This shows that when return is moderately negative or positive, the conventional leverage effect is exhibited and its magnitude is as strong as the two-regime models. However, in regime 1, it is estimated to be -0.118. When there is an extreme negative return, the leverage effect is much weaker even if it exists significantly. This may be due to a wait-and-see investing strategy by investors during turmoil periods. When stock price drops beyond a certain level, investors become cautious (more risk averse) and might choose a wait-and-see investing strategy. This would lead to the weaker leverage effect. In regime 3, ρ is estimated to be close to zero ($\hat{\rho}_3 = -0.003$) and it is insignificant. When investors observe that stock price rises beyond a certain level, they become optimistic (risk seeking) and might choose a more active investing strategy. As a result, there might be more trading that could increase volatility. Hence, this effect may neutralize the conventional leverage effect. As discussed in Section 2, this is why 3 regimes are introduced to differentiate the time varying leverage effects that are dependent upon the sign and magnitude of lagged returns.

Three figures of the third row in Figure 4 show the estimated posterior densities of ρ_1, ρ_2, and ρ_3. They clearly indicate that the leverage effect is different for each regime. The posterior densities of ρ_1 and ρ_2 are centered at negative values that are different from each other. The estimated posterior density of ρ_3 is centred at zero.

Third, the volatility level parameter μ is estimated to be the highest in regime 1 and to be the lowest in regime 3. When return has a extreme negative value, volatility is the highest. When return is all positive, volatility is the lowest. The volatility persistence coefficient β is estimated to be similar for each regime. $\hat{\beta}_1, \hat{\beta}_2$ and $\hat{\beta}_3$ are $0.953, 0.964$ and 0.959, respectively.

For comparison of SV models, we used the deviance information criterion (DIC) proposed by Spiegelhalter et al. (2002). Berg et al. (2004) demonstrated that model selection can be more easily done by the DIC. It combines a Bayesian measure of fit with a measure of model complexity and can be expressed as

$$DIC = \bar{D} +_{PD} = D(\bar{\theta}) + 2_{PD},$$

where $\bar{D}$, a Bayesian measure of model fit, is defined as the posterior expectation of the deviance, $_{PD}$ is a measure of complexity (penalty term for increasing model complexity) defined as the difference between the posterior expectation of the deviance $\bar{D}$ and the deviance evaluated at the posterior mean of the parameters $D(\bar{\theta})$. Thus, $\bar{D}(= D(\bar{\theta}) +_{PD})$, a Bayesian measure of model fit, already contains a penalty term for model complexity. Therefore, the DIC can be divided into a pure measure of fit $D(\bar{\theta})$ plus a measure of complexity 2_{PD}. See Berg et al (2004) for detailed explanation. Table 6 provides the comparison results by DIC. Our TRSV model shows a reasonably good fit compared to other SV models. Our model exhibits the best fit in terms of $D(\bar{\theta})$, but is heavily penalized by its large number of parameters. While the SVL model achieves the smallest value of DIC, the difference between SVL and our model is negligible.

3.3 Result for Microsoft

We now consider the stock return series of Microsoft Corporation. For SV, SVT and SVTL models, we had 200,000 iterations and first 20,000 samples for SV and SVT models were discarded as a burn-in period. We discarded the first 60,000 iterations in SVL model as a burn-in period. For the SVL and TRSV models, a total of 250,000 and 300,000 iterations were drawn, respectively, and first 40,000 and 100,000 samples were discarded as a burn-in period. We collected every 20th iteration for all models.

Table 7 provides the estimation result of the basic SV model. The autoregressive coefficient β is estimated to be 0.942, which is lower than that in the S&P 500 Index case. The volatility persistence of the individual firm turns out to be lower than the stock index.

Table 8 presents the estimation result of the SVL model. The correlation coefficient ρ is estimated to be -0.254. As in the stock index case, return and volatility has a negative relationship. However, the leverage effect is much weaker compared to that in the stock index. The autoregressive coefficient β is estimated to be 0.935, which is similar to that in the basic SV model.

Table 9 reports the estimation result of the SVT model. When stock return is negative, the volatility level parameter μ_0 is estimated to be -8.005. When stock return is non-negative, μ_1 is estimated to be -9.787. This implies that the level of volatility is much higher when return is negative. The volatility persistence parameter β_0 and β_1 are estimated to be 0.925 and 0.954, respectively. It is interesting to note that the volatility persistence is estimated to be higher when stock return is non-negative.

Table 10 provides the estimation result of the SVTL model. The correlation coefficient ρ is estimated to be -0.297, which is similar to that in the SVL model. The volatility persistence parameter β_0 and β_1 are estimated to be 0.930 and 0.945. While the difference between $\hat{\beta}_0$ and $\hat{\beta}_1$ is not substantial, as in the SVT model, it indicates that volatility persistence is higher when stock return is non-negative. Compared to the SVT model, it is also interesting to note that the volatility level parameter μ is estimated to be higher when stock return is non-negative. When the model incorporates both threshold and leverage effects, the volatility level parameter is estimated to be higher when stock return is non-negative.

The estimation result of our triple regime SV model is provided in Table 11. The convergence diagnostics by Geweke (1992) in Table 11 show that the Markov chains converged well. Figures 6-8 give the trace of the MCMC iterates after burn-in period, the autocorrelations of the draw sequences and the estimated posterior densities of all parameters. As in the S&P 500 case, we observed the high speed of convergence. The autocorrelations of the iterates decayed very quickly. Figure 9 provides the plot of the posterior mean of MCMC iterates for h_t.

First, for our TRSV model, the parameter τ_1 is estimated to be -0.0082, which is the 0.25 quantile of the demeaned return series. When return is lower than its 25 percent quantile, it belongs to regime 1. The parameter τ_2 is estimated to be 0.0139, which is 0.85 quantile of the return series. Hence, when return is between its 25 percent quantile and 85 percent quantile, it belongs to regime 2. When return is larger than its 85 percent quantile, it is in regime 3. Therefore, regime 1 contains 25% extreme negative returns, regime 2 includes 60% moderate negative/positive returns and regime 3 has 15% extreme positive returns. Compared to the S&P 500 Index case where regimes 1, 2 and 3 contain 14%, 74%, and 12% of return series, respectively, now regime 1 includes more return series while regime 2 contains less return series in the MSFT.

Second, in regime 2, the correlation coefficient ρ is estimated to be -0.401, which is lower than the estimates in SVL and SVTL models. This indicates that when return is moderately negative or positive the leverage effect is stronger compared to that in the two-regime models. However, in both regime 1 and regime 2, it is estimated to be close to zero and also insignificant. When there is a large negative or positive return, the conventional leverage effect disappears. Therefore, if this feature in both regime 1 (25%) and regime 3 (15%) is ignored and only two regimes are allowed in SV models, the leverage effect appears to be weaker as in the SVL and SVTL models. Especially, when there is a large negative lagged return, we can interpret the weak leverage effect as follows. A large negative return leads to increased debt/equity ratio and investors will expect increased future return since blue chip stocks like Microsoft have less risk compared to other stocks during the periods of financial crisis. Investors expect that the stock price for MSFT will bounce, and current price is too low considering all the financial aspects of the Microsoft including debt/equity ratio.

Third, the volatility level parameter μ is estimated to be the highest in regime 2 and to be the lowest in regime 3. When the return is all positive (regime 3), volatility is the lowest. The volatility persistence coefficient β is estimated to be the largest in regime 1 ($\hat{\beta}_1, \hat{\beta}_2$ and $\hat{\beta}_3$ are $0.979, 0.944$ and 0.945, respectively).

Table 12 shows that our TRSV model achieves the lowest DIC, which implies that our TRSV model fit the data best. The DIC of our TRSV model is -13,690, which is substantially lower compared to other SV models. The second best model in terms of DIC is the SVL model and its DIC is -13,618. The SVTL model has a similar DIC as the SVL model

4 Conclusion

The paper investigates a new stochastic volatility model that accommodates three regimes and both threshold and leverage effects. We find evidence that the relationship between stock return and volatility depends on the magnitude of return as well as its sign. In both the S&P 500 Index and MSFT, the results show that the leverage effect is estimated to be different in each three regime. In both cases, when stock price drops (or rises) beyond a certain level, the conventional leverage effect either disappear or becomes much weaker. In regime 3 (large positive return), the leverage effect disappeared in both cases. In regime 1 (large negative return), the leverage effect weakly appeared for the index ($\hat{\rho} = -0.118$), while it disappeared for the MSFT. In regime 2 (moderate negative or positive return), the conventional leverage effect appeared in both cases. Comparing the leverage effects between the index and the MSFT cases, the individual firm showed much weaker leverage effect than the index. In regime 2, the correlation coefficient ρ was estimated to be -0.787 and -0.401 for the index and the MSFT, respectively. Compared to existing SV models, our model fits the data well, supporting the idea of allowing three regimes in our model.

The paper provides empirical evidences that the conventional leverage effect either disappears or becomes much weaker when stock return is largely positive or negative. We conjecture that it is because investors behave differently when stock prices show rapid changes. It would be desirable to further investigate the reason why the leverage effect either disappears or becomes weaker when stock return is largely positive or negative. Moreover, there has been active research on multivariate stochastic volatility models (Omori and Ishihara 2012) and it will be interesting to investigate a multivariate stochastic volatility model that accommodates three regimes. We leave these for future works.

A Tables and Figures

Table 1. Estimation results of the basic SV model for S&P 500

Parameters	Posterior Mean	Std errors	5%	95%	Convergence Diagnostics
μ	-9.3851	0.2652	-9.8277	-8.9485	-0.3970
β	0.9838	0.0047	0.9754	0.9910	-0.6349
σ	0.0377	0.0078	0.0271	0.0527	0.3635

Note: Values in the fifth and the sixth columns are the 5^{th} and the 95^{th} quantile, respectively. The last column in the table indicates the convergence diagnostic by Geweke (1992).

Table 2. Estimation results of the SVL model for S&P 500

Parameters	Posterior Mean	Std errors	5%	95%	Convergence Diagnostics
μ	-9.3354	0.1252	-9.5392	-9.1273	1.4670
β	0.9711	0.0044	0.9637	0.9781	0.8821
ρ	-0.7747	0.0355	-0.8277	-0.7106	-0.7078
σ	0.0660	0.0091	0.0512	0.0813	-0.2663

Note: Same as Table 1.

Table 3. Estimation results of the SVT model for S&P 500

Parameters	Posterior Mean	Std errors	5%	95%	Convergence Diagnostics
μ_0	-5.1997	0.9065	-6.5174	-3.5536	0.4153
μ_1	-13.9036	0.9300	-15.5326	-12.4276	-0.5469
β_0	0.9651	0.0078	0.9514	0.9768	0.1171
β_1	0.9711	0.0064	0.9595	0.9803	0.3598
σ	0.0429	0.0086	0.0306	0.0576	-0.3977

Note: Same as Table 1.

Table 4. Estimation results of the SVTL model for S&P 500

Parameters	Posterior Mean	Std errors	5%	95%	Convergence Diagnostics
μ_0	-8.9003	0.7228	-10.0175	-7.6882	-1.1850
μ_1	-9.8412	0.8283	-11.2518	-8.5689	-0.0277
β_0	0.9655	0.0128	0.9437	0.9859	-1.1710
β_1	0.9739	0.0107	0.9554	0.9907	1.3250
ρ	-0.7349	0.0482	-0.8076	-0.6522	0.3766
σ	0.0638	0.0121	0.0465	0.0871	0.2109

Note: Same as Table 1.

Table 5. Estimation results of the TRSV model for S&P 500

Parameters	Posterior Mean	Std errors	5%	95%	Convergence Diagnostics
μ_1	-4.8548	0.9552	-6.2907	-3.2648	-0.3030
μ_2	-9.1412	0.5346	-9.7198	-7.9391	-1.0340
μ_3	-14.7316	1.4485	-17.3304	-12.6117	0.9524
β_1	0.9526	0.0149	0.9267	0.9752	0.5759
β_2	0.9642	0.0107	0.9461	0.9823	-0.3281
β_3	0.9588	0.0099	0.9412	0.9732	-0.2021
ρ_1	-0.1183	0.0586	-0.2041	-0.0130	-0.3657
ρ_2	-0.7867	0.0586	-0.8688	-0.6789	1.7930
ρ_3	-0.0029	0.0526	-0.0903	0.0816	0.0091
τ_1	-0.0103	0.0021	-0.0129	-0.0065	-1.1610
τ_2	0.0111	0.0010	0.0091	0.0121	0.0070
σ	0.0706	0.0122	0.0495	0.0919	-1.6930

Note: Same as Table 1.

Table 6. Comparison results of SV models for S&P 500

Models	DIC	$\bar{D}$	$D(\bar{\theta})$	p_D
SV_0	-15541.8	-15663.0	-15784.2	121.2
SVL	-15611.6	-15718.0	-15824.3	106.3
SVT	-15592.5	-15720.6	-15848.7	128.1
SVTL	-15609.6	-15722.8	-15836.1	113.3
TRSV	-15606.6	-15731.6	-15856.6	125.0

Note: DIC is the deviance information criterion. $\bar{D}$ is the posterior expectation of the deviance. $D(\bar{\theta})$ is the deviance evaluated at the posterior mean of the parameters. p_D is a measure of complexity defined as the difference between $\bar{D}$ and $D(\bar{\theta})$.

Table 7. Estimation results of the basic SV model for MSFT

	Posterior				Convergence
Parameters	Mean	Std errors	5%	95%	Diagnostics
μ	-8.5958	0.1196	-8.7869	-8.4016	-0.5096
β	0.9423	0.0124	0.9205	0.9616	-1.3150
σ	0.0941	0.0193	0.0651	0.1306	0.9173

Note: Same as Table 1.

Table 8. Estimation results of the SVL model for MSFT

	Posterior				Convergence
Parameters	Mean	Std errors	5%	95%	Diagnostics
μ	-8.5971	0.1094	-8.7733	-8.4162	1.0870
β	0.9350	0.0123	0.9143	0.9544	0.8534
ρ	-0.2538	0.0584	-0.3456	-0.1558	-0.4792
σ	0.1085	0.0194	0.0782	0.1408	-0.6357

Note: Same as Table 1.

Table 9. Estimation results of the SVT model for MSFT

Parameters	Posterior Mean	Std errors	5%	95%	Convergence Diagnostics
μ_0	-8.0054	0.3541	-8.4981	-7.4000	-0.0803
μ_1	-9.7865	0.8813	-11.4150	-8.7482	0.6480
β_0	0.9246	0.0224	0.8882	0.9615	1.2020
β_1	0.9542	0.0216	0.9150	0.9851	0.0400
σ	0.0986	0.0167	0.0734	0.1287	-0.7250

Note: Same as Table 1.

Table 10. Estimation results of the SVTL model for MSFT

Parameters	Posterior Mean	Std errors	5%	95%	Convergence Diagnostics
μ_0	-9.0454	0.5608	-10.1259	-8.3228	-0.0984
μ_1	-8.0954	0.6430	-9.1198	-7.0244	-0.4398
β_0	0.9298	0.0249	0.8889	0.9699	0.2518
β_1	0.9452	0.0242	0.9039	0.9835	-1.1010
ρ	-0.2970	0.0942	-0.4526	-0.1462	0.3662
σ	0.1084	0.0202	0.0764	0.1469	0.4348

Note: Same as Table 1.

Table 11. Estimation results of the TRSV model for MSFT

Parameters	Posterior Mean	Std errors	5%	95%	Convergence Diagnostics
μ_1	-9.8687	1.8444	-12.7187	-7.2345	-0.8808
μ_2	-7.1248	0.8178	-8.1636	-5.5307	1.0620
μ_3	-12.7589	1.2759	-14.9801	-10.7404	-1.6170
β_1	0.9790	0.0149	0.9496	0.9964	1.3380
β_2	0.9444	0.0180	0.9128	0.9726	0.6361
β_3	0.9448	0.0154	0.9168	0.9669	0.9998
ρ_1	-0.0353	0.0521	-0.1164	0.0507	-0.0494
ρ_2	-0.4009	0.1637	-0.6517	-0.1002	0.5930
ρ_3	-0.0007	0.0419	-0.0696	0.0685	1.7330
τ_1	-0.0082	0.0027	-0.0131	-0.0051	-0.1677
τ_2	0.0139	0.0017	0.0113	0.0170	0.8141
σ	0.1207	0.0207	0.0895	0.1583	0.0053

Note: Same as Table 1.

Table 12. Comparison results of SV models for MSFT

Models	DIC	$\bar{D}$	$D(\bar{\theta})$	p_D
SV_0	-13599.1	-13755.8	-13912.6	156.7
SVL	-13617.8	-13777.4	-13936.9	159.5
SVT	-13608.2	-13767.7	-13927.1	159.5
SVTL	-13615.4	-13775.0	-13934.5	159.6
TRSV	-13690.2	-13855.1	-14020.0	164.9

Note: Same as Table 6.

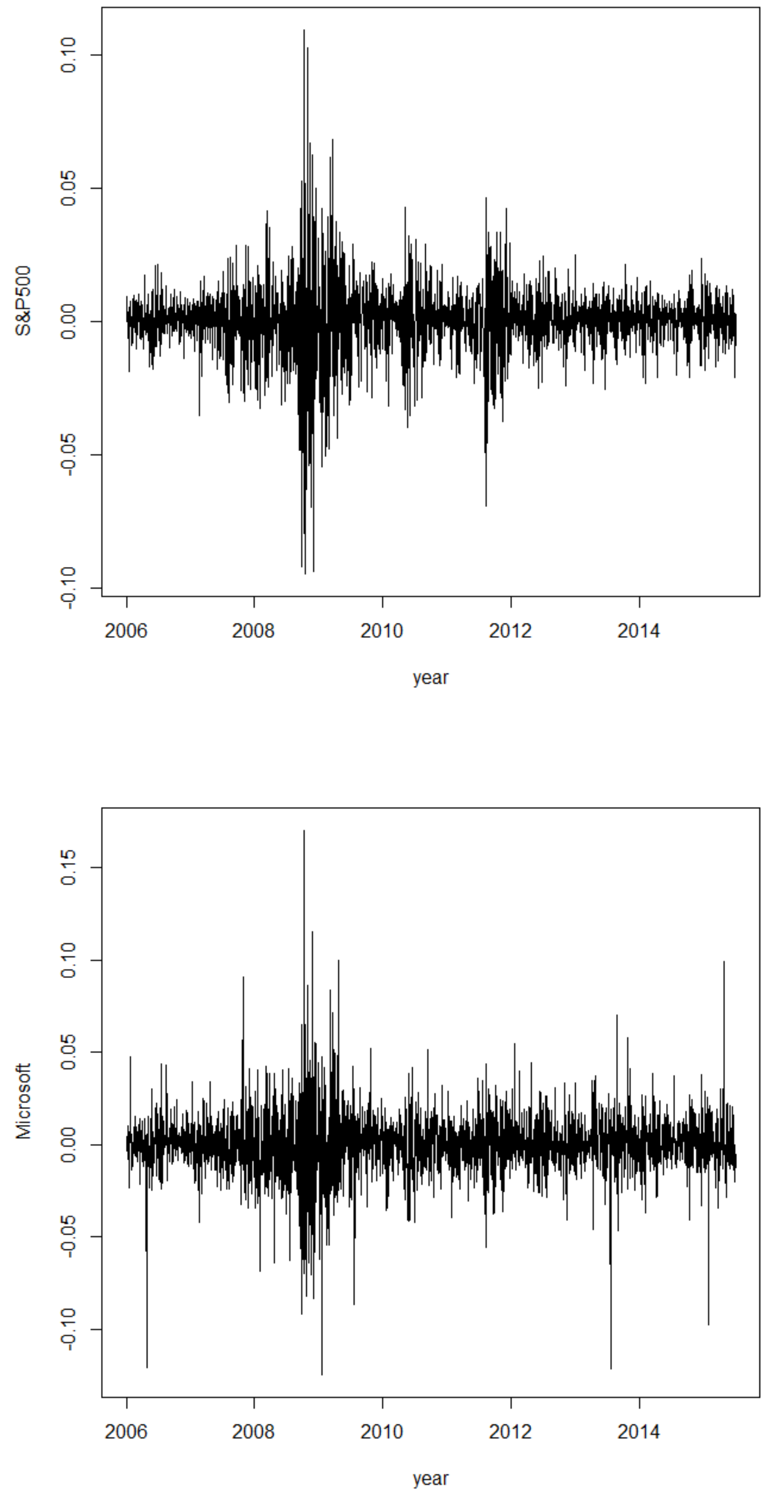

Figure 1. Plots of stock return series: S&P 500 Index and MSFT

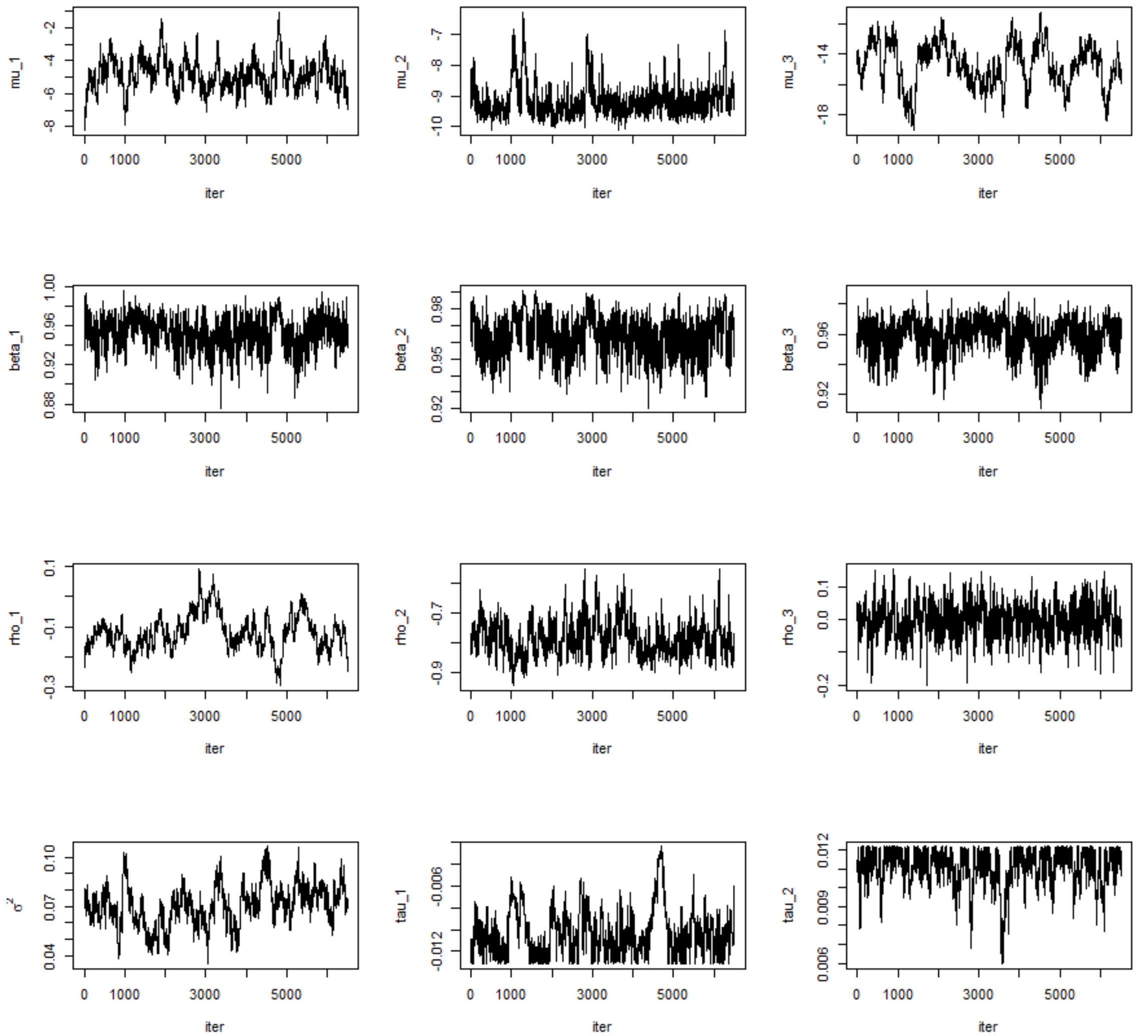

Figure 2. Plots of the 10,000 MCMC iterates obtained from the TRSV model fitting of S&P 500

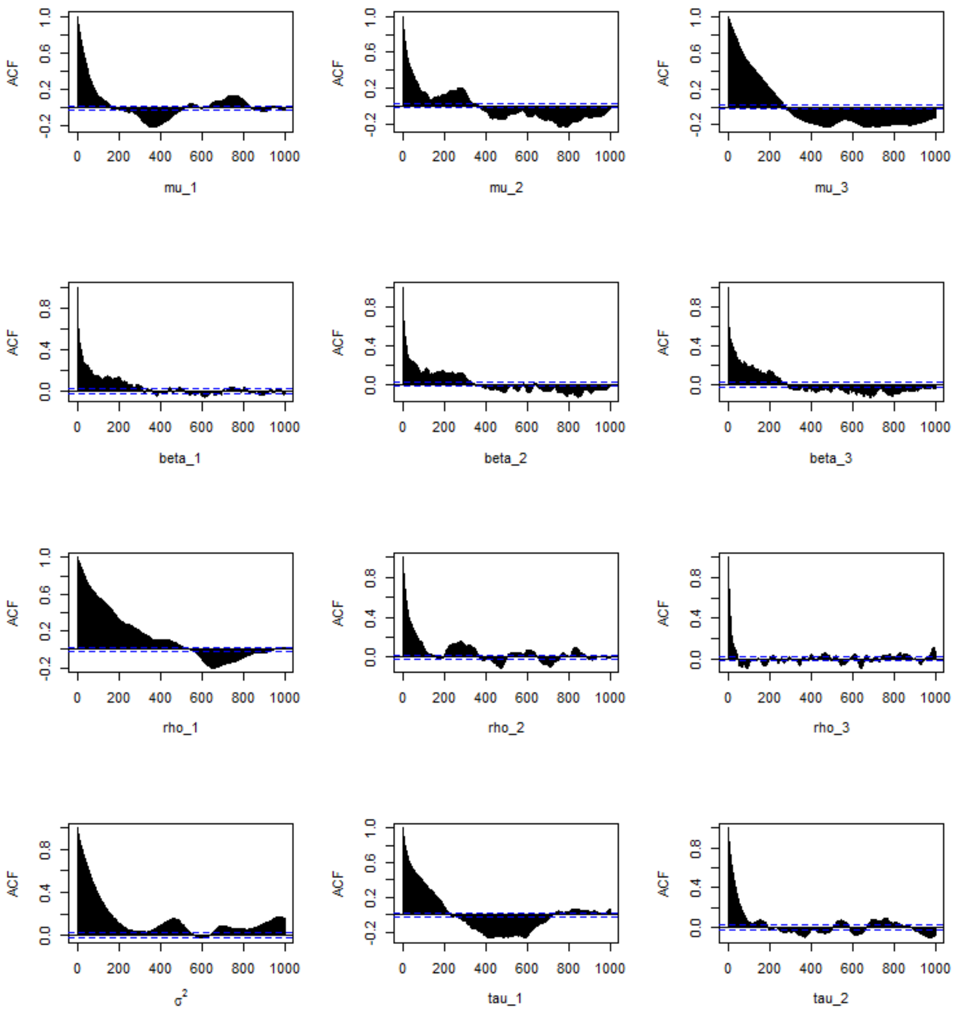

Figure 3. Autocorrelation of the MCMC iterates for the TRSV model fitting of S&P 500

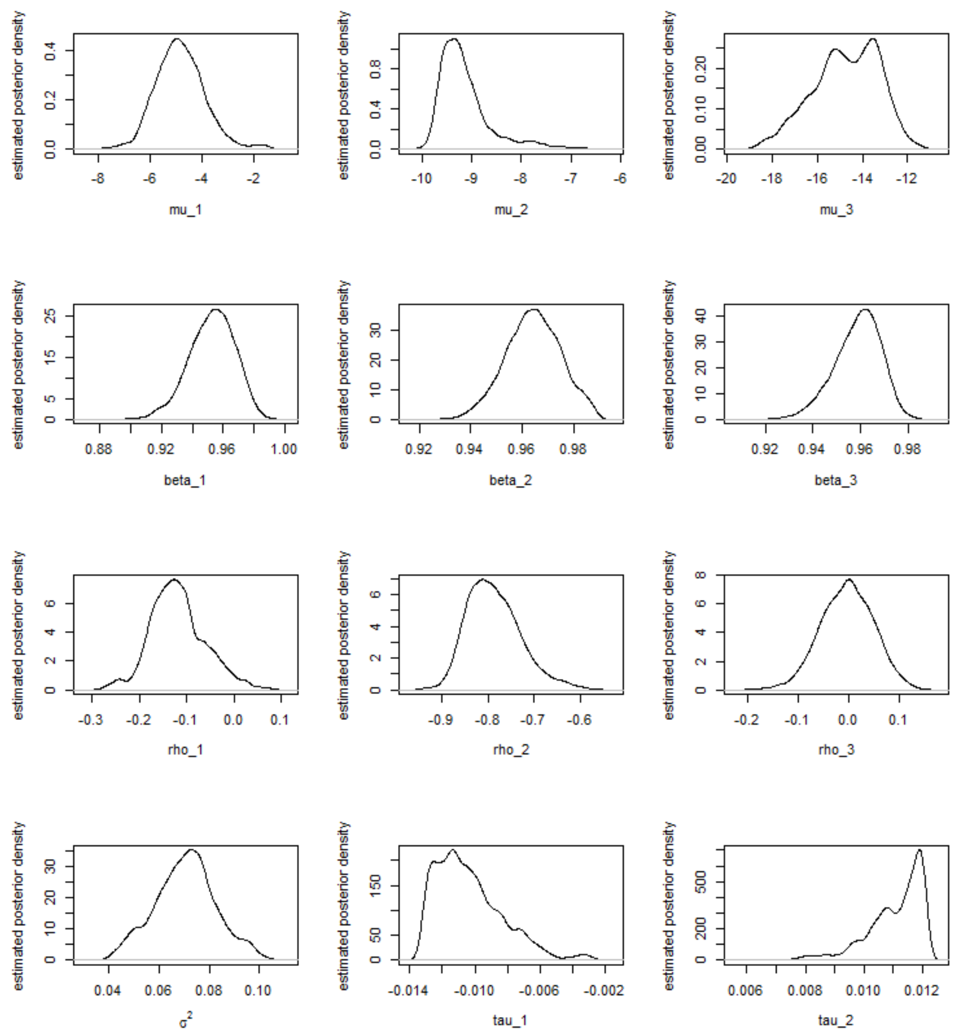

Figure 4. Estimated posterior densities for the TRSV model fitting of S&P 500

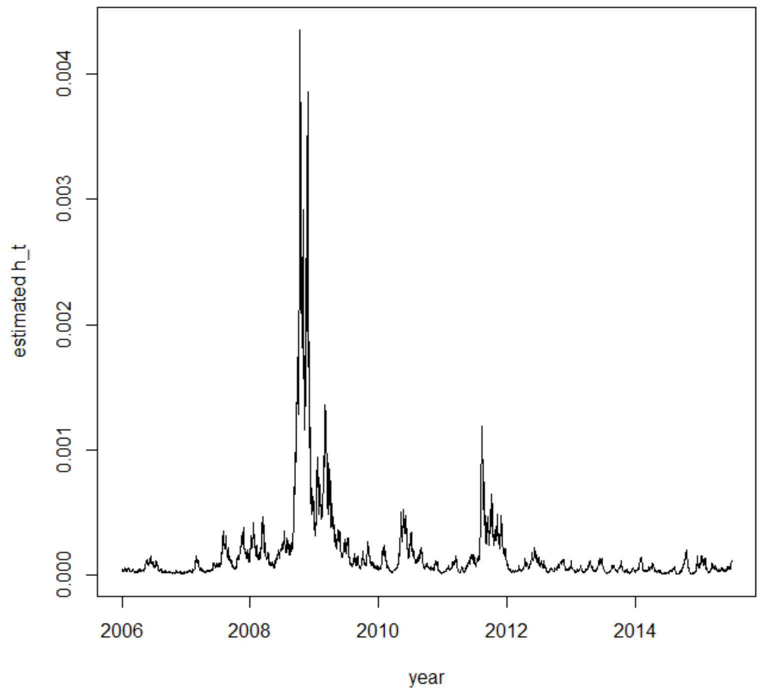

Figure 5. Estimated h_t for the TRSV model fitting of S&P 500

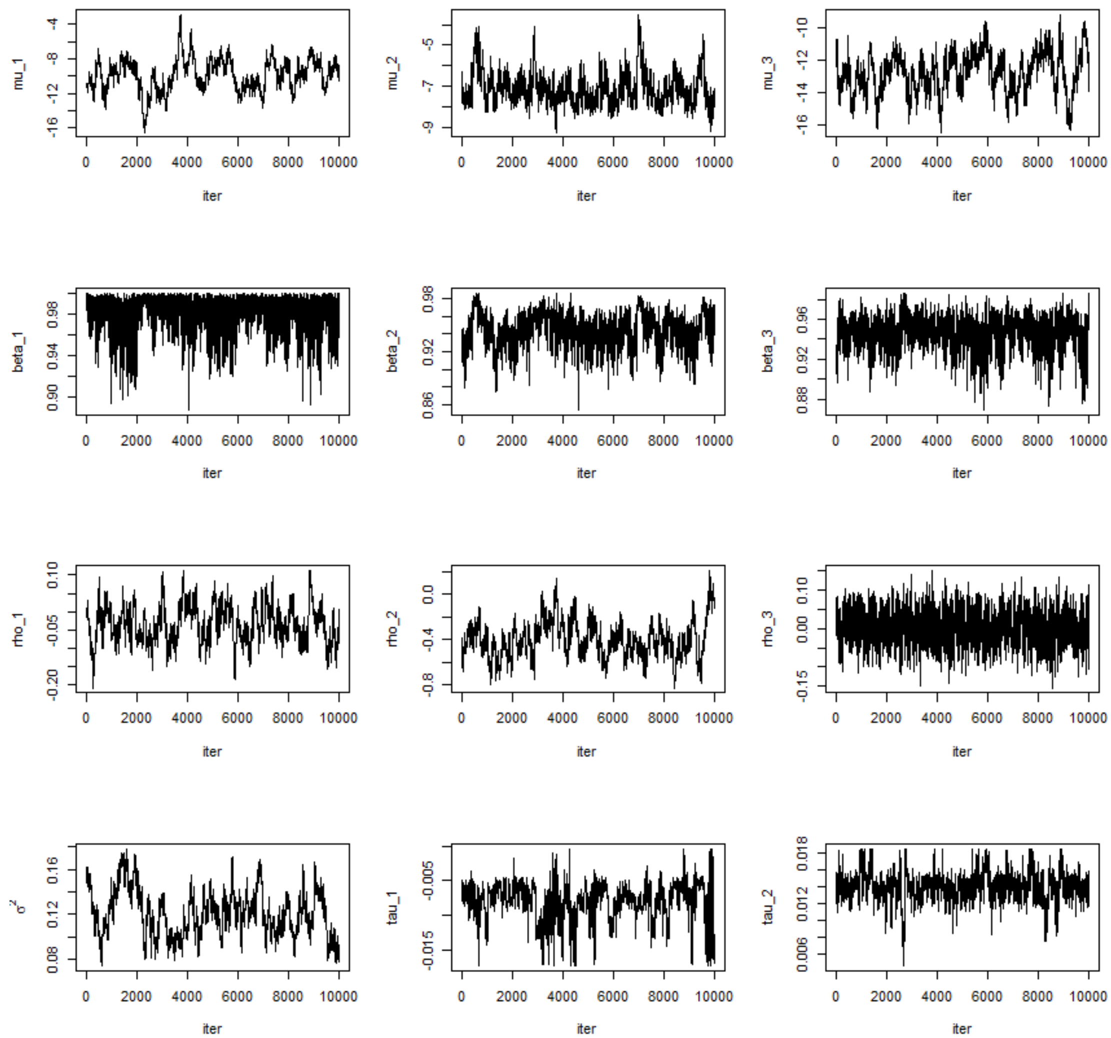

Figure 6. Plots of the 10,000 MCMC iterates obtained from the TRSV model fitting of MSFT

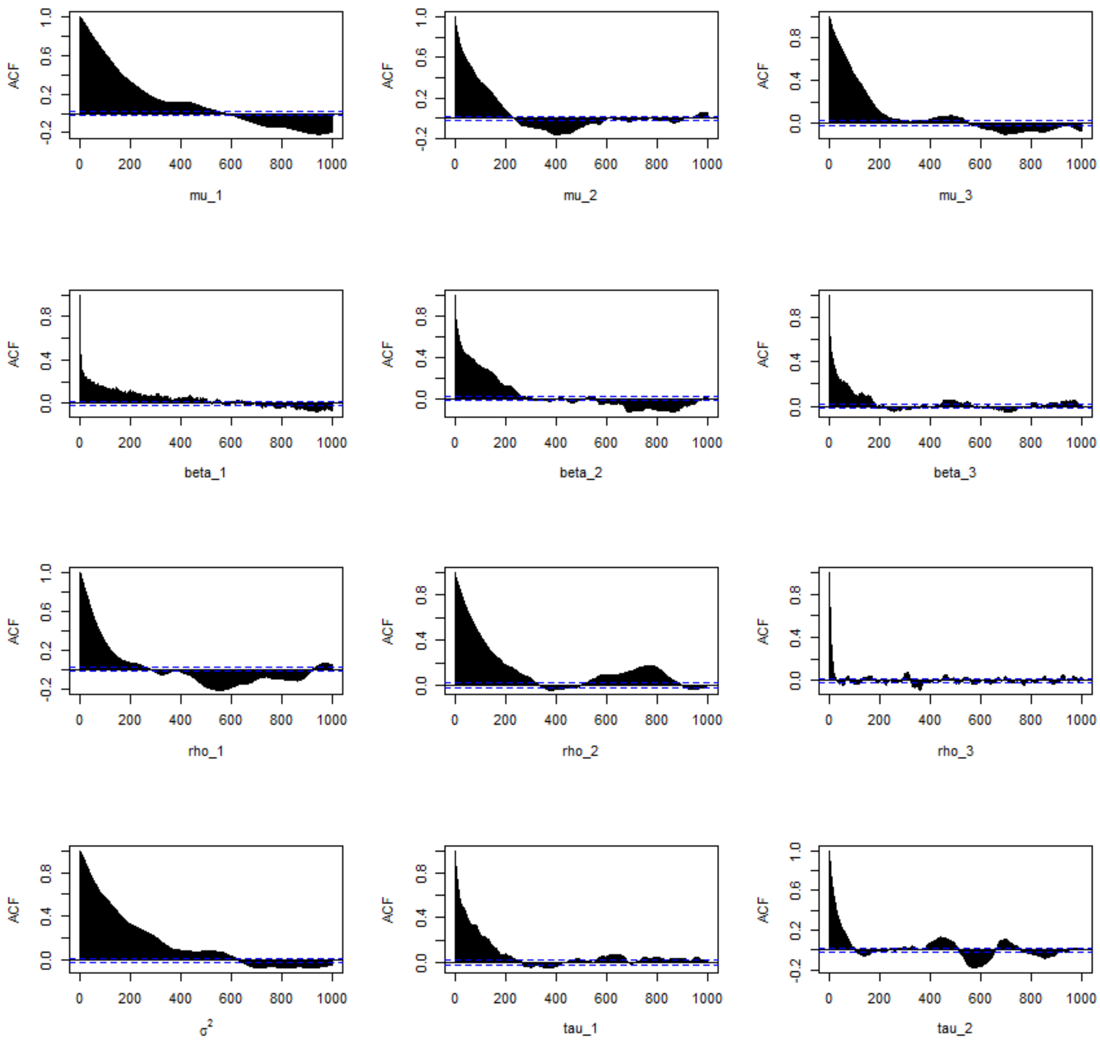

Figure 7. Autocorrelation of the MCMC iterates for the TRSV model fitting of MSFT

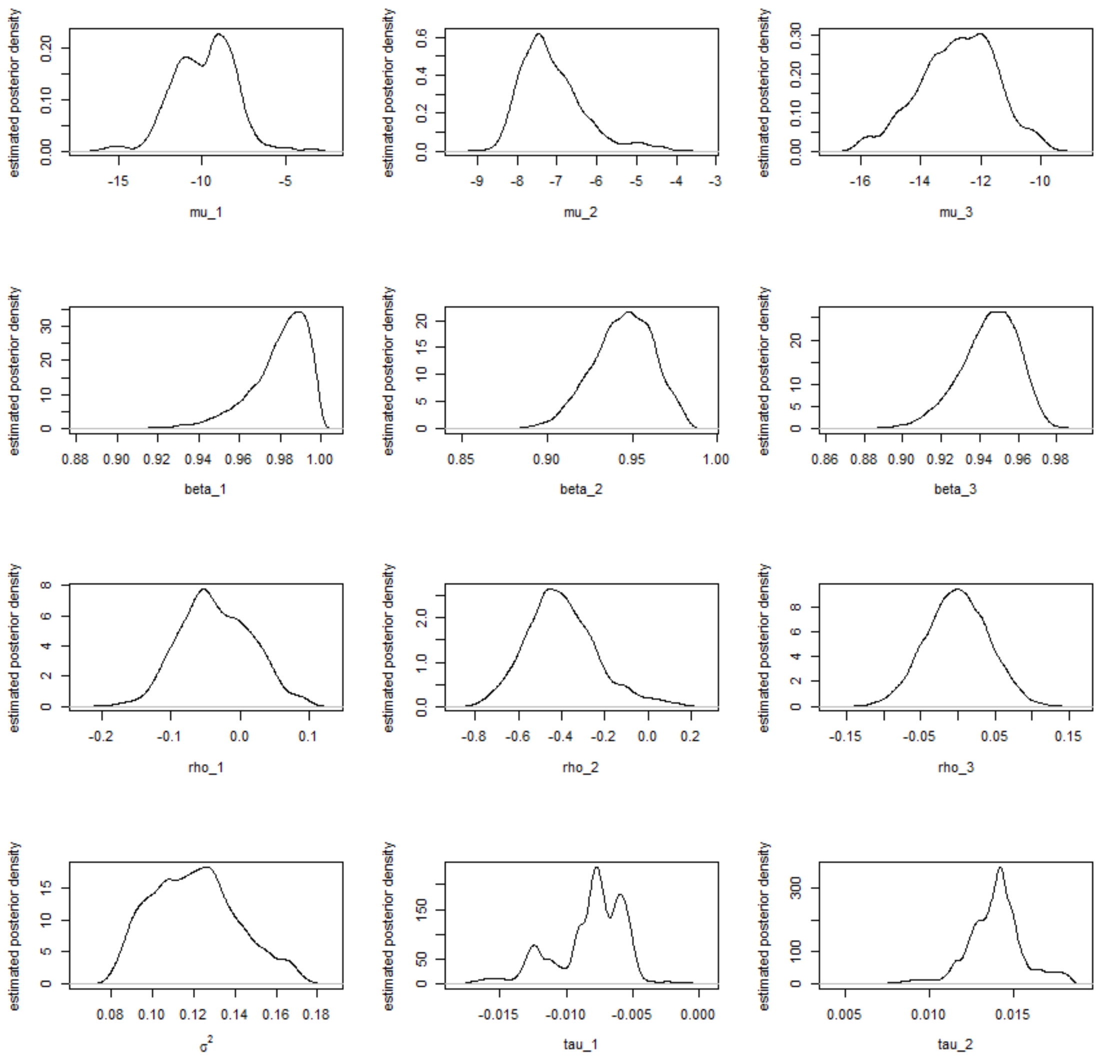

Figure 8. Estimated posterior densities for the TRSV model fitting of MSFT

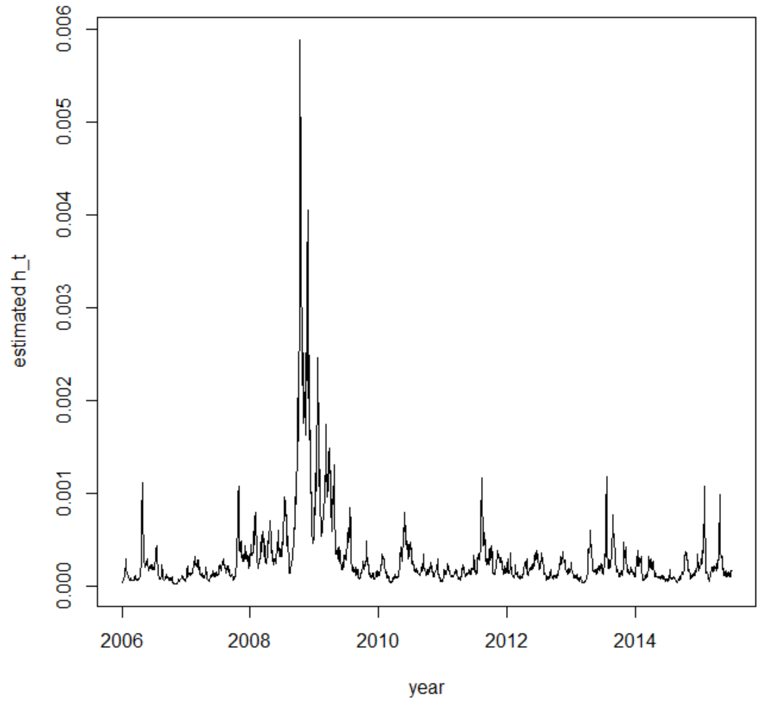

Figure 9. Estimated h_t for the TRSV model fitting of MSFT

References

Aït-Sahalia, Y., J. Fan and Y. Li (2013) The leverage effect puzzle: Disentangling sources of bias at high frequency, *Journal of Financial Economics*, 109, 224–249.

Andersen, T.G., H.-J. Chung and B.E. Sørensen (1999) Efficient method of moments estimation of a stochastic volatility model: A Monte Carlo study, *Journal of econometrics*, 91, 61-87.

Andersen, T.G., B.E. Sørensen (1996) GMM estimation of a stochastic volatility model: a Monte Carlo study, *Journal of Business & Economic Statistics*, 14, 328-352.

Asai, M. and M. McAleer (2006) Asymmetric multivariate stochastic volatility, *Econometric Reviews*, 25, 453-473.

Bandi, F.M. and R. Reno (2012) Time-varying leverage effects. *Journal of Econometrics* 169, 94–113.

Berg, A., R. Meyer and J. Yu (2004) Deviance information criterion for comparing stochastic volatility models. *Journal of Business & Economic Statistics*, 22, 107-120.

Chib, S. and E. Greenberg (1995) Understanding the Metropolis-Hastings algorithm, *The American Statistician*, 49, 327-335.

Chib, S., F. Nardari and N. Shephard (2002) Markov chain Monte Carlo methods for stochastic volatility models, *Journal of Econometrics*, 108, 281-316.

Christensen, B.J., M. Nielson and J. Zhu (2015) The impact of financial crises on the risk–return tradeoff and the leverage effect, *Economic Modelling*, 49, 407-418.

Danielsson, J. (1994) Stochastic volatility in asset prices: Estimation with simulated maximum likelihood, *Journal of Econometrics*, 64, 375-400.

Daouk, H. and D. Ng (2011) Is unlevered firm volatility asymmetric?, *Journal of Empirical Finance*, 18, 634-651.

Duffie, D. and K.J. Singleton (1993) Simulated moments estimation of Markov models of asset prices, *Econometrica*, 61, 929-952.

Durbin, J. and S.J. Koopman (1997) Monte Carlo maximum likelihood estimation for non-Gaussian state space models, *Biometrika*, 84, 669-684.

Engle, R.F. and V.K. Ng (1993). Measuring and testing the impact of news on volatility. *Journal of Finance 48*, 1749-1778.

Gallant, A.R. and G. Tauchen (1996) Which moments to match?, *Econometric Theor*y, 12, 657-681.

Geweke, J. (1992) Evaluating the accuracy of sampling-based approaches to the calculation of posterior moments, In: Bernardo, J.M., J.P. Berger, A.P. Dawid, A.F.M. Smith (Eds.), *Bayesian Statistics*, vol. 4, Oxford University Press, Oxford, 169-193.

Glosten, L.R., R. Jagannathan and D. Runkle (1993) On the relation between the expected value and the volatility of nominal excess returns on stocks, *Journal of Finance,* 48, 1779-1801.

Harvey, A., E. Ruiz and N. Shephard (1994) Multivariate stochastic variance models, *The Review of Economic Studies*, 61, 247-264.

Harvey, A.C. and N. Shephard (1996) The estimation of an asymmetric stochastic volatility model for asset returns. *Journal of Business & Economic Statistics*, 14, 429-434.

Jacquier, E., N.G. Polson and P.E. Rossi (1994) Bayesian analysis of stochastic volatility models, *Journal of Business & Economic Statistics*, 12, 69-87.

Kim, S., N. Shephard and S. Chib (1998) Stochastic volatility: likelihood inference and comparison with ARCH models. *Review of Economic Studies*, 65, 361-393.

Linton, O., Y. Whang and Y. Yen (2016) A nonparametric test of a strong leverage hypothesis, *Journal of Econometrics*, 194, 153-186.

Melino, A. and S.M. Turnbull (1990) Pricing foreign currency options with stochastic volatility, *Journal of Econometrics*, 45, 239-265.

Nelson, D.B. (1991) Conditional heteroskedasticity in asset returns: A new approach, *Econometrica*, 59, 347-370.

Omori, Y. and T. Ishihara (2012), Multivariate Stochastic Volatility Model, in *Handbook of Volatility Models and Their Applications,* eds. L. Bauwens, C. Hafner and S. Laurent, Wiley, 175-195.

Patton, A.J. and K. Sheppard (2015) Good volatility, bad volatility: Signed jumps and the persistence of volatility, *The Review of Economics and Statistics*, 94, 683-697.

Pagan, A.R. and G.W. Schwert (1990a). Alternative models for conditional stock volatility. *Journal of Econometrics, 45*, 267-290.

Poon, S-H and C.W.J. Granger (2003) Forecasting volatility in financial markets: a review. *Journal of Economic Literature*, 41, 478-539.

Ritter, C. and M.A. Tanner (1992) Facilitating the Gibbs sampler: the Gibbs stopper and the griddy-Gibbs sampler, *Journal of the American Statistical Association*, 87, 861-868.

Sandmann, G. and S.J. Koopman (1998) Estimation of stochastic volatility models via Monte Carlo maximum likelihood, *Journal of Econometrics*, 87, 271-301.

Smith, D.R. (2009) Asymmetry in stochastic volatility models: threshold or correlation. *Studies in Nonlinear Dynamics & Econometrics*, 13, Issue 3. Article 1.

So, M.K.P., W.K. Li and K. Lam (2002) A threshold stochastic volatility model. *Journal of Forecasting*, 21, 473-500.

Sørensen, M. (2000) Prediction-based estimating functions, *The Econometrics Journal*, 3, 123-147.

Spiegelhalter, D.J., N.G. Best, B.P. Carlin and A. van der Linde (2002) Bayesian measures of model complexity and fit (with discussion). *Journal of Royal Statistical Society*, Series B 64, 583-639.

Taylor, S.J. (1986) *Modeling financial time series.* Wiley, New York.

Tierney, L. (1992) Exploring posterior distributions using Markov chains, Discussion paper, DTIC Documnet.

Tierney, L. (1994) Markov chains for exploring posterior distributions, *The Annals of Statistics*, 22, 1701-1728.

Wang, D.C., and P.A. Mykland (2014) The estimation of leverage effect with high frequency data. *Journal of the American Statistical Association,* 109, 197–215.

Wu, X-Y and H-L. Zhou (2014) A triple-threshold leverage stochastic volatility model. *Studies in Nonlinear Dynamics & Econometrics*, 19, Issue 4. 483-500.

Xu, D. (2010) A threshold stochastic volatility model with realized volatility, Working Paper, University of Waterloo.

Yu, J. (2005) On leverage in a stochastic volatility model. *Journal of Econometrics*, 127, 165-178.

Yu, J. (2012) A semiparametric stochastic volatility model. *Journal of Econometrics*, 167, 473-482.