Quantile Dependence between Stock Markets and Its Application in Volatility Forecasting[*]

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December 2016

Abstract

This paper examines quantile dependence between international stock markets and evaluates its use for improving volatility forecasting. First, we analyze quantile dependence and directional predictability between the US stock market and stock markets in the UK, Germany, France and Japan. We use the cross-quantilogram, which is a correlation statistic of quantile hit processes. The detailed dependence between stock markets depends on specific quantile ranges and this dependence is generally asymmetric. Second, we consider a simple quantile-augmented volatility model that accommodates the quantile dependence and directional predictability from the US market to these other markets. The quantile-augmented volatility model provides superior in-sample and out-of-sample volatility forecasts.

Keywords: Quantile, Cross-quantilogram, Spillover, Volatility Forecasting.

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1 Introduction

In many circumstances, investors are interested in dependence between financial markets such as dependence between international stock markets, dependence between currency markets, dependence between stock markets and bond markets or dependence between stock markets and commodity markets. It is essential for investors to have an understanding of the dependence between financial markets because this can be used to improve asset allocation and risk management. Therefore, volatility spillover, co-movement and contagion of financial markets have been extensively investigated in the literature. Researchers typically adopted a vector autoregressive model, a multivariate generalized autoregressive conditional heteroskedasticity (GARCH) model or a combination of both models to analyze volatility spillover, co-movement and contagion of financial markets (Baele (2005), Dungey et al. (2005), Forbes and Rigobon (2002), Karolyi (1995), King et al. (1994) and the references therein). Additionally, a copula model or a combination of a copula and an existing multivariate model has been used to investigate dependence between financial markets (Garcia and Tsafack (2011), Lee and Long (2009), and Rodriguez (2007), among others).

While these existing methods generally depend on parametric modeling of conditional variance, conditional correlation or copula of multivariate financial time series, researchers recently introduced some methods that do not require any modeling and focus directly on the quantile dependence of financial time series (Baruník and Kley (2015), Cappiello et al. (2014), Han et al. (2016), Li et al. (2015), Schmitt et al. (2015), and Sim and Zhou (2015)). These works provide various new methods to measure quantile dependence that is not captured by classical measures based on linear correlation. Some methods such as that used in Cappiello et al. (2014) test

contagion or constant correlation between financial time series, which can provide useful implications for asset allocation. However, little research has explored beyond basic measurement of quantile dependence between financial time series to investigate how to directly make use of measured quantile dependence in volatility forecasting, asset allocation or risk management.

The main motivation of this paper is to address this gap. We first measure detailed quantile dependence between stock markets and examine quantile-based directional predictability between stock markets. Using the quantile-based dependence and directional predictability, we introduce and evaluate a method to improve volatility forecasting in each stock market.

We consider the daily S&P 500 index, FTSE 100 index, DAX index, CAC 40 index and Nikkei 250 index and examine quantile dependence between the US stock return and stock return series for the UK, Germany, France and Japan, i.e. quantile dependence between US-UK, US-Germany, US-France or US-Japan bivariate stock market returns. To examine detailed quantile-based relationships between stock markets, we adopt the cross-quantilogram recently proposed by Han et al. (2016). The cross-quantilogram is a correlation statistic of quantile hit processes and measures dependence between the quantile range of one time series and the quantile range of the other time series. Therefore, it can provide quantile-based dependence between two financial markets. One can set up a cross-quantilogram for specific quantile ranges of interest or for an arbitrary large lag, and it is simple to interpret these results.

The results based on the cross-quantilogram show the following. First, negative spillover (left-tail dependence between stock markets) is stronger than positive spillover (right-tail dependence between stock markets). The cross-quantilogram has higher values and remains significant for larger lags when we consider left-tail dependence between stock markets. Second, there exists stronger quantile dependence or

directional predictability from the US stock market to the UK, Germany, France and Japan markets than the other way around. Third, when stock returns are devolatized and standardized residuals are used, directional predictability remains significant only at the first lag in the tail parts from the US market to other markets (UK, Germany, France or Japan), but it disappears from other markets (UK, Germany, France or Japan) to the US market.

Using these findings, we consider a simple way to improve volatility forecasting. In particular, we modify a volatility model to exploit the quantile-based directional predictability from the US market to markets in the UK, Germany, France and Japan. In a volatility model for stock markets in the UK, Germany, France and Japan, we introduce an additional multiplicative component that can be predicted from a tail event in the US stock market. We show that such quantile-augmented volatility models provide superior in-sample and out-of-sample volatility forecasts. We also find that our multiplicative model provides better volatility forecasts than the usual additive GARCH-X model.

The rest of the paper is organized as follows. Section 2 explains the cross-quantilogram and related Box-Ljung type test statistic. Sections 3 provides the data description and results on quantile dependence between stock markets. It presents results of auto-quantilogram and cross-quantilogram for stock return series and the standardized residual. Section 4 presents the application of quantile dependence to volatility forecasting and Section 5 concludes the paper.

2   Econometric Tool

Linton and Whang (2007) introduced the (auto-) quantilogram to measure dependence in different parts of the distribution of a stationary time series based on the

correlogram of quantile hits. Han et al. (2016) developed a multivariate version called the cross-quantilogram. The cross-quantilogram can be used 1) to measure quantile dependence between two series, 2) to test directional predictability between two series, and 3) to test model specification. They proposed and investigated the stationary bootstrap procedure and a self-normalized approach to construct the confidence intervals of the cross-quantilogram.

As explained in Linton and Whang (2007) and Han et al. (2016), the advantages of the cross-quantilogram are as follows: 1) it is simple to interpret, 2) no moment condition is required for time series, 3) it captures the properties of a joint distribution, 4) it can consider arbitrary lags. The second advantage is particularly important when we use the cross-quantilogram to analyze financial time series. While commonly used models such as multivariate GARCH models in general assume the existence of finite fourth moments of time series, it is well known that finite fourth moments do not exist for most stock return or exchange rate return series due to heavy tails.

We let $q_{i,t}(\tau_i)$ be either τ_i conditional or unconditional quantile of $y_{i,t}$. The cross-quantilogram measures dependence between two events $\{y_{1,t} < q_{1,t}(\tau_1)\}$ and $\{y_{2,t-k} < q_{2,t-k}(\tau_2)\}$ for an arbitrary pair of $\tau = (\tau_1, \tau_2)'$ and a positive integer k. In the literature, $\{1[y_{i,t} < q_{i,t}(\cdot)]\}$ is called the quantile-hit or quantile-exceedance process for $i = 1, 2$, where $1[\cdot]$ denotes the indicator function.

The cross-quantilogram is the cross-correlation of the quantile-hit processes and is defined as

$$\rho_\tau(k) = \frac{E\left[\psi_{\tau_1}\left(y_{1,t} - q_{1,t}(\tau_1)\right)\psi_{\tau_2}\left(y_{2,t-k} - q_{2,t-k}(\tau_2)\right)\right]}{\sqrt{E\left[\psi_{\tau_1}^2\left(y_{1,t} - q_{1,t}(\tau_1)\right)\right]}\sqrt{E\left[\psi_{\tau_2}^2(y_{2,t-k} - q_{2,t-k}(\tau_2))\right]}} \tag{1}$$

for $k = 0, \pm 1, \pm 2, \ldots$, where

$$\psi_{\tau_i}(y_{i,t} - q_{i,t}(\tau_i)) = 1[y_{i,t} < q_{i,t}(\tau_i)] - \tau_i.$$

Its sample counterpart is

$$\hat{\rho}_\tau(k) = \frac{\sum_{t=k+1}^T \psi_{\tau_1}(y_{1,t} - \hat{q}_{1,t}(\tau_1))\psi_{\tau_2}(y_{2,t-k} - \hat{q}_{2,t-k}(\tau_2))}{\sqrt{\sum_{t=k+1}^T \psi_{\tau_1}^2(y_{1,t} - \hat{q}_{1,t}(\tau_1))}\sqrt{\sum_{t=k+1}^T \psi_{\tau_2}^2(y_{2,t-k} - \hat{q}_{2,t-k}(\tau_2))}},$$

where $\hat{q}_{i,t}(\tau_i)$ is the estimate of either τ_i conditional or unconditional quantile of $y_{i,t}$. As an example, Figure 1 provides a pair of events: $\{y_{1,t} < q_{1,t}(\tau_1)\}$ for $\tau_1 = 0.05$ and $\{y_{2,t-k} < q_{2,t-k}(\tau_2)\}$ for $\tau_2 = 0.5$. Given $y_{2,t-k}$ is located below its median, the cross-quantilogram $\rho_\tau(k)$ is zero if the probability of $y_{1,t}$ being located below its 0.05 quantile is the same as 0.05.

Instead of two events $\{y_{1,t} < q_{1,t}(\tau_1)\}$ and $\{y_{2,t-k} < q_{2,t-k}(\tau_2)\}$, one may be interested in measuring the dependence between two events $\{q_{1,t}(\tau_1^l) < y_{1,t} < q_{1,t}(\tau_1^h)\}$ and $\{q_{2,t-k}(\tau_2^l) < y_{2,t-k} < q_{2,t-k}(\tau_2^h)\}$ for arbitrary quantile ranges $[\tau_1^l, \tau_1^h]$ and $[\tau_2^l, \tau_2^h]$. Figure 2 provides various events $\{q_{i,t}(\tau_i^l) < y_{i,t} < q_{i,t}(\tau_i^h)\}$ for different quantiles for τ_i^l and τ_i^h. To obtain the dependence of such events, one can use an alternative version of the cross-quantilogram that is defined by replacing $\psi_{\tau_i}(y_{it} - q_{i,t}(\tau_i))$ in (1) with

$$\psi_{[\tau_i^l, \tau_i^h]}(y_{it} - q_{i,t}([\tau_i^l, \tau_i^h])) = 1[q_{i,t}(\tau_i^l) < y_{it} < q_{i,t}(\tau_i^h)] - (\tau_i^h - \tau_i^l).$$

See footnote 4 in Han et al. (2016). This alternative version could be easier to interpret and therefore we will adopt this alternative version of the cross-quantilogram in this paper.

If $\rho_\tau(k) = 0$, there is no dependence or directional predictability from an event $\{q_{2,t-k}(\tau_2^l) \leq y_{2,t-k} \leq q_{2,t-k}(\tau_2^h)\}$ to an event $\{q_{1,t}(\tau_1^l) \leq y_{1,t} \leq q_{1,t}(\tau_1^h)\}$. If $\rho_\tau(k) \neq 0$, there exists quantile dependence or directional predictability between two events. If $\rho_\tau(k) > 0$, it is more likely for $y_{1,t}$ to be located in the range $[q_{1,t}(\tau_1^l), q_{1,t}(\tau_1^h)]$ when $y_{2,t-k}$ is located in the range $[q_{2,t-k}(\tau_2^l), q_{2,t-k}(\tau_2^h)]$. If $\rho_\tau(k) < 0$, it is less likely for $y_{1,t}$ to be located in the range $[q_{1,t}(\tau_1^l), q_{1,t}(\tau_1^h)]$ when $y_{2,t-k}$ is located in the range

$[q_{2,t-k}(\tau_2^l), q_{2,t-k}(\tau_2^h)]$. The stationary bootstrap inference procedure is still valid for this alternative version, as mentioned in Han et al. (2016) and, therefore, we will use it to construct confidence bands.

Using the cross-quantilogram, we can conduct related Portmanteau tests. Suppose that $\tau \in \mathcal{T}$ and p are given. One may be interested in testing

$$
\begin{aligned}
H_0 &: \quad \rho_\tau(1) = \cdots = \rho_\tau(p) = 0, \\
H_1 &: \quad \rho_\tau(k) \neq 0 \text{ for some } k \in \{1, \ldots, p\}.
\end{aligned}
$$

For this test, the Box-Pierce type test statistic $\hat{Q}_\tau^{(p)} = T \sum_{k=1}^{p} \hat{\rho}_\tau^2(k)$ can be used. We will use the Box-Ljung version $\check{Q}_\tau^{(p)} = T(T+2) \sum_{k=1}^{p} \hat{\rho}_\tau^2(k)/(T-k)$ in this paper because it has better finite sample performance for a large p and a small sample size. Han et al. (2016) also analyze the sup-version test statistic over a set of quantiles and the partial cross-quantilogram.

3 Quantilogram Analysis

3.1 Data and Setup

We investigate quantile dependence and directional predictability between the US stock market and stock markets in the UK, Germany, France and Japan, i.e. quantile dependence and directional predictability between US-UK, US-Germany, US-France and US-Japan bivariate stock market returns. We consider the daily S&P 500 index, FTSE 100 index, DAX index, CAC 40 index and Nikkei 250 index. To calculate the cross-quantilogram between the US stock return and the stock return series for the UK, Germany, France and Japan, we only consider days t for which we have observations from both indices for each pair. The sample period and sample size for

each pair of indices is given in Table 1.[1] We consider samples until the end of 2007 so that strict stationarity holds for the data. We demean each stock return series by subtracting its sample mean.

We let τ_i denote a quantile range $\left[\tau_1^l, \tau_1^h\right]$ in this section. The quantile range of stock return τ_i is set to be [0,0.05], [0.05,0.1], [0.1,0.2], [0.2,0.4], [0.4,0.6], [0.6,0.8], [0.8,0.9], [0.9,0.95] or [0.95,1]. We first let $\tau_1 = \tau_2$ for the next two subsections and consider the case with $\tau_1 \neq \tau_2$ later. We let lag $k = 1, \ldots, 20$. We use the stationary bootstrapping procedure by Politis and Romano (1994) to obtain confidence intervals based on 1,000 bootstrap replicates. The tuning parameter is chosen by adapting the rule suggested by Politis and White (2004) (and later corrected in Patton et al. (2009)).

3.2   Auto-Quantilogram and Cross-Quantilogram

We first examine the auto-quantilogram in the US stock market and the UK stock market. The results for the German, French or Japanese stock market are in general similar to those for the UK stock market and, therefore, we do not include them in the paper. Figures 3(a) and 3(b) show the auto-quantilogram and the Box-Ljung test statistic for the S&P 500 index return series. The auto-quantilogram is significantly positive at some lags for τ_1=[0,0.05], [0.4,0.6] or [0.95,1.0], which makes the Box-Ljung test statistic in Figure 3(b) significant for the same quantile range τ_1.

Figures 4(a) and 4(b) present the auto-quantilogram and the Box-Ljung test statistic for the FTSE 100 index return series. The results of the UK stock market are in general similar to those of the US stock market. For both tail parts (τ_1=[0,0.05] or [0.95,1.0]) and the mid-range (τ_1=[0.4,0.6]), the auto-quantilogram is significantly positive for some lags.

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[1]The data set is from realized library 0.1 by the Oxford-Man Institute.

Next, we investigate the cross-quantilogram between the US stock market and the UK stock market. Figures 5(a) and 5(b) provide the cross-quantilogram and the Box-Ljung test statistic from the US stock market to the UK stock market, i.e., $y_{1,t}$ is the FTSE 100 index return and $y_{2,t-k}$ is the S&P 500 index return. This shows that there exists directional predictability from the US market to the UK market for various quantile ranges. When we consider only the first lag, $k = 1$, the cross-quantilogram is significantly positive for τ_1=[0,0.05], [0.05,0.1], [0.1,0.2], [0.9,0.95] or [0.95,1.0].

It is not surprising to note that the quantile dependence is asymmetric. For the left-tail (τ_1=[0,0.05]), the cross-quantilogram exhibits much higher values and it is significant for larger lags. This implies that when there is a very large negative loss in the US stock market, it is more likely that there is also a very large loss in the UK stock market for quite a long time. Table 3 provides the value of $\hat{\rho}_{\tau_1}(1)$, the cross-quantilogram at the first lag, for both tail parts; it is 0.25 for the left-tail (τ_1=[0,0.05]) and 0.13 for the right-tail (τ_1=[0.95,1.0]). This implies that the negative spillover (risk spillover) is stronger than the positive spillover. Such an asymmetric relationship is in accordance with what we commonly observe in international stock markets.

Figures 6(a) and 6(b) present the cross-quantilogram and the Box-Ljung test statistic from the UK stock market to the US stock market, i.e., $y_{1,t}$ is the S&P 500 index return and $y_{2,t-k}$ is the FTSE 100 index return. Compared to the results in Figures 5(a) and 5(b), the dependence is much weaker. The cross-quantilogram in general has a lower value and is significant at some lags only for τ_1=[0,0.05], [0.4,0.6] or [0.95,1.0]. The cross-quantilogram from the UK market to the US market exhibits similar patterns to the auto-quantilogram for the US market in Figure 3(a).

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3.3 Results of Devolatized Return Series

The results in the previous subsection show that dependence or predictability still exists from the UK stock market to the US stock market despite it being much weaker than the case from the US market to the UK market. However, the auto-quantilogram in the US market exhibits similar patterns to the cross-quantilogram from the UK market to the US market, while it is obviously different from the cross-quantilogram from the US market to the UK market. Therefore, the quantile dependence from the UK stock return to the US stock return could be an artifact due to persistence and synchronicity in the marginal volatilities of the two stock return series. As discussed in Section 3 in Davis et al. (2013), this phenomenon is similar to the well-known issue with the cross-correlation function of linear bivariate time series. Unless one or all time series are whitened, the cross-correlation may appear to be spuriously significant (see Chapter 11 in Brockwell and Davis (1991)).

Hence, in this subsection, we devolatize each stock return series and examine the cross-quantilogram using standardized residuals. For each return series, we estimate the GJR-GARCH(1,1) model:

$$
\begin{aligned}
y_{i,t} &= \sigma_{i,t}\varepsilon_{i,t}, \\
\sigma_{i,t}^2 &= \omega + \alpha y_{i,t-1}^2 + \gamma y_{i,t-1}^2 I(y_{i,t-1} < 0) + \beta \sigma_{i,t-1}^2.
\end{aligned}
$$

We adopt the GJR-GARCH model to accommodate the asymmetric relationship between stock return and volatility. The innovation $\varepsilon_{i,t}$ is is assumed to be iid (0,1) and therefore the standardized residual $\hat{\varepsilon}_{i,t} = y_{i,t}/\hat{\sigma}_{i,t}$ is presumed to be serially uncorrelated. Testing serial correlation in the standardized residual is one of the most common ways to check model specification in the literature. Table 2 reports the 'usual' Ljung-Box Q-statistic based on auto-correlations of $\hat{\varepsilon}_{i,t}$ or $\hat{\varepsilon}_{i,t}^2$. For all stock

return series, the p-values of the Ljung-Box Q-statistic for lag 10 or 20 are larger than 0.05. This shows that $\hat{\varepsilon}_{i,t}$ or $\hat{\varepsilon}_{i,t}^2$ are serially uncorrelated and suggests that the GJR-GARCH model is an appropriate volatility model for this return series.

Now we use the standardized residual instead of the stock return series and conduct quantilogram analysis. Figures 7(a)-8(b) provide the auto-quantilogram and the Box-Ljung test statistic using the standardized residual $\hat{\varepsilon}_{i,t}$ for the US market or the UK market. The auto-quantilogram is insignificant in most cases for both stock markets, which is in accordance with the results of the 'usual' Ljung-Box Q-statistic on $\hat{\varepsilon}_{i,t}$ or $\hat{\varepsilon}_{i,t}^2$ in Table 2 and suggests the GJR-GARCH model is appropriate for modeling each stock return series.

Figures 9(a) and 9(b) present the cross-quantilogram and the Box-Ljung test statistic from the US market to the UK market using the standardized residual, i.e. $y_{1,t}$ is the standardized residual for the FTSE 100 index return and $y_{2,t-k}$ is that for the S&P 500 index return. The cross-quantilogram has a large positive value at the first lag for the left-tail (τ_1=[0,0.05]), while it is mostly insignificant in the rest of the cases. Even after devolatizing the returns series, there still exists directional predictability from the US market to the UK market in the left-tail. Figures 10(a) and 10(b) provide the cross-quantilogram and the Box-Ljung test statistic from the UK market to the US market using the standardized residual, i.e. $y_{1,t}$ is the standardized residual for the S&P 500 index return and $y_{2,t-k}$ is that for the FTSE 100 index return. The cross-quantilogram is insignificant in almost all cases and consequently the Box-Ljung test statistic is insignificant in all cases.

When we devolatize only one stock return series, the results are in general similar. For example, when $y_{1,t}$ is the standardized residual for the FTSE 100 index return and $y_{2,t-k}$ is the S&P 500 index return itself, predictability still exists at the first lag for the left-tail from the US market to the UK market. However, when $y_{1,t}$ is the

standardized residual for the S&P 500 index return and $y_{2,t-k}$ is the FTSE 100 index return itself, no predictability exists from the UK market to the US market in all quantile ranges. It should be emphasized that the directional predictability from the US market depends on quantile ranges (i.e., exists only in the left-tail). In the next section, we will exploit such a quantile dependence from the US market to the UK market to improve volatility forecasting in the UK stock market.

When one or both stock return series is devolatized, directional predictability still appears from the US market to the UK market in the left tail, but disappears from the UK market to the US market in all quantile ranges. This could be due to the dominance of the US stock market. Another possibility is the difference in stock market opening times. The stock market opening times are Japan (00:00-06:00), UK/Germany/France (08:00-16:30) and US (14:30-21:00) in GMT. There are two hours of overlap between the European and US stock market opening times. One may surmise that a shock in the UK market on day t will be transmitted to the US market on the same day and, consequently, directional predictability will disappear from the UK market to the US market at the first lag. However, this does not make sense considering that the US-Japan case presented in Tables 2 and 3 shows similar results as the US-UK case despite no overlap between the US and Japan stock market opening times. We conjecture that the market dominance of the US causes large significant values of the cross-quantilogram at the first lag for tails from the US market to each stock market in Europe and Japan.

When we replace the UK stock market with the German or French stock market, the cross-quantilogram exhibits similar patterns as the US-UK case. Table 3 provides the cross-quantilograms at the first lag from the US stock market to each stock market and Table 4 presents those from each stock market to the US stock market. For example, when we consider the US-Germany case, we observe the following: 1)

dependence is stronger for the case from the US market to the German market than the other way around, 2) the negative spillover is stronger than the positive spillover, 3) when the standardized residual is used, directional predictability still exists in both tails from the US market to the German market, but disappears from the German market to the US market.

There is an interesting difference in the US-Japan case. The positive spillover from the US market to the Japanese market is stronger than the negative spillover, i.e., $\hat{\rho}_\tau(1) = 0.18$ for $\tau_1=[0,0.05]$ and $\hat{\rho}_\tau(1) = 0.21$ for $\tau_1=[0.95,1.0]$, whereas the negative spillover is stronger from the US market to three European markets. Figures 11(a) and 11(b) present the cross-quantilogram from the US market to the Japanese market. In Figure 11(a), the cross-quantilogram is also significantly positive at the first lag for $\tau_1=[0.9,0.95]$ while it is insignificant for $\tau_1=[0.05,0.1]$. When we use the standardized residuals from the GJR-GARCH model, Figure 11(b) shows that the cross-quantilogram is significantly positive for both tails at the first lag.

3.4   Results of Cross-Quantile Ranges

Instead of letting $\tau_1 = \tau_2$, we now consider the case with $\tau_1 \neq \tau_2$. We let the quantile range of the US stock market τ_2 be either [0,0.05] or [0.95,1.0]. We set the quantile range of the UK stock market τ_1 to be [0,0.05], [0.05,0.1], [0.1,0.2], [0.2,0.4], [0.4,0.6], [0.6,0.8], [0.8,0.9], [0.9,0.95] or [0.95,1] as in previous subsections.

First, we examine dependence and directional predictability from the left-tail event in the US market to various quantile ranges of the UK stock market. Figure 12(a) presents the cross-quantilogram from the US market to the UK market, i.e., $y_{1,t}$ is the FTSE 100 index return, $y_{2,t-k}$ is the S&P 500 index return and $\tau_2=[0,0.05]$. The first plot in the first row in Figure 12(a) is identical to that in Figure 5(a) where

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$\tau_1 = \tau_2 = [0,0.05]$. For mid-quantile ranges of the UK market ($\tau_1 = [0.2,0.4]$, $[0.4,0.6]$ or $[0.6,0.8]$), the cross-quantilogram is significantly negative for some lags. This means that it is less likely for the UK stock return to be located in mid-quantile ranges when there is a large loss in the US market at day $t - k$. For the right-tail of the UK stock market ($\tau_1 = [0.95,1]$), the cross-quantilogram is close to zero and insignificant at the first lag but it is mostly significantly positive from the second lag to the last lag. This could be due to the bouncing effect after a large negative shock. It is interesting to note that values of the cross-quantilogram are higher in the right-tail than in the left-tail from the second lag, while the value is very high in the left-tail only at the first lag.

Second, we consider the case from the right-tail event in the US market. Figure 12(b) presents the cross-quantilogram from the US market to the UK market, i.e., $y_{1,t}$ is the FTSE 100 index return, $y_{2,t-k}$ is the S&P 500 index return and $\tau_2 = [0.95,1]$. The last plot in the third row in Figure 12(b) is identical to that in Figure 5(a) where $\tau_1 = \tau_2 = [0.95,1]$. In general, the dependence is weaker than the case in Figure 12(a). On various quantile ranges of the UK stock market return, a large negative shock in the US stock market has a stronger influence than a large positive shock. For $\tau_1 = [0.9,0.95]$, the cross-quantilogram is significantly positive at the first lag. The figure shows that, when there is a large gain in the US stock market, it is more likely for the UK stock market to have a large or a relatively large gain on the next day.

Next, we use the standardized residuals from the GJR-GARCH model and examine the same cross-quantile range aspects. When the standardized residuals are used, the cross-quantilogram is mostly insignificant except for some quantile ranges at the first lag. Figure 13(a) considers the left-tail case corresponding to Figure 12(a). At the first lag, the cross-quantilogram is significantly positive for $\tau_1 = [0,0.05]$, $[0.05,0.1]$ or $[0.1,0.2]$. Figure 13(b) presents the right-tail case corresponding to Figure 12(b).

The cross-quantilogram is mostly close to zero and insignificant.

4 Application in Volatility Forecasting

4.1 Quantile-Augmented Volatility Model

In this section, we consider a method that uses the findings in the previous section to improve volatility forecasting. The cross-quantilogram analysis in the previous section shows that there exists directional predictability from a tail event in $y_{2,t-1}$, i.e., US stock return at day $t-1$ for $\tau_2 = [0, 0.05]$ or $[0.95, 1]$, to the standardized residual $\hat{\varepsilon}_{1,t}$ in each market in the UK, Germany, France and Japan. This result suggests that we can decompose $\hat{\varepsilon}_{1,t}$ into two parts; one is a predictable component from a tail event in the US market and the other is an unpredictable component. It will be more desirable to accommodate such a predictable component from a tail event in the US market in modeling volatility in each stock market in the UK, Germany, France and Japan.

It is possible to decompose the standardized residual in a multiplicative way such that $\hat{\varepsilon}_{1,t} = \sqrt{\hat{f}_{1,t}}\hat{\eta}_{1,t}$ where $f_{1,t}$ is the predictable component from a tail event in $y_{2,t-1}$ and $\eta_{1,t}$ is an unpredictable component. Using this, we consider the following volatility model for stock return series $y_{1,t}$ of each market in the UK, Germany, France and Japan;

$$y_{1,t} = \sqrt{h_{1,t}f_{1,t}}\eta_{1,t}$$

where $h_{1,t}$ is a base volatility model such as the GJR-GARCH model, $f_{1,t}$ is a function of a tail event in $y_{2,t-1}$ and $\eta_{1,t}$ is iid (0,1). The return series in each market has three multiplicative components. The first component $h_{1,t}$ is a function of past values of $y_{1,t}$ and it is possible to specify it as an another GARCH-type model. We can model

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the second component $f_{1,t}$ in various ways including nonparametric methods. In this paper, we consider the following simple specification:

$$f_{1,t}(\delta) = \delta_0 + \delta_1 y_{2,t-1}^2 I(y_{2,t-1} \leq q_2(0.05)) + \delta_2 y_{2,t-1}^2 I(y_{2,t-1} \geq q_2(0.95)) \qquad (2)$$

where $y_{2,t}$ is the return series in the US stock market and $q_2(0.05)$ or $q_2(0.95)$ is 0.05 or 0.95 quantile of $y_{2,t}$, respectively. In this manner, the conditional variance of $y_{1,t}$ is augmented as

$$\sigma_{1,t}^2 = h_{1,t} \times f_{1,t}.$$

If the stock index return in US is below the 5% quantile or above the 95% quantile, a positive value of δ_1 or δ_2 will make the volatility in each stock market higher on the next day. We call this model the *quantile-augmented* volatility model (QA model). Compared to a base volatility model, the QA model accommodates additional information from the US market in the previous day (particularly a tail event). We expect that this augmented model will provide better volatility forecasts than a base volatility model.

Another way to accommodate the directional predictability from the US market to each stock market in volatility modeling is to adopt the following additive GARCH-X model;

$$y_{1,t} = \sqrt{h_{1,t}} \eta_{1,t} \qquad (3)$$

where

$$h_{1,t} = \omega + \alpha y_{1,t-1}^2 + \gamma y_{1,t-1}^2 I(y_{t-1} < 0) + \beta h_{1,t-1}$$
$$+ \delta_1 y_{2,t-1}^2 I(y_{2,t-1} \leq q_2(0.05)) + \delta_2 y_{2,t-1}^2 I(y_{2,t-1} \geq q_2(0.95))$$

and $\eta_{1,t}$ is *iid* (0,1). The GARCH-X model is a typical way to accommodate exogenous covariates in volatility modeling (see Han and Kristensen (2015) and references

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therein).

Now we discuss the estimation method of the QA model. We can rearrange the model

$$y_{1,t} = \sqrt{h_{1,t}\left(\theta\right) f_{1,t}\left(\delta\right)}\eta_{1,t} \text{ for } \eta_{1,t} \sim iid(0,1),$$

into

$$y_{1,t}^2/h_{1,t}\left(\theta\right) = f_{1,t}\left(\delta\right) + u_{1,t}$$

where $u_{1,t} = f_{1,t}\left(\delta\right)\left(\eta_{1,t}^2 - 1\right)$. Here $u_{1,t}$ is a Martingale difference sequence. The estimation procedure is as follows:

1. Estimate θ in the base model $h_{1,t}\left(\theta\right)$, for example the GJR-GARCH model, using the quasi-maximum likelihood estimation (QMLE) method from

$$y_{1,t} = \sqrt{h_{1,t}\left(\theta\right)}\varepsilon_{1,t} \text{ for } \varepsilon_{1,t} \sim iid(0,1).$$

2. Rescale the squared return and estimate δ in the following model using the least squares method

$$y_{1,t}^2/h_{1,t}(\hat{\theta}) = f_{1,t}\left(\delta\right) + u_{1,t}.$$

3. Use the estimates from the previous steps and obtain

$$\hat{\sigma}_{1,t}^2 = h_{1,t}(\hat{\theta}) \times f_{1,t}(\hat{\delta}).$$

4.2 Forecast Evaluation Method

We evaluate the within-sample and out-of-sample predictive power of the quantile-augmented volatility model. We will compare the within-sample and out-of-sample forecasts of the base model (GJR-GARCH, $\hat{\sigma}_{t,base}^2 = \hat{h}_t$) and the QA model ($\hat{\sigma}_{t,QA}^2 = \hat{h}_t \times \hat{f}_t$). To evaluate the volatility forecast, we adopt the following standard procedure.

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First, we use the realized kernel as a proxy for actual volatility. Barndorff-Nielsen et al. (2008) introduced the realized kernel and it has some robustness to market microstructure noise. The realized kernels of the return series are available in the 'Oxford-Man Institute's realised library' database produced by Heber et al. (2009).

Second, we use the QLIKE loss function defined as

$$L(\hat{\sigma}_t^2, \sigma_t^2) = \frac{\sigma_t^2}{\hat{\sigma}_t^2} - \log \frac{\sigma_t^2}{\hat{\sigma}_t^2} - 1 \tag{4}$$

where (σ_t^2) is the proxy for actual volatility and $(\hat{\sigma}_t^2)$ is the within-sample or out-of-sample volatility forecast. Even if realized measures are known to be better measures, they are imperfect and noisy proxies for actual volatility. There has been research on loss functions that are robust to the use of a noisy volatility proxy (see Hansen and Lunde (2006), Patton (2010) and Patton and Sheppard (2009)). Patton (2010) shows that the QLIKE loss function is robust and, in particular, Patton and Sheppard (2009) show in their simulation study that the QLIKE loss function has the highest power.

Third, the significance of any difference in the QLIKE loss is tested via a Diebold-Marinao and West (DMW) test. See Diebold-Marinao (1995) and West (1996). A DMW statistic is computed using the difference in the losses of two models

$$
\begin{aligned}
d_t &= L(\hat{\sigma}_{t,base}^2, \sigma_t^2) - L(\hat{\sigma}_{t,QA}^2, \sigma_t^2) \\
DMW_T &= \frac{\sqrt{T}\bar{d}_T}{\sqrt{\widehat{avar}\left(\sqrt{T}\bar{d}_T\right)}}
\end{aligned}
\tag{5}
$$

where $\bar{d}_T$ is the sample mean of d_t and T is the number of forecasts. The asymptotic variance of the average is computed using a Newey-West variance estimator with the number of lags set to $\left[T^{1/3}\right]$. If DMW_T is positive, it means that our quantile-augmented model has a smaller loss than the base model. The DMW test for equal

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predictability is for

$$H_0 : \mathbb{E}\left[d_t\right] = 0$$

and the asymptotic distribution of the test statistic is standard normal under the null hypothesis.

4.3 Forecast Evaluation Results

We first compare fitted values of volatility for the entire sample period. Table 5 shows the DMW test results for each series. In all cases, the DMW test statistics are positive and statistically significant at the 1% level. This shows that our quantile-augmented model significantly outperforms the GJR-GARCH model.

Next we compare one-step ahead out-of-sample forecasts. We adopt the rolling window procedure with a moving window of eight years (2016 days) and produce one-step ahead out-of-sample forecasts. The forecast period and number of forecasts for each series are given in Table 6.

Table 5 shows the DMW test results for the out-of-sample forecasts. The results are similar to those for the in-sample comparison. The quantile-augmented model significantly outperforms the GJR-GARCH model. Both in-sample and out-of-sample comparison results show that a simple augmented model using quantile dependence and directional predictability from the US market can significantly improve volatility forecasting.

Remark 1: Instead of a tail event in $y_{2,t-1}$, one may use a tail event in $\hat{\varepsilon}_{2,t-1}$ that is the standardized residual of the GJR-GARCH model for $y_{2,t-1}$. Accordingly, we can adjust $f_{1,t}$ as follows:

$$f_{1,t}\left(\delta\right) = \delta_0 + \delta_1 \hat{\varepsilon}_{2,t-1}^2 I(\hat{\varepsilon}_{2,t-1} \leq q_2\left(0.05\right)) + \delta_2 \hat{\varepsilon}_{2,t-1}^2 I(\hat{\varepsilon}_{2,t-1} \geq q_2\left(0.95\right))$$

where $q_2(0.05)$ or $q_2(0.95)$ are 0.05 or 0.95 quantile of $\hat{\varepsilon}_{2,t}$, respectively. We still obtain similar results. For all cases, the quantile-augmented model significantly outperforms the base model in both in-sample and out-of-sample forecasts.

Remark 2: We consider two different base models instead of the GJR-GARCH model and conduct the same in-sample and out-of-sample forecast evaluations. One is the GJR-GARCH model with t-distribution, in which the innovation $\varepsilon_{1,t}$ follows the t-distribution. The other is the HEAVY model by Shephard and Sheppard (2010). Specifically, we use their HEAVY-r model:

$$
\begin{aligned}
y_{1,t} &= \sigma_{1,t}\varepsilon_{1,t} \\
\sigma_{1,t}^2 &= \omega + \beta\sigma_{1,t-1}^2 + \pi RM_{1,t-1}
\end{aligned}
$$

where $RM_{1,t}$ is the realized volatility measure of $y_{1,t}$ at time t. Shephard and Sheppard (2010) and Hansen et al. (2012) show that this GARCH-X type model using a realized volatility measure as the covariate performs better than the standard GARCH model. Following Shephard and Sheppard (2010), we use the realized kernel as $RM_{1,t}$. Tables 7 and 8 show the results of in-sample and out-of-sample forecast comparisons using the alternative base models. They show that the quantile-augmented approach still significantly improves volatility forecasting.

Remark 3: We consider the additive GARCH-X model given in (3). When we compare in-sample forecasts, the additive GARCH-X model provides lower QLIKE losses than the GJR-GARCH model, but the DMW test statistics are in general insignificant. This shows that the additive GARCH-X model is not as effective as the multiplicative approach in our model.

Remark 4: We apply the same quantile-augmented approach in volatility mod-

eling of the US stock return. For the US stock return $y_{2,t}$, we consider

$$y_{2,t} = \sqrt{h_{2,t} f_{2,t}} \eta_{2,t}$$

where $h_{2,t}$ is the GJR-GARCH model, $\eta_{2,t}$ is iid (0,1) and

$$f_{2,t} = \delta_0 + \delta_1 y_{1,t-1}^2 I(y_{1,t-1} \leq q_1(0.05)) + \delta_2 y_{1,t-1}^2 I(y_{1,t-1} \geq q_1(0.95)).$$

$y_{1,t}$ is the stock return of one of the markets in the UK, Germany, France and Japan, and $q_1(0.05)$ and $q_1(0.95)$ are the 0.05 and 0.95 quantile of $y_{1,t}$, respectively. Since the cross-quantilogram analysis in Section 3 shows that there is no quantile dependence or directional predictability from each market (UK, Germany, France or Japan) to the US market after devolatizing, there is no reason to expect that the quantile-augmented model outperforms the base model in this case. When we compare in-sample forecasts, the quantile-augmented model does not provide any significant improvement: DMW test statistics are either insignificantly positive or significantly negative. This confirms that the quantile-augmented approach should be based on the quantile dependence or directional predictability revealed in cross-quantilogram analysis.

5   Conclusion

The paper examines quantile dependence and directional predictability between international stock markets and investigates how to apply these measures in volatility forecasting. We consider dependence between the US stock return and stock return series in the UK, Germany, France and Japan, i.e., quantile dependence between US-UK, US-Germany, US-France and US-Japan bivariate stock market returns. The results based on the cross-quantilogram show that the negative spillover is in general much stronger than the positive spillover, while, exceptionally, positive spillover is

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stronger in the case from the US stock market to the Japanese stock market. We apply the cross-quantilogram on standardized residuals as well as stock return series. There exists directional predictability from the US stock market to markets in the UK, Germany, France and Japan. In particular, tail events in the US stock market influence these stock markets. However, when standardized residuals are used, there is no directional predictability from markets in the UK, Germany, France and Japan to the US market. Using these results on quantile dependence and directional predictability, we consider a simple method to improve volatility forecasting in stock markets in the UK, Germany, France and Japan. The quantile-augmented volatility model significantly improves both in-sample and out-of-sample volatility forecasting, which is robust to the choice of a base volatility model.

Recently, researchers have developed various methods to measure quantile dependence between financial time series. This paper considers a simple method to make use of quantile dependence in order to improve volatility forecasting. The information provided on detailed quantile dependence can be used for various purposes, such as modeling univariate or multivariate volatility and estimating value at risk. We leave it as future work to develop more sophisticated methods that apply quantile dependence in asset allocation and risk management.

A   Tables and Figures

Table 1. Sample period and sample size for each pair of stock return series

Pair of indices	Sample period (sample size)
FTSE - S&P 500	21 Oct. 1997 - 31 Dec. 2007 (2470)
DAX - S&P 500	3 Jan. 1996 - 28 Dec. 2007 (2907)
CAC - S&P 500	3 Jan. 1996 - 31 Dec. 2007 (2908)
Nikkei - S&P 500	8 Jan. 1996 - 27 Dec. 2007 (2763)

Table 2. Results of the 'usual' Ljung-Box Q-statistic

		S&P	FTSE	DAX	CAC	Nikkei
$\hat{\varepsilon}_t$	p-value of Q(10)	0.19	0.67	0.99	0.11	0.63
	p-value of Q(20)	0.17	0.55	0.86	0.39	0.80
$\hat{\varepsilon}_t^2$	p-value of Q(10)	0.81	0.59	0.08	0.31	0.39
	p-value of Q(20)	0.70	0.28	0.19	0.25	0.35

Note: The table reports the Ljung-Box Q-statistic on $\hat{\varepsilon}_t$ or $\hat{\varepsilon}_t^2$, where $\hat{\varepsilon}_t$ is the standardized resisidual from the GJR-GARCH model.

Table 3. Cross-quantilograms at the first lag from the US market to other markets

	$\tau_1 \ (= \tau_2)$	FTSE	DAX	CAC	Nikkei
Return	$[0, 0.05]$	0.25*	0.17*	0.20*	0.18*
	$[0.95, 1]$	0.13*	0.12*	0.11*	0.21*
Std. residual	$[0, 0.05]$	0.16*	0.14*	0.14*	0.14*
	$[0.95, 1]$	0.04	0.06*	0.03	0.15*

Note: The table reports $\hat{\rho}_\tau(1)$, a sample cross-quantilogram at the first lag, from the US stock market to other stock markets, i.e., $y_{1,t}$ is the return series of FTSE, DAX, CAC or Nikkei and $y_{2,t-1}$ is the S&P 500 index return. The second and third rows are the cases where stock return series are used. The fourth and fifth rows are the cases where standardized residuals from the GJR-GARCH model are used. * indicates singificance at the 5% level.

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Table 4. Cross-quantilograms at the first lag from other markets to the US market

	$\tau_1 \ (= \tau_2)$	FTSE	DAX	CAC	Nikkei
Return	[0, 0.05]	0.06*	0.08*	0.06*	0.05*
	[0.95, 1]	0.03	0.06*	0.06	-0.01
Std. residual	[0, 0.05]	0.02	0.01	-0.00	0.02
	[0.95, 1]	-0.02	-0.01	0.01	-0.04*

Note: The table reports $\hat{\rho}_\tau(1)$, a sample cross-quantilogram at the first lag, from each stock market to the US stock market, i.e., $y_{1,t}$ is the S&P 500 index return and $y_{2,t-1}$ is the return series of FTSE, DAX, CAC or Nikkei. Same as Table 3.

Table 5. DMW test results for GJR-GARCH

	FTSE	DAX	CAC	Nikkei
In-sample	3.29**	3.13**	4.03**	3.59**
Out-of-sample	7.67**	10.88**	12.90**	10.01**

Note: The table reports the DMW statistics given in (5). The base model is the GJR-GARCH model. ** indicates that the null hypothesis of equal predictability between the base model and the quantile-augmented model is rejected at the 1% significance level.

Table 6. Out-of-sample forecast period and number of forecasts

Index	Forecast period (number of forecasts)
FTSE	2 Mar. 2006 - 31 Dec. 2007 (454 forecasts)
DAX	1 June 2004 - 31 Dec. 2007 (891 forecasts)
CAC	2 June 2004 - 31 Dec. 2007 (892 forecasts)
Nikkei	29 Oct. 2004 - 31 Dec. 2007 (746 forecasts)

Note: The table reports the out-of-sample forecast period and number of forecasts for each return series. For each return series, one-step ahead out-of-sample forecasts are produced via the rolling window procedure with a moving window of eight years (2016 days).

Table 7. DMW test results for GJR-GARCH with t-distribution

	FTSE	DAX	CAC	Nikkei
In-sample	2.96**	2.85**	3.96**	2.91**
Out-of-sample	7.28**	8.71**	12.56**	8.98**

Note: The base model is the GJR-GARCH model with t-distribution. Same as Table 5.

Table 8. DMW test results for HEAVY

	FTSE	DAX	CAC	Nikkei
In-sample	3.78**	3.20**	4.86**	2.89**
Out-of-sample	7.22**	8.19**	10.84**	8.43**

Note: The base model is the HEAVY-r model by Shephard and Sheppard (2010). Same as Table 5.
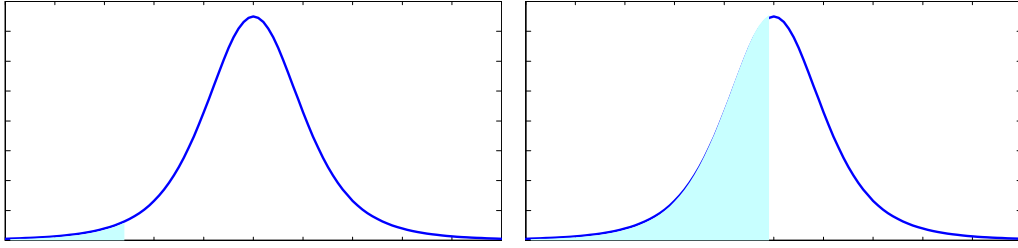
Figure 1. Event $\{y_{i,t} < q_{i,t}(\tau_i)\}$. The left figure describes an event $\{y_{1,t} < q_{1,t}(\tau_1)\}$ for $\tau_1 = 0.05$ and the right figure provides an event $\{y_{2,t-k} < q_{2,t-k}(\tau_2)\}$ for $\tau_2 = 0.5$.
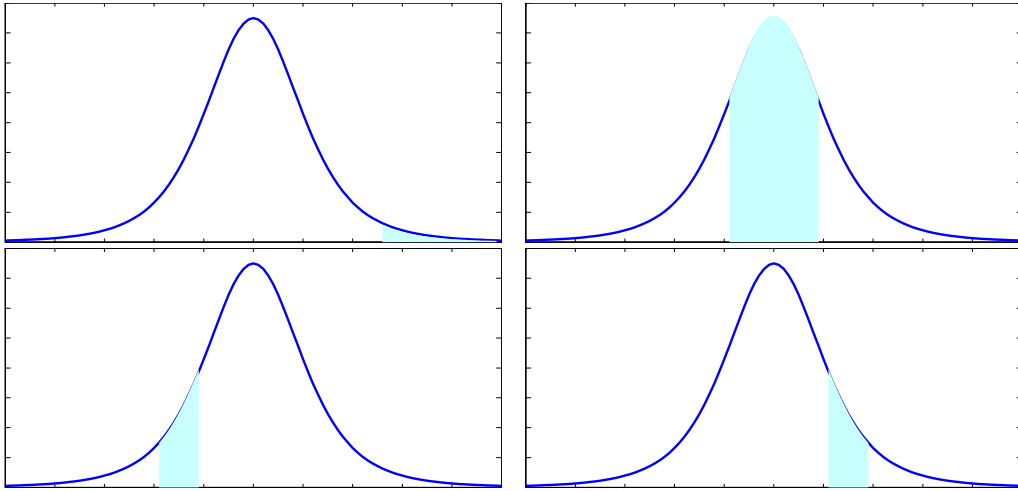
Figure 2. Event $\{q_{i,t}(\tau_i^l) < y_{i,t} < q_{i,t}(\tau_i^h)\}$. The figures describe various events $\{q_{i,t}(\tau_i^l) < y_{i,t} < q_{i,t}(\tau_i^h)\}$ for different quantiles for τ_i^l and τ_i^h. The top left figure provides a right-tail event and the top right figure gives a mid-range event. The bottom figures present events for the left and right shoulders of the distribution.
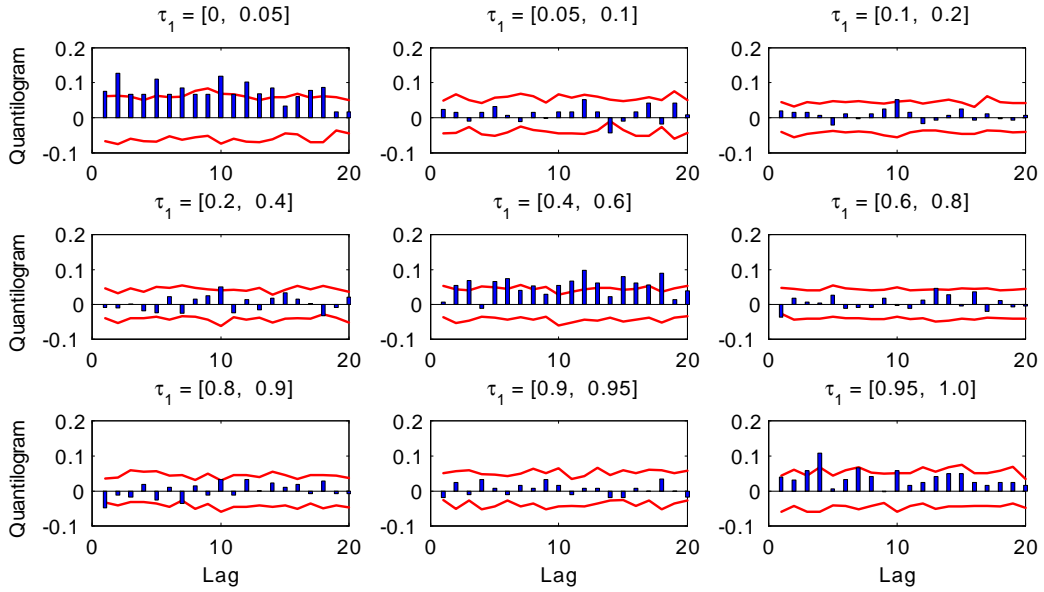
Figure 3(a). [US] Auto-quantilogram $\hat{\rho}_\tau(k)$ of the S&P 500 index return series. τ_1 is the quantile range. Bar graphs describe sample cross-quantilograms and lines are the 95% bootstrap confidence intervals centered at zero.
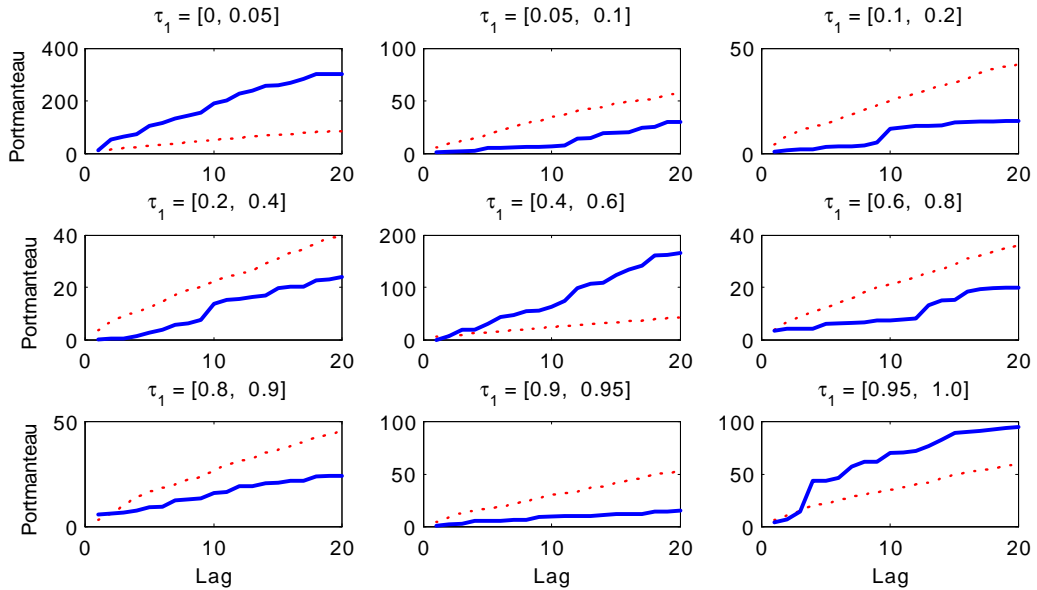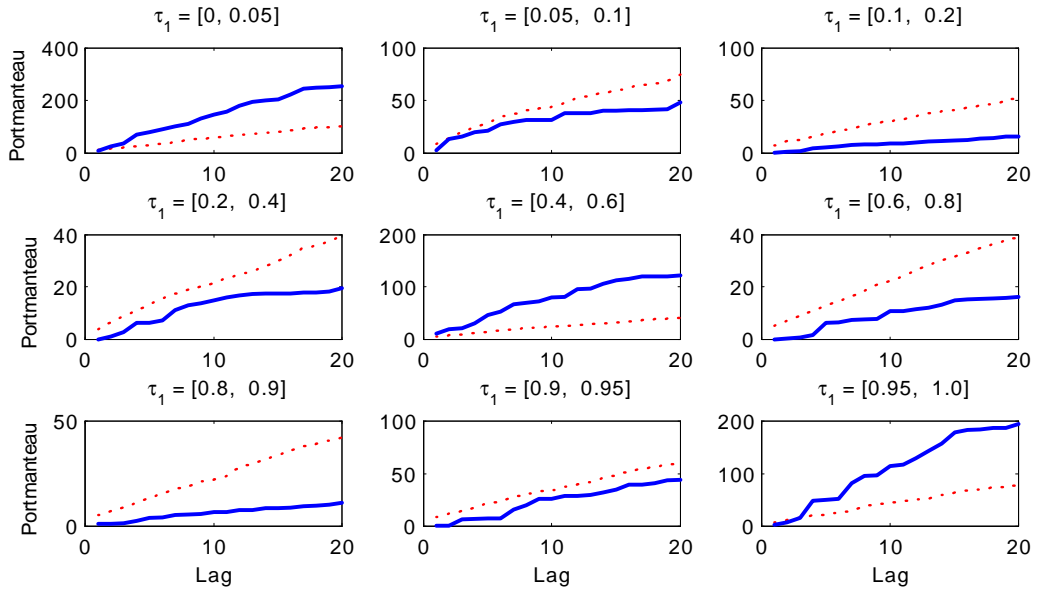

Figure 3(b). [US] Box-Ljung test statistic $\hat{Q}_\tau^{(p)}$ for each lag p using $\hat{\rho}_\tau(k)$. Same as Figure 1(a). The dashed lines are the 95% bootstrap confidence intervals centered at zero.
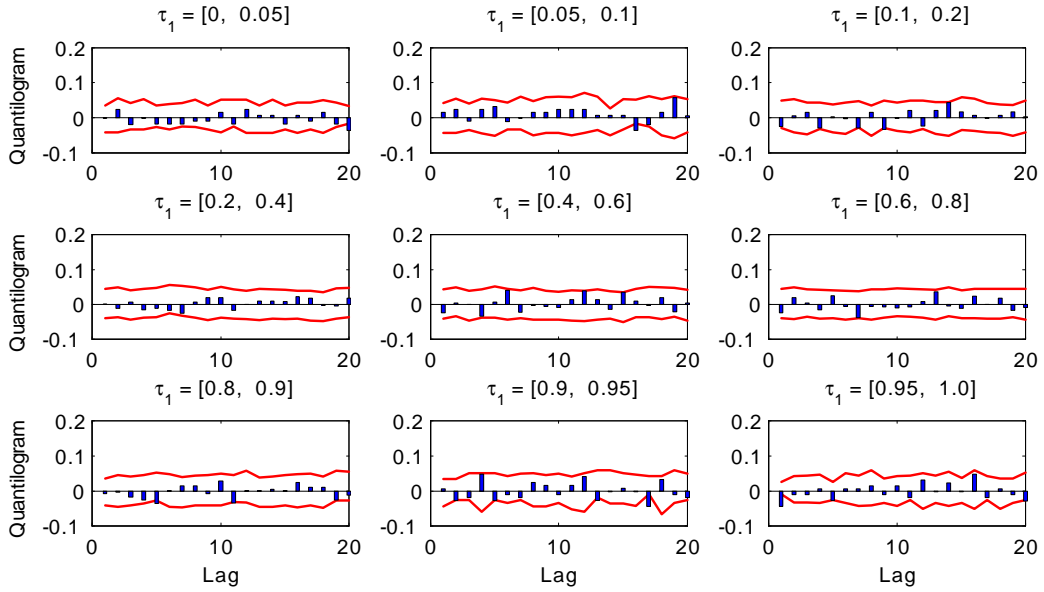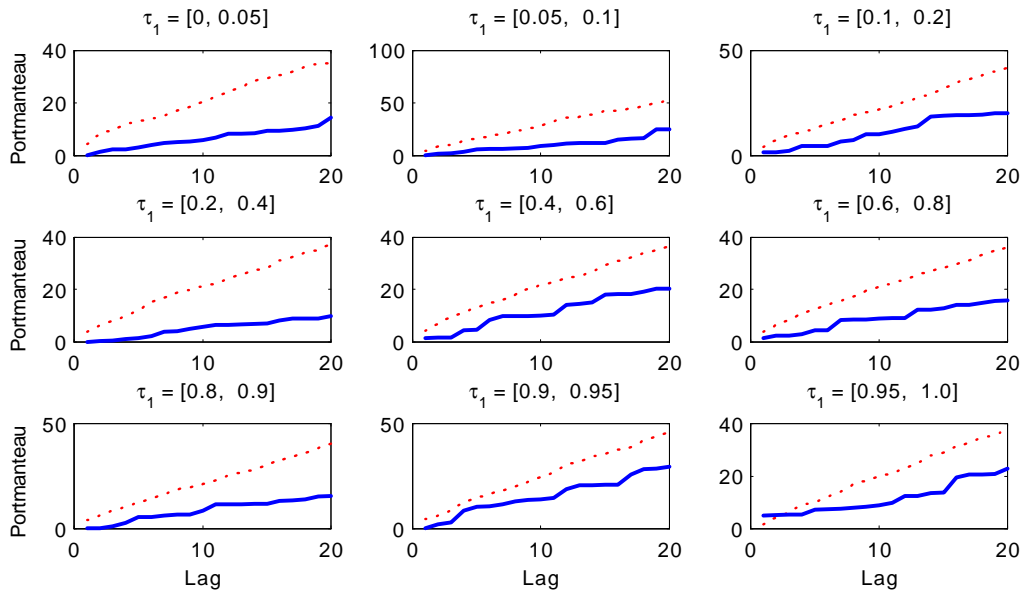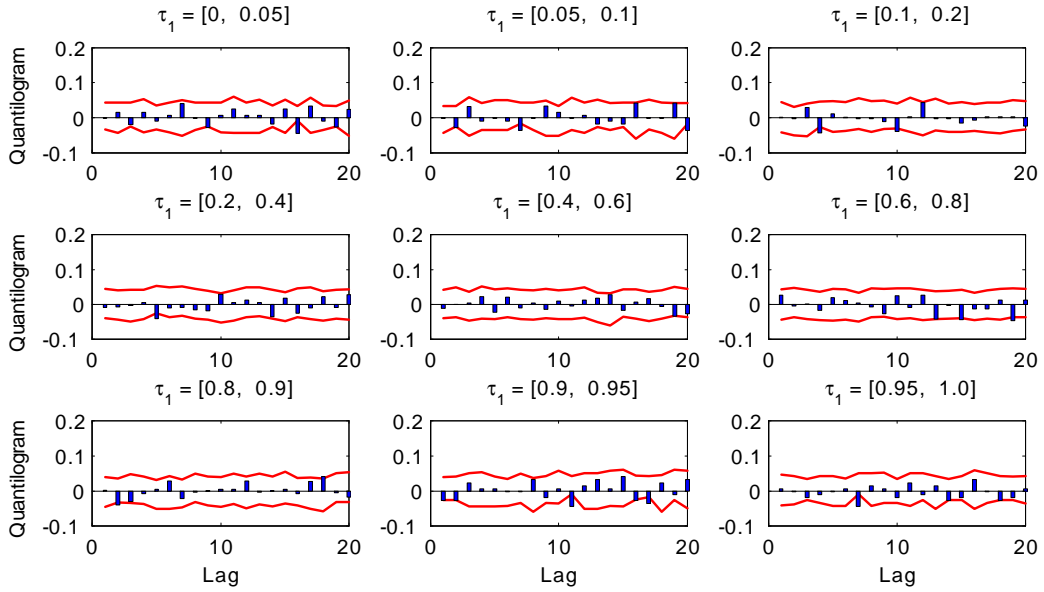
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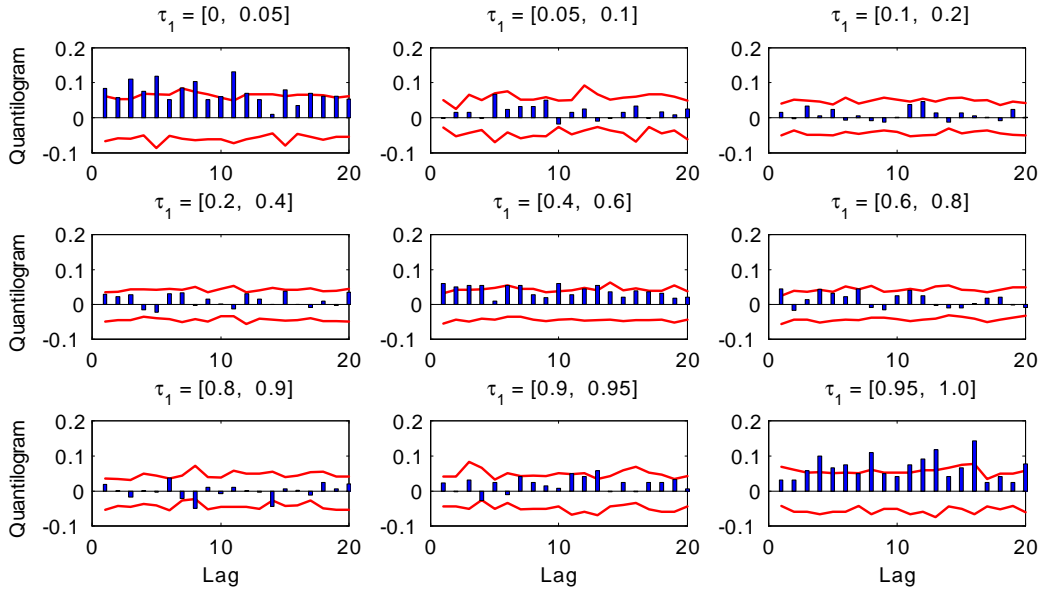
Figure 4(a). [UK] Auto-quantilogram $\hat{\rho}_\tau(k)$ of the FTSE index return series. Same as Figure 1(a).
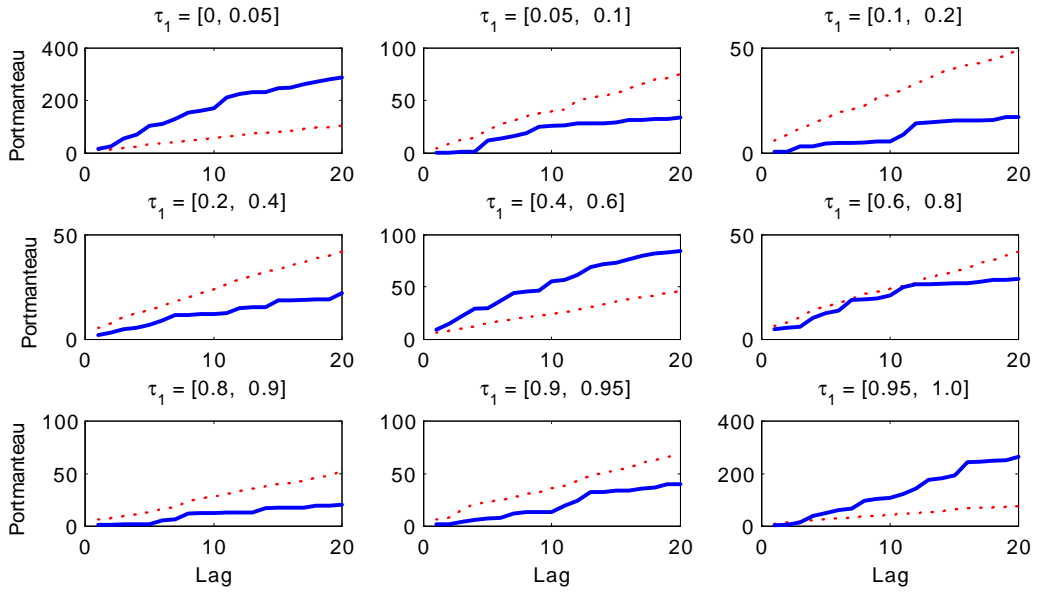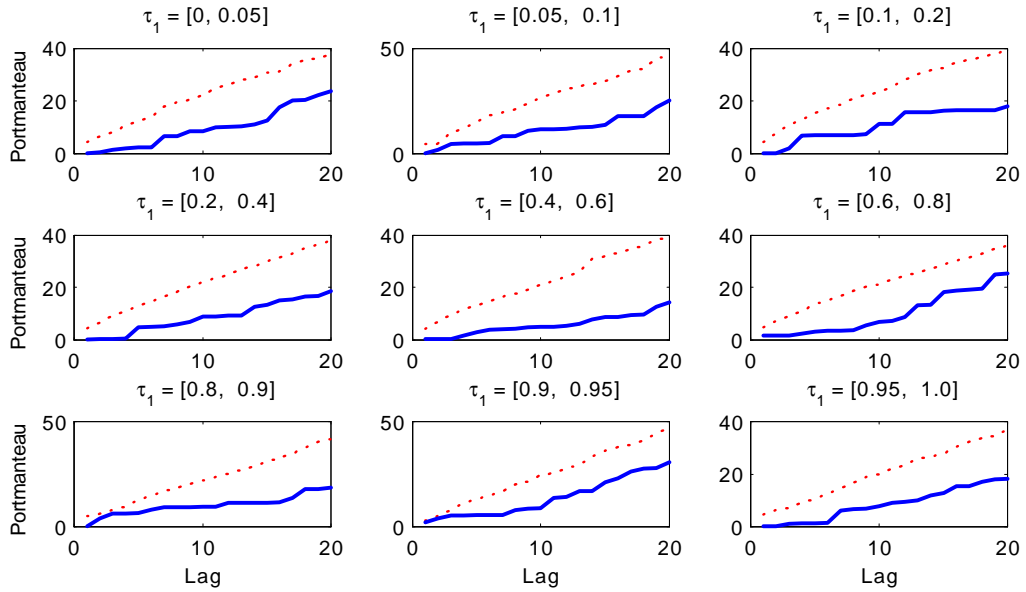

Figure 4(b). [UK] Box-Ljung test statistic $\hat{Q}_\tau^{(p)}$ for each lag p using $\hat{\rho}_\tau(k)$. Same as Figure 1(b).
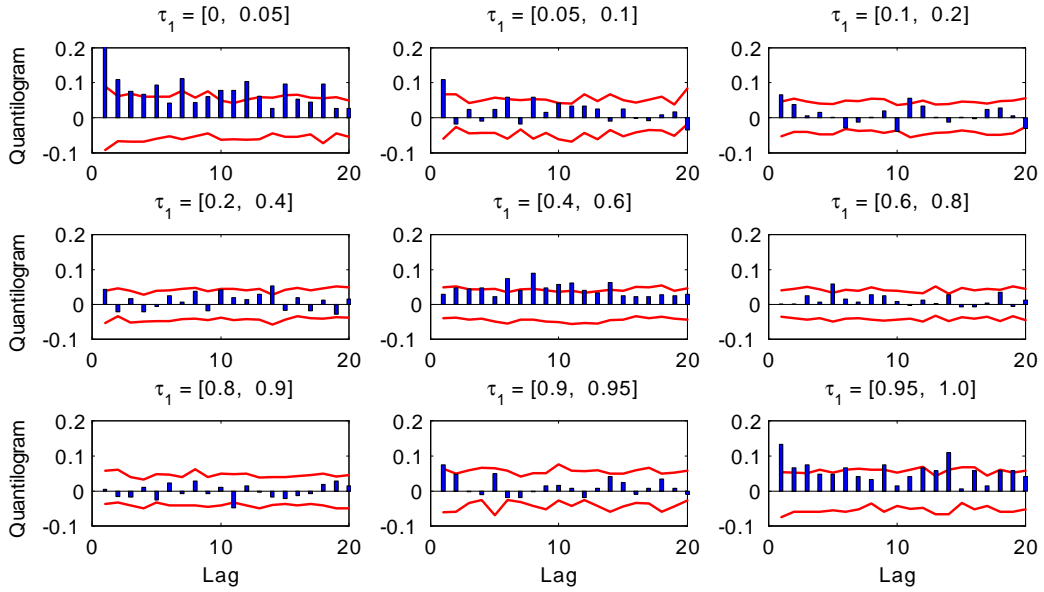
Figure 5(a). [US to UK] Cross-quantilogram $\hat{\rho}_\tau(k)$ to detect directional predictability from US to UK. $\tau_1=\tau_2$. Same as Figure 1(a).
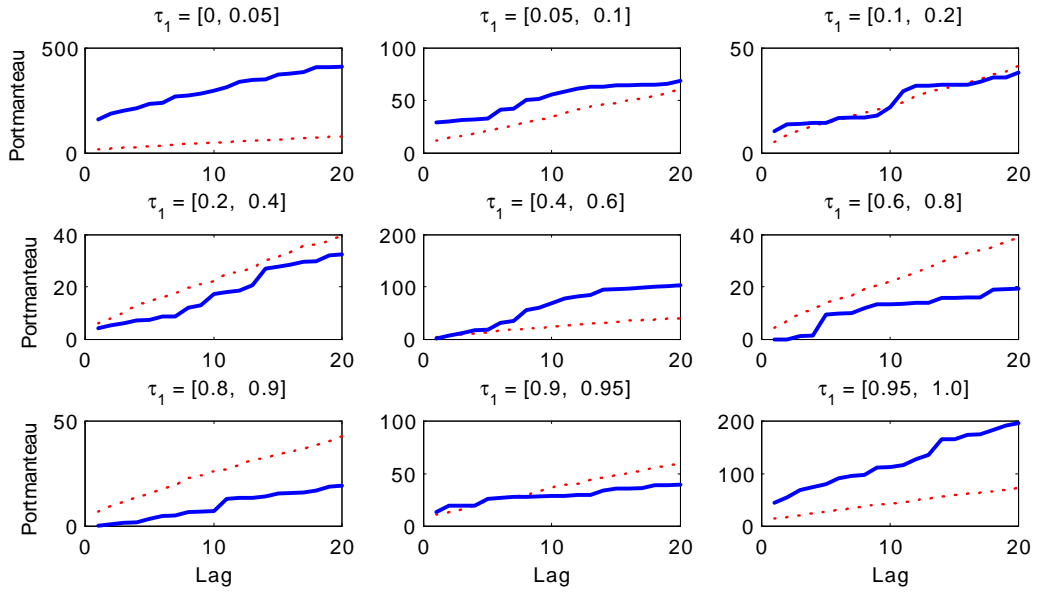
Figure 5(b). [US to UK] Box-Ljung test statistic $\hat{Q}_\tau^{(p)}$ for each lag p using $\hat{\rho}_\tau(k)$. Same as Figure 1(b).
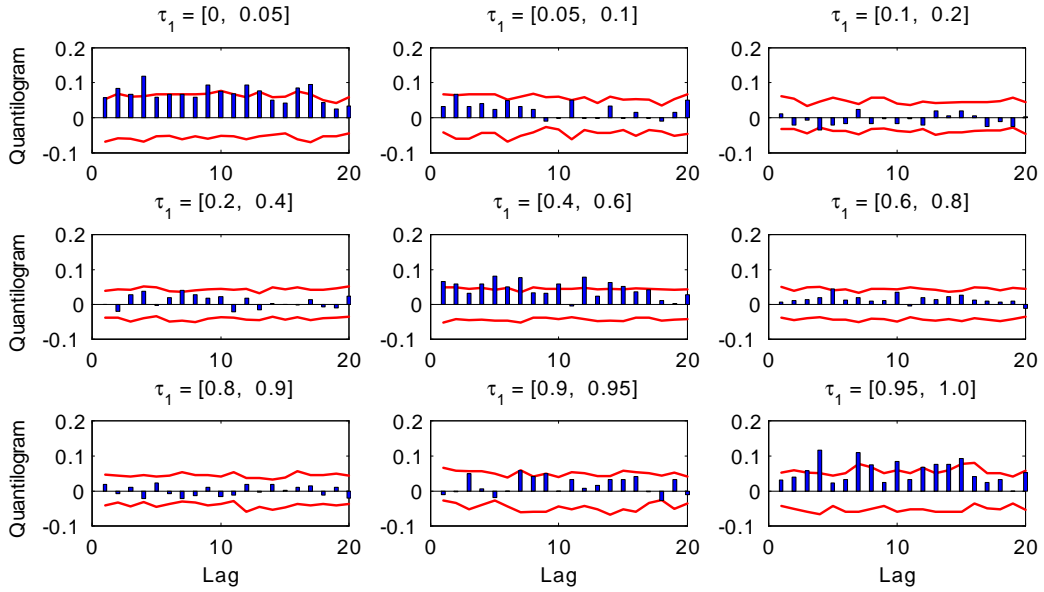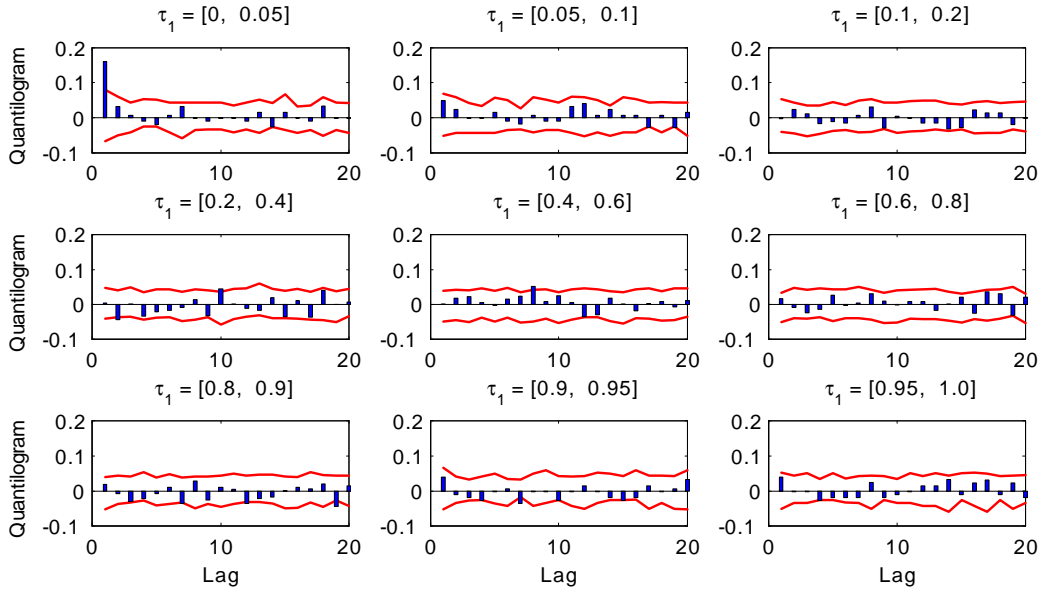
Figure 6(a). [UK to US] Cross-quantilogram $\hat{\rho}_\tau(k)$ to detect directional predictability from UK to US $\tau_1 = \tau_2$. Same as Figure 1(a).

Figure 6(b). [UK to US] Box-Ljung test statistic $\hat{Q}_\tau^{(p)}$ for each lag p using $\hat{\rho}_\tau(k)$. Same as Figure 4(b).

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Figure 7(a). [US, std. residual] Auto-quantilogram $\hat{\rho}_\tau(k)$ of the S&P 500 index return series using the standardized residual from the GJR-GARCH model. $\tau_1 = \tau_2$. Same as Figure 1(a).
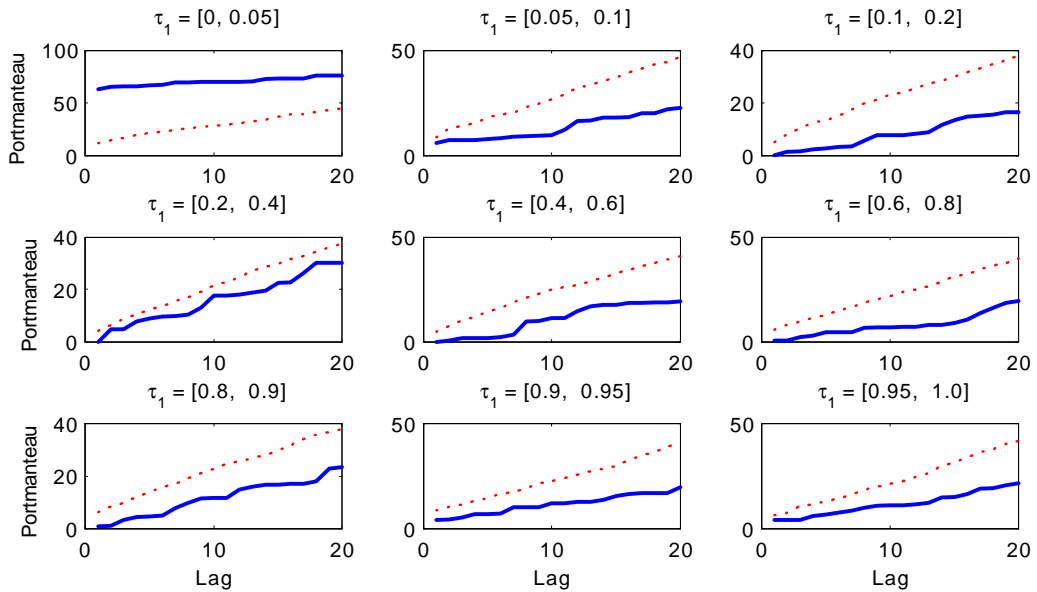

Figure 7(b). [US, std. residual] Box-Ljung test statistic $\hat{Q}_\tau^{(p)}$ for each lag p using $\hat{\rho}_\tau(k)$. Same as Figure 1(b).
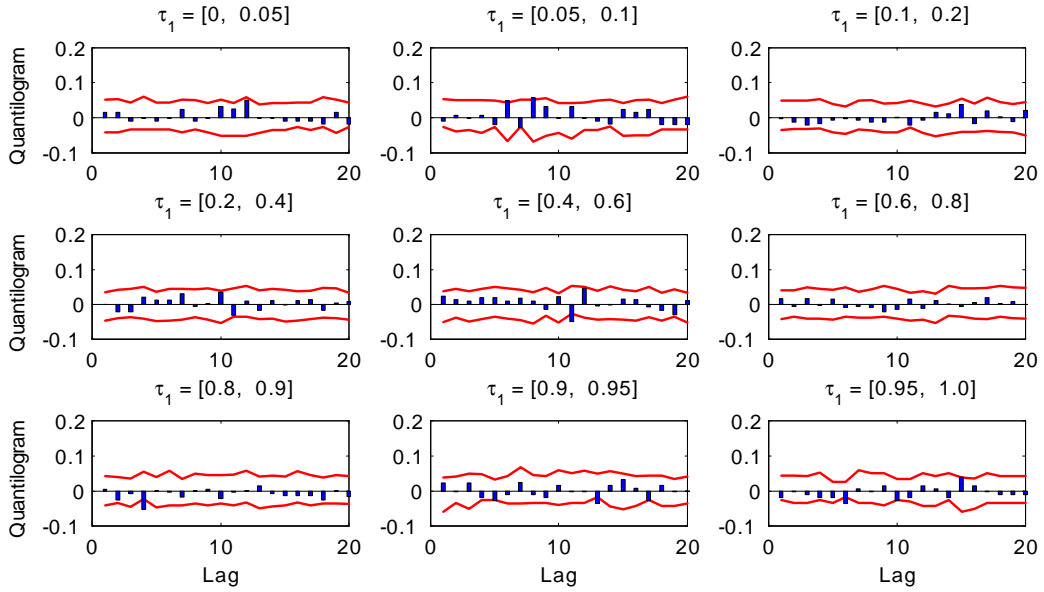
Figure 8(a). [UK, std. residual] Auto-quantilogram $\hat{\rho}_\tau(k)$ of the FTSE index return series using the standardized residual from the GJR-GARCH model. $\tau_1 = \tau_2$. Same as Figure 1(a).

Figure 8(b). [UK, std. residual] Box-Ljung test statistic $\hat{Q}_\tau^{(p)}$ for each lag p using $\hat{\rho}_\tau(k)$. Same as Figure 1(b).

Figure 9(a). [US to UK, std. residual] Cross-quantilogram $\hat{\rho}_\tau(k)$ to detect directional predictability from US to UK using the standardized residual from the GJR-GARCH model. $\tau_1 = \tau_2$. Same as Figure 1(a).
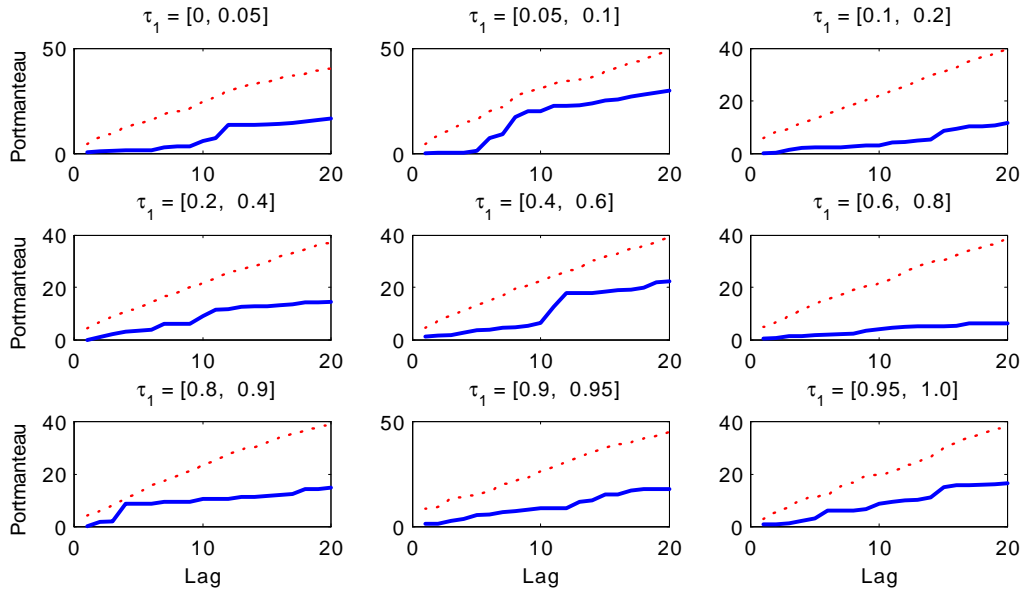

Figure 9(b). [US to UK, std. residual] Box-Ljung test statistic $\hat{Q}_\tau^{(p)}$ for each lag p using $\hat{\rho}_\tau(k)$. Same as Figure 1(b).
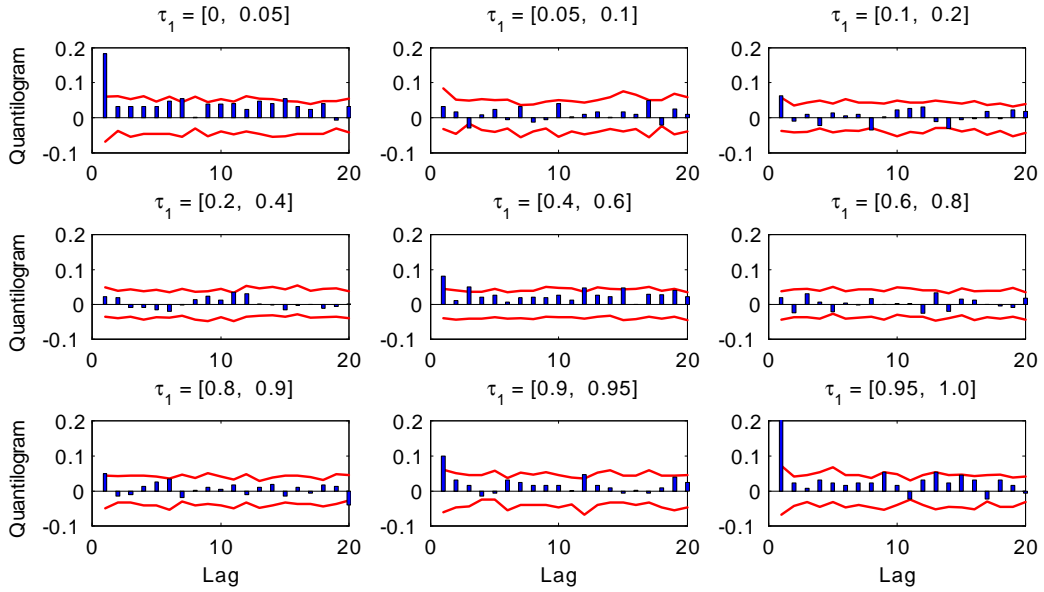
Figure 10(a). [UK to US, std. residual] Cross-quantilogram $\hat{\rho}_\tau(k)$ to detect directional predictability from UK to US using the standardized residual from the GJR-GARCH model. $\tau_1 = \tau_2$. Same as Figure 1(a).

Figure 10(b). [UK to US, std. residual] Box-Ljung test statistic $\hat{Q}_\tau^{(p)}$ for each lag p using $\hat{\rho}_\tau(k)$. Same as Figure 1(b).

Figure 11(a). [US to Japan] Cross-quantilogram $\hat{\rho}_\tau(k)$ to detect directional predictability from US to Japan. $\tau_1 = \tau_2$. Same as Figure 1(a).
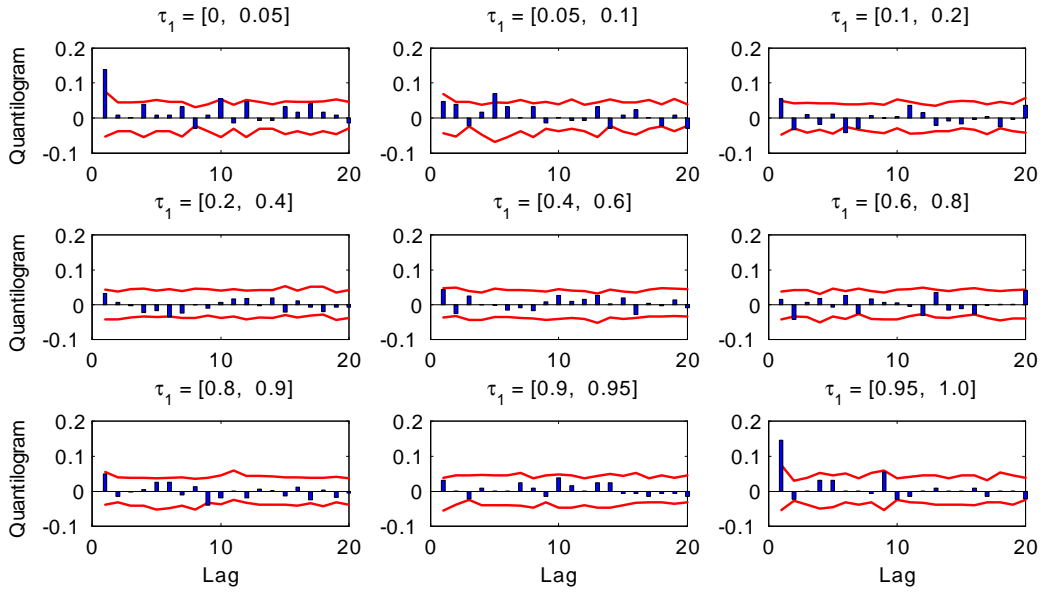

Figure 11(b). [US to Japan, std. residual] Cross-quantilogram $\hat{\rho}_\tau(k)$ to detect directional predictability from US to Japan using the standardized residual from the GJR-GARCH model. $\tau_1 = \tau_2$. Same as Figure 1(a).
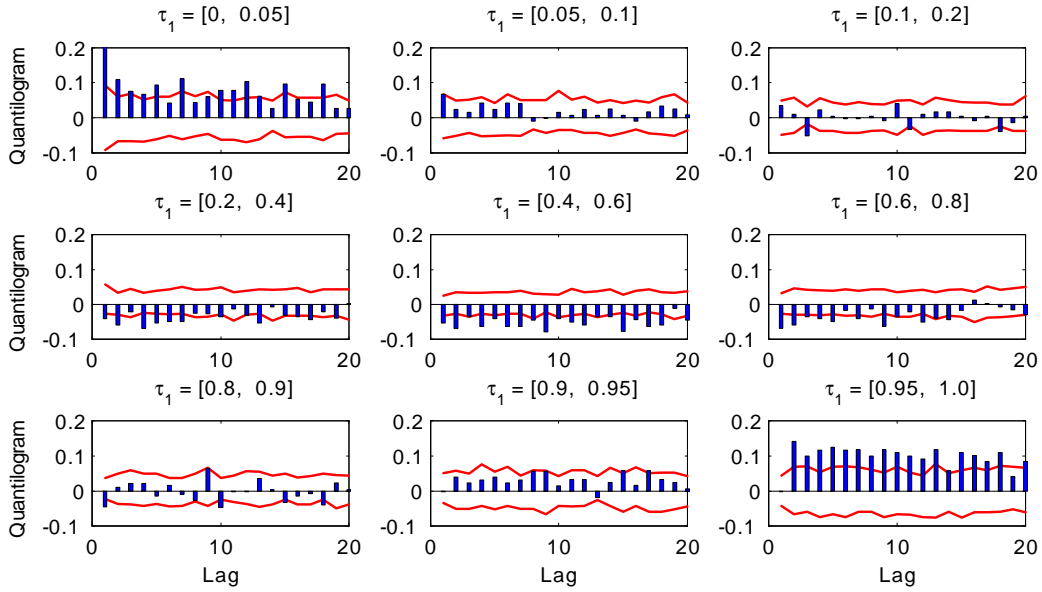
Figure 12(a). [US to UK, from the left tail] Cross-quantilogram $\hat{\rho}_\tau(k)$ to detect directional predictability from US to UK. $\tau_1 \neq \tau_2$ and $\tau_2 = [0, 0.05]$ where τ_2 is the quantile range of US the stock return. Same as Figure 1(a).
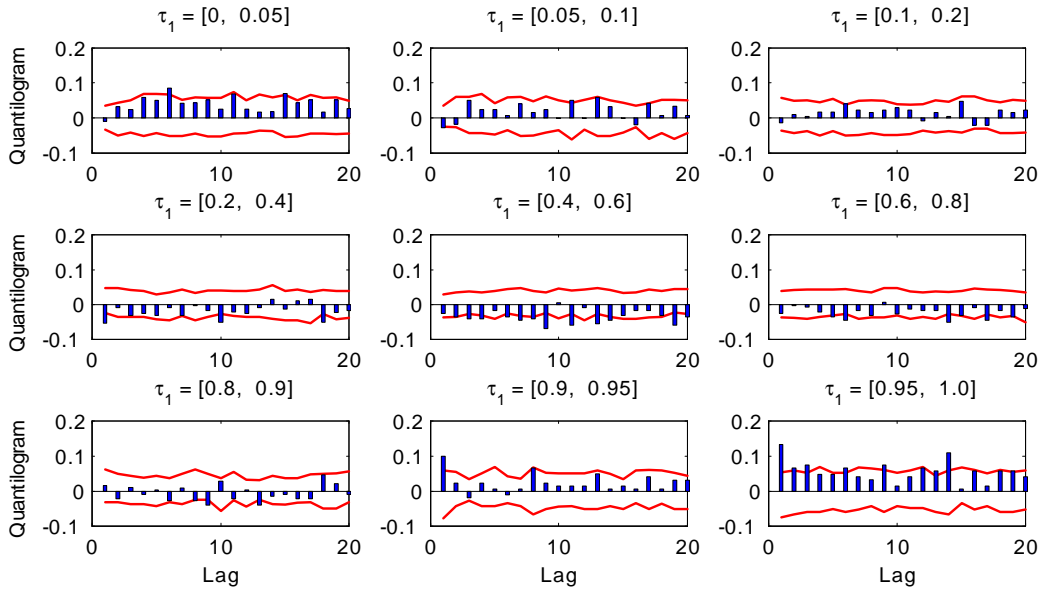

Figure 12(b). [US to UK, from the right tail] Cross-quantilogram $\hat{\rho}_\tau(k)$ to detect directional predictability from US to UK. $\tau_1 \neq \tau_2$ and $\tau_2 = [0.95, 1]$ where τ_2 is the quantile range of the US stock return. Same as Figure 1(a).
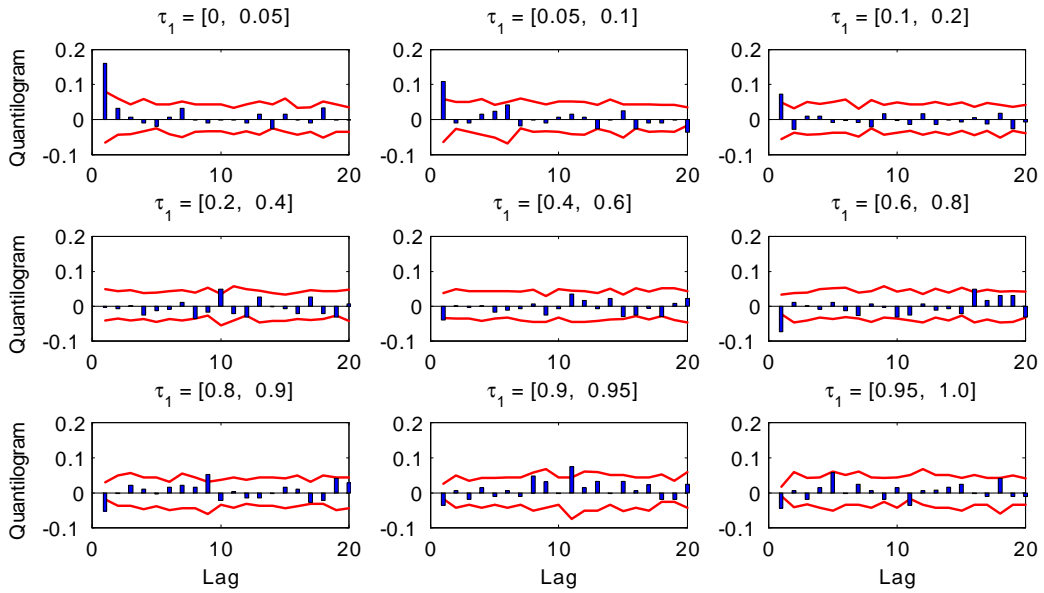
Figure 13(a). [US to UK, std. residual, from the left tail] Cross-quantilogram $\hat{\rho}_\tau(k)$ from US to UK using the standardized residual from the GJR-GARCH model. $\tau_1 \neq \tau_2$ and $\tau_2 = [0, 0.05]$ where τ_2 is for the US stock return. Same as Figure 1(a).
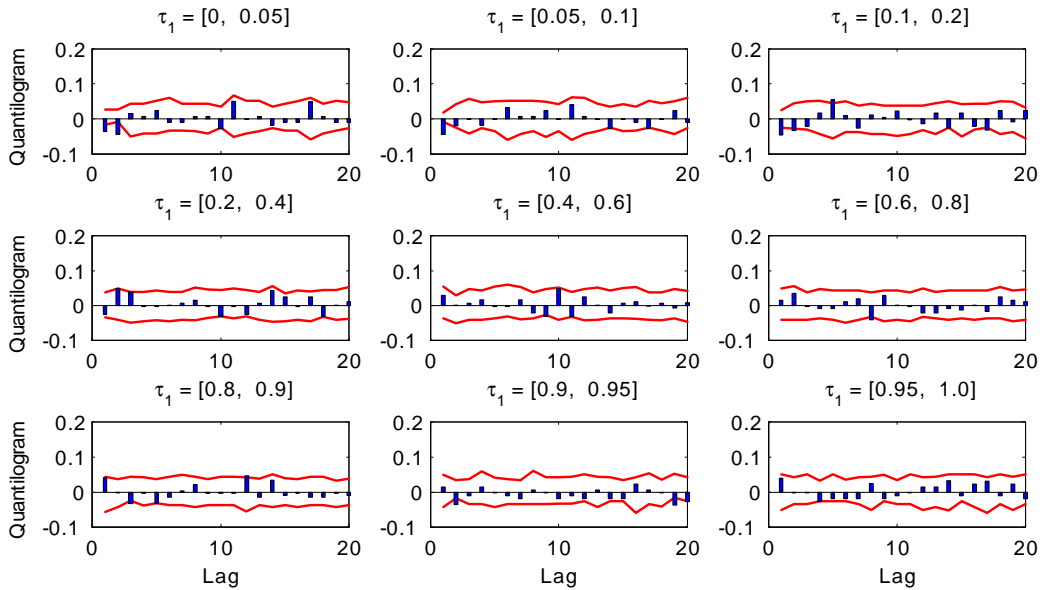

Figure 13(b). [US to UK, std. residual, from the right tail] Cross-quantilogram $\hat{\rho}_\tau(k)$ from US to UK for $\tau_1 \neq \tau_2$ and $\tau_2 = [0.95, 1]$. Same as Figure 13(a).

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