Money, Asset Prices and the Liquidity Premium

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ABSTRACT

This paper examines the effect of monetary policy on the market value of the liquidity services that financial assets provide, known as the liquidity premium. Money supply and nominal interest rates have positive effects on the liquidity premium, but asset supply has a negative effect. This implies that liquid financial assets are substantive substitutes for money, and that the opportunity cost of holding money plays a key role in explaining variation in the liquidity premium and thus in asset prices. The higher cost of holding money due to higher money growth rates leads to the higher liquidity premium of liquid assets. My empirical analysis with U.S. Treasury data during the period from 1947 to 2007 confirms the theoretical predictions. The theory also proposes that the liquidity properties of assets can cause negative nominal yields in equilibrium when the cost of holding money is low and liquid assets are scarce. I present the suggestive empirical findings in the U.S. and Switzerland to support this prediction.

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1 Introduction

Investors value financial assets not only for their intrinsic value, the present value of the expected dividend stream, but also for their liquidity: the ability to help facilitate transactions. For instance, U.S. Treasuries are often used as collateral in a secured credit market through repurchase agreements, are easily sold for cash in secondary asset markets, and sometimes are even used directly as a means of payment. Accordingly, many liquid financial assets are priced above their fundamental values, and their prices are higher than those of illiquid assets with comparable safety and maturity characteristics: liquid assets bear liquidity premia. It is well known that liquidity premia account for much of the variation in liquid asset prices observed in financial markets.[1]

The objective of this paper is to examine the effect of monetary policy on liquidity premia (and asset prices) and to explore a liquidity-based explanation for the negative bond yields that have been recently observed in some countries. This analysis is important for investigating the effects of various monetary policy tools of central banks. These include open market operations that aim to adjust the supplies of money and bonds circulated in the economy (and thus to control short-term interest rates) as well as unconventional monetary policy to directly change the quantity of liquid assets traded in financial markets. In addition, since negative nominal yields have never been observed until recently, it is true that our understanding of this phenomenon is limited at best.

More precisely, the questions addressed in this paper are the following: how do the liquidity properties of assets (other than fiat money) affect their prices?; do these prices include liquidity premia?; how and how much do changes in the money and bond supplies affect these liquidity premia?; and can the liquidity properties of bonds explain the negative yields recently observed? I develop a monetary search model, building on Lagos and Wright (2005), in which assets also have certain liquidity properties, as fiat money does, in order to address these questions. This theoretical framework is not only tractable, but also allows me to model financial assets as objects that compete with fiat money as facilitators of exchange, when there exist frictions such as anonymity and limited commitment among agents in the economy.[2] In this environment, asset prices bear liquidity premia, and monetary policy affects these premia by changing the opportunity cost of holding money.

[1] See Amihud, Mendelson, and Pedersen (2006), Duffie, Gârleanu, and Pedersen (2005), and Krishnamurthy and Vissing-Jorgensen (2012), among others, for details. In particular, the first one reviews both the theories and empirical studies about liquidity-based asset pricing.

[2] In a recent survey paper, titled "Micro-foundations of Money: Why they Matter", Waller (2015) identifies endogenous asset liquidity as the first in a series of valuable lessons that macroeconomists have learned from the recent monetary-search literature.

Specifically, I extend the baseline framework of Lagos and Wright (2005) by including a risk-free government bond in addition to fiat money. The government bond as well as money is used as a medium of exchange in a goods market characterized by frictions such as anonymity and limited commitment. Agents are willing to pay a premium for the bond's ability to help them carry out transactions in the frictional market. Accordingly, not only bond supply but also money supply affects the bond's liquidity premium and thereby its price. Importantly, the key concept that links money supply and the liquidity premium is the opportunity cost of holding money. An increase in the money growth rate raises the inflation rate under fully flexible prices. This results in higher nominal interest rates through the Fisher effect, and thus the higher cost of holding money.[3] Then, agents replace more costly fiat money with liquid bonds which are also useful in the exchange process. This implies the greater bond demand, which leads to higher liquidity premia and, ultimately, higher bond prices (lower bond yields). Consequently, the money growth rate has a positive influence on the bond prices. In contrast to this, bond supply negatively affects the bond prices because the marginal benefit from using the bonds as a medium of exchange decreases as its supply increases.

In an empirical exercise, I document these theoretical predictions by showing that changes in the growth rate of Narrow Money[4] significantly and positively affect the yield spreads between liquid and illiquid financial assets, whereas changes in the liquid asset supply negatively affect. I use three-month U.S. Treasury Bills as a proxy for the liquid bonds introduced in the model, and measure the liquidity premia on Treasury Bills by the yield spreads between Treasury Bills and financial assets with different (lower) liquidity but similar characteristics in terms of safety and maturity. The examples include three-month AA-rated Commercial Papers, three-month general collateral Repurchase Agreement, and three-month bankers' acceptance. As explained in Krishnamurthy and Vissing-Jorgensen (2012) and Nagel (2014), where these data are used first to measure liquidity premia, these financial assets are comparably safe but are not as liquid as the Treasury Bills of similar maturities; therefore one can reasonably argue that the spreads between the yields on these assets and the Treasury Bills of similar maturities reflect differences in the market value of liquidity, i.e., liquidity premia. In addition, a monetary aggregate, Narrow Money, is used as a proxy for money, because it includes only components that can be used as

[3] A one-time injection of money into the economy or a change in the level of money supply does not affect the liquidity premium and any other real variables in the model, but the money growth rate does. Hence, money is neutral but not superneutral here. However, unlike money, a one-time injection of bonds has a substantial impact on real variables. I will show relevant theoretical findings in Section 2.5.

[4] Narrow Money includes nonbank public currency used as a medium of exchange, deposits held at Federal Reserves Banks, and reserve adjustment magnitude, which is"adjustments made to the monetary base due to changes in the statutory reserve requirements". See http://research.stlouisfed.org/publications/review/03/09/0309ra.xls for more details.

a direct medium of exchange, as implied by the theory, unlike other broader monetary aggregates such as M1 and M2. Furthermore, the demand for Narrow Money has a stable and negative relationship with its holding cost, or nominal interest rates over the sample period from 1955 to 2007 as required by the theory. The empirical tests with these data confirm the positive effect of the money growth rate on liquidity premia and the negative effect of bond supply. Specifically, a one percentage point increase in the growth rate of Narrow Money causes an increase of around 3.5 bps on average in the liquidity premia of three-month Treasury Bills. Moreover, this test result supports the theoretical arguments that the Treasuries are substantive substitutes for money, and that the opportunity cost of holding money plays a key role in explaining variations in the liquidity premium and thus in asset prices.

My model also suggests that negative nominal bond yields can arise as an equilibrium phenomenon under certain conditions. More precisely, the liquidity properties of bonds can lead to negative yields when there exist transactions where bonds are used as a medium of exchange through repurchase agreement contracts but where money would not be used, and, in particular, when the opportunity cost of holding money (or nominal interest rates) is low and bond supply is scarce. Under these circumstances, the liquidity benefits from bonds are highly valued, and thereby the probability of negative yields due to liquidity properties increases. Interestingly, the negative yields of liquid bonds can emerge even though nominal short-term interest rates are slightly positive, or there exist other illiquid assets with non-negative yields.

I present suggestive empirical findings in Switzerland and the United States to support the liquidity-based explanation for negative nominal bond yields. It is well-known that there always exists a strong demand for Swiss and U.S. government bonds (rather than the Swiss Franc and the U.S. dollar) from not only domestic investors but also investors outside these countries, unlike other countries' government bonds, because of their high liquidity or usefulness for Repo financing in financial markets. For Switzerland, we observed negative Treasury yields over the several years since 2008. In fact, a huge decline in the supply of liquid government bonds was markedly observed in Switzerland around the period of the financial crisis, starting in the last quarter of 2008, and the supply remained historically low until 2015. Moreover, short-term nominal interest rates were positive but close to zero during most years since 2008.[5] According to the aforementioned theoretical and empirical results, these observations strongly suggest that the liquidity of Swiss government bonds was highly valued, and thereby their high

[5] The negative Treasury yields were observed in some European countries such as Germany, Italy and France; however, they emerged after the European Central Bank lowered its monetary policy target interest rate to a negative level. See Section 4 for details.

liquidity premia caused negative yields in Switzerland. Similar changes and negative Treasury yields were also observed in the U.S. in September 2015. Notice that the existence of negative nominal yields is often considered anomalous, because it is hard to reconcile through the lens of traditional monetary models. However, my model of asset liquidity can help rationalize these observations.[6]

The monetary search literature similarly shows that the liquidity premium is a primary factor of variation in the prices of liquid financial assets, and that their supply is negatively correlated with the liquidity premium, whereas the money supply is positively correlated. Also, the key mechanism for this relationship is the opportunity cost of holding money. As a pioneer theoretical paper, Geromichalos, Licari, and Suarez-Lledo (2007) set up a Lagos-Wright type of monetary search framework with a real asset and theoretically show that the money growth rate increases the liquidity premium in the economy where neither money nor asset is plentiful. They derive this result from the model where asset is a perfect substitute for money in transactions in a decentralized market. Also, several papers on this substitution relationship between money and financial assets deliver similar results. Examples include Rocheteau and Wright (2005a), Lagos (2010), Lester, Postlewaite, and Wright (2012), Jacquet and Tan (2012), Williamson (2012), Carapella and Williamson (2015), Geromichalos and Herrenbrueck (2016), and Geromichalos, Lee, Lee, and Oikawa (2016). Aruoba, Waller, and Wright (2011) calibrate a money search model to examine how money supply affects capital formation in a similar way. However, to the best of my knowledge, the literature has not empirically tested the effect of money supply on liquidity premia. The empirical analysis in my paper is one of the primary contributions to the literature. Also, it has not much investigated how and under what conditions liquidity premia can cause negative yields on liquid assets. He, Huang, and Wright (2008) and Sanches and Williamson (2010) present how negative nominal rates arise, but safety, instead of liquidity, causes them in a theoretical model where currency is vulnerable to theft, whereas other assets can not be stolen. Rocheteau, Wright, and Xiao (2016) set up a model, which is similar to my model, in order to show that the liquidity properties of assets can cause a negative interest rate. The economic insight about how negative interest rates emerge in my paper follows their findings. However, they do not much discuss empirical relevance of their model that my paper shows.

In the finance literature, there are papers that study the negative effect of bond supply on liquidity premia. For example, Krishnamurthy and Vissing-Jorgensen (2012) show a strong negative relationship

[6] One of the important lessons we've learned from asset liquidity is that it can shed light on existing asset-related puzzles from a new perspective and provide a liquidity-based theory of asset pricing. Examples include Lagos (2010), Geromichalos, Herrenbrueck, and Salyer (2013), Geromichalos and Simonovska (2011), and Jung and Lee (2015).

between the U.S. Treasury supply and its convenience yield (its premia for both liquidity and safety).[7] Greenwood, Hanson, and Stein (2015) show that the T-bill supply has a negative effect on its liquid premium. Similarly, Longstaff (2004), Vayanos (2004), Brunnermeier (2009) and Krishnamurthy (2010) investigate liquidity premia, but focus on short time periods, such as during the recent financial crises. In fact, they all set up the models without money; therefore, they do not study the effect of money supply on liquidity premia even if liquid bonds can play a role as substitutes for money. As a result, the opportunity cost of holding money does not account for liquidity premia, either. Meanwhile, Nagel (2014) studies how this substitution relationship between money and liquid bonds affects the liquidity premium through monthly changes in the opportunity cost of holding money, which is represented by the federal funds rate. The paper shows that the federal funds rate is positively correlated with the liquidity premium and that bond supply only has a transitory effect on the liquidity premium. However, the model in Nagel (2014) focuses on the short-run effect of the opportunity cost. Also, it does not distinguish the effect of bond supply from that of money supply, even though changes in the federal funds rate can reflect changes in both supplies at the same time. Importantly, in this paper, I separate these effects and examine each of them. Moreover, I show that the effect of changes in the money growth rate is different from that of changes in the level of money supply.

The rest of the paper is organized as follows. Section 2 lays out a theoretical model to be tested in Section 3, in which I describe the data used in the empirical work and test the results from the theory. In Section 4, I discuss negative yields on liquid bonds with the theoretical and empirical results from the previous sections. Section 5 concludes.

2 Model

2.1 Physical Environment

Time is discrete and continues forever. There is a discount factor between periods, $\beta \in (0, 1)$. Each period is divided into two sub-periods. A decentralized market (henceforth DM) with frictions opens in the first sub-period, and a perfectly competitive or centralized market (henceforth CM) follows. The frictions are characterized by anonymity, limited commitment and bilateral bargaining trade among

[7] Krishnamurthy and Vissing-Jorgensen (2012) define the convenience yield as the premia for both safety and liquidity attributes of financial assets such as U.S. Treasuries, and show that the Treasury supply has a negative effect on the convenience yield.

agents. As a result, unsecured credit is not allowed in transactions, and exchange must be $quid\ pro\ quo$ or needs secured credit. There are two divisible and nonstorable consumption goods: goods produced and consumed in the CM (henceforth $CM\ goods$) and goods in the DM (henceforth $DM\ goods$). There are two types of agents; buyers and sellers. Each of them is populated with a continuum of the unit and lives forever. Their permanent identities are determined by the roles they play in the DM. While sellers produce, sell, and do not consume the DM goods, buyers consume and do not produce. Their preferences in period t are given by

$$\begin{aligned} \text{Buyers}: \quad \mathcal{U}(x_t, h_t, q_t) =& u(q_t) + U(x_t) - h_t \\ \text{Sellers}: \quad \mathcal{V}(x_t, h_t, q_t) =& -c(q_t) + U(x_t) - h_t \end{aligned}$$

where x_t is consumption of CM goods, q_t consumption of DM goods, h_t hours worked to produce CM goods, and $c(q_t)$ a cost of production of q_t. As in a standard model, U and u are twice continuously differentiable with $U' > 0, u' > 0, U'' < 0, u'' < 0, u(0) = 0, u'(0) = \infty$, and $u'(\infty) = 0$. Also, I assume that $c(q_t) = q_t$ for simplicity without loss of generality. Let $q^* \equiv \{q : u'(q) = 1\}$, i.e., it denotes the optimal consumption level in the DM. Also, assume that there exists $x^* \in (0, \infty)$ such that $U'(x^*) = 1$ and $U(x^*) > x^*$.

There are two types of assets: fiat money and a one-period real government bond. They are perfectly divisible and storable. Agents can purchase any amount of money and government bonds at the ongoing price ϕ_t and ψ_t in the CM, respectively. Money grows at the rate of μ: $M_{t+1} = (1 + \mu)M_t$. I assume that $\mu > \beta - 1$, but also consider the limit case where $\mu \to \beta - 1$, i.e., the case where the money growth rate approaches closely the Friedman rule later. If μ is positive, new money is injected into buyers in the CM; but if μ is negative, new money is withdrawn through lump-sum transfers. A government bond issued in period t delivers one unit of CM goods in period $t + 1$, and its supply in period t is A_t. Since I will focus on stationary equilibria, A_t is fixed at A. The government (a consolidated authority) budget constraint is

$$G_t + T_t - \phi_t \mu M_t + A(1 - \psi_t) = 0,$$

where G_t is government expenditure, T_t is a lump-sum transfer or tax, $\phi_t \mu M_t$ is seigniorage of new money injection, and $A(1 - \psi_t)$ is government debt service.

Now, I describe more details about activities in each sub-period. First, I describe the second subperiod, where the CM opens. Both buyers and sellers consume and produce a CM good. They work or use their assets, money (m) and government bonds (a), which they are holding from the previous period in order to consume and to pay back the credit made in the previous period. Also, they trade money and bonds among all agents to re-balance the portfolios they will bring to the next period. They have access to technology that turns one unit of labor into one unit of CM goods.

Next, the DM opens in the first subperiod. Each buyer is matched with a seller bilaterally and vice versa. A buyer makes a take-it–or-leave-it (henceforth $TIOLI$) offer to a seller to establish the terms of trade.[8] Since buyers are anonymous and have limited commitment, a medium of exchange (henceforth MOE) is required in transactions. Both money and government bonds can serve as media of exchange. Precisely, the DM is divided into two sub-markets, DM1 and DM2, depending on what type of MOE can be used. In the DM1, sellers accept only a direct MOE. Both assets are used as a direct MOE, but, unlike money, only a fraction $g \in (0, 1)$ of bonds can serve as a direct MOE. Here g is an illiquidity parameter of government bonds, reflecting the fact that government bonds are not as liquid as money as a direct MOE. Intuitively, it implies that it can take time and cost for bonds to play a role like money does in exchange in the DM1.[9] On the other hand, in the DM2, sellers accept only collateralized credit (or loans), i.e., secured credit as a MOE, and bonds are used as collateral for credit. The credit is paid back in CM goods in the forthcoming CM. Also, a portion $h \in (0, 1)$ of bonds can be used only as collateral; therefore buyers always have incentives to pay back their credits. This h is the so-called Loan to Value (henceforth LTV) ratio and is also related to the haircut, which is defined by 1 minus the LTV, following the standard approach in finance. All buyers and sellers visit the DM1 and the DM2 with probabilities θ and $1 - \theta$, respectively. Figure 1 summarizes the events within each period.

2.2 Value Functions

First, I describe the value function of a representative buyer who enters the CM with money (m), bonds (a) and a collateralized credit (ℓ) made last period, since it is the buyer who makes primary decisions on

[8] I could assume that they negotiate the terms of trade through a Kalai bargaining protocol, where the buyers' bargaining power is less than one. However, since the bargaining protocol is not critical to derive most of the interesting results of the paper, I use the simplest setup here by assuming that buyers make a TIOLI offer to their trading partners.

[9] The parameter, g, can be implicitly interpreted as an indicator of how developed or how liquid a secondary asset market is, where bonds are exchanged for money. For instance, less trading friction in the secondary market implies higher g because it means that bonds are more easily converted to money, and then sellers are more willing to accept bonds as a means of payment, and vice versa. In addition, if there are more buyers for bonds due to the developed institution, including the high-quality trading platform technology of secondary markets, bonds can provide better liquidity services.

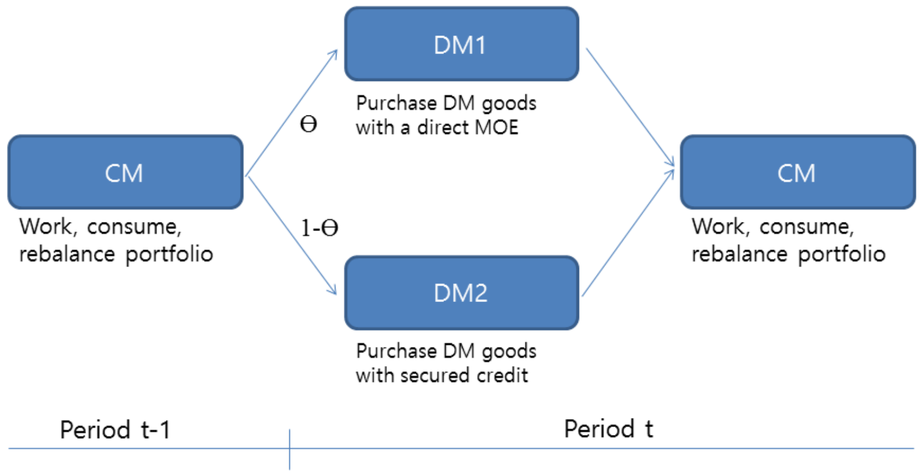

Figure 1: Market Timing

interesting results from the model. The value function of the buyer is

$$W^B(\mathbf{w}_t, \ell_t) = \max_{x_t, h_t, \mathbf{w}_{t+1}} \left\{ U(x_t) - h_t + \beta \mathbb{E}\left[V^B(\mathbf{w}_{t+1}) \right] \right\} \tag{1}$$
$$\text{s.t. } x_t + \boldsymbol{\phi}_t' \mathbf{w}_{t+1} = h_t + \boldsymbol{\phi}_t \mathbf{w}_t - \ell_t + T$$

where $\mathbf{w}_t = (m_t, a_t)$, $\boldsymbol{\phi}_t' = (\phi_t, \psi_t)$, and $\boldsymbol{\phi}_t = (\phi_t, 1)$. ℓ_t stands for the collateralized loan which was made in the last subperiod, and so must be paid back in the form of CM goods. T_t is a lump-sum transfer to the buyer. V^B represents the buyer's value function in the next period DM. It can be easily verified that $x_t = x^*$ at the optimum. Substituting h_t in the budget constraint into the value function W^B yields

$$W^B(\mathbf{w}_t, \ell_t) = \boldsymbol{\phi}_t \mathbf{w}_t - \ell_t + \Lambda_t^B \tag{2}$$

where $\Lambda_t^B \equiv U(x^*) - x^* + T_t + \max_{\mathbf{w_{t+1}}} \{ -\boldsymbol{\phi}_t' \mathbf{w}_{t+1} + \beta \mathbb{E}\left[V^B(\mathbf{w}_{t+1}) \right] \}$. Notice that the value function in the CM is linear in choice variables due to the quasi-linearity of $\mathcal{U}$, as in the standard models based on Lagos and Wright (2005). Consequently, the optimal choices of the buyer do not depend on current state variables.

Next, consider a representative seller with money, bonds, and a collateralized loan to enter the CM. The loan is paid back by the counterpart buyer whom the seller met in the previous DM.

$$W^S(\mathbf{w}_t, \ell_t) = \max_{x_t, h_t} \left\{ U(x_t) - h_t + \beta \mathbb{E}\left[V^S(\mathbf{0}) \right] \right\}$$
$$\text{s.t. } x_t = h_t + \boldsymbol{\phi}_t \mathbf{w}_t + \ell_t$$

where V^S denotes the seller's value function in the DM. Notice that $\mathbf{w}_{t+1} = \mathbf{0}$ for the seller. Since the seller does not consume any goods in the DM, he/she has no incentive to bring money and bonds to the next period DM, when the money holding cost is strictly positive because $\mu > \beta - 1$.[10] It is also easily verified that $x_t = x^*$ at the optimum as in the case of the buyer. Replacing h_t in the value function yields

$$W^S(\mathbf{w}_t, \ell_t) = \boldsymbol{\phi}_t \mathbf{w}_t + \ell_t + \Lambda_t^S \tag{3}$$

where $\Lambda_t^S \equiv U(x^*) - x^* + \beta \mathbb{E}[V^S(0)]$.

Next, the DM opens. Buyers and sellers visit the DM1 with the probability θ and the DM2 with the probability $1 - \theta$. Also, all the agents are matched in each DM. Hence, the expected value function of a buyer with portfolio $\mathbf{w}_t$ in the DM is given by

$$\mathbb{E}\left[V^B(\mathbf{w}_t)\right] = \theta\left[u(q_t^1) + W^B(\mathbf{w}_t - \boldsymbol{p}_t, 0)\right] + (1-\theta)\left[u(q_t^2) + W^B(\mathbf{w}_t, \ell_t)\right] \tag{4}$$

where $\boldsymbol{p}_t = (p_t^m, p_t^a)$ is a portfolio exchanged for DM goods in a meeting with a seller in the DM1, and ℓ_t is a collateralized loan made in the DM2. q_t^1 (q_t^2) represents the quantity traded in the DM1 (DM2). The terms of trades in each market are determined by bargaining in pairwise meetings, which Section 2.3 describes.

The value function of a seller is similar except for the fact that the seller does not bring any money and bonds to the DM for transactions.

$$\mathbb{E}\left[V^S(\mathbf{0})\right] = \theta\left[-q_t^1 + W^S(\boldsymbol{p}_t, 0)\right] + (1-\theta)\left[-q_t^2 + W^S(\mathbf{0}, \ell_t)\right]$$

$-q_t^1$ and $-q_t^2$ represent the efforts that sellers exert to produce the DM goods traded for $\boldsymbol{p}_t$ and ℓ_t in the DM1 and DM2.

2.3 Bargaining Problems in the DM

There are two sub-markets in the DM: DM1 and DM2, depending on what means of payment can be used in transactions. First, consider a meeting in the DM1 where a buyer with portfolio $\mathbf{w}_t$ meets with a seller. The seller accepts both money and bonds as a MOE. However, only a fraction g of bonds can be accepted. The buyer makes a TIOLI offer to the seller to establish the terms of trade: the quantity of

[10]See Rocheteau and Wright (2005b) for the precise and careful proof.

DM goods and a total payment of money and bonds exchanged between them. Specifically, the buyer maximizes his/her surplus under the seller's participation constraint and his/her budget constraint. Then, the bargaining problem is expressed by

$$\max_{q_t^1, \boldsymbol{p}_t} \left\{ u(q_t^1) + W^B(\mathbf{w}_t - \boldsymbol{p}_t, 0) - W^B(\mathbf{w}_t, 0) \right\} \tag{5}$$
$$\text{s.t.} - q_t^1 + W^S(\boldsymbol{p}_t, 0) - W^S(\mathbf{0}, 0) = 0,$$

and the effective budget constraint $\boldsymbol{p}_t \leq \widetilde{\mathbf{w}}_t$, and $\widetilde{\mathbf{w}}_t \equiv (m_t, g \cdot a_t)$. Notice that, since bonds are not as liquid as money in the DM1, i.e., only a fraction $g \in (0, 1)$ of bonds can be used as a MOE, the real value of the effective budget is less than that of the total budget. I will consider an extreme case where $g \to 1$ later to discuss under what conditions negative nominal yields emerge. Substituting (2) and (3) into (5) simplifies the bargaining problem as follows.

$$\max_{q_t^1, \boldsymbol{p}_t} \left\{ u(q_t^1) - \boldsymbol{\phi}_t \boldsymbol{p}_t \right\} \tag{6}$$
$$\text{s.t.} - q_t^1 + \boldsymbol{\phi}_t \boldsymbol{p}_t = 0,$$

and $\boldsymbol{p}_t \leq \widetilde{\mathbf{w}}_t$. The following lemma summarizes the terms of trade as the solutions to the bargaining problem.

Lemma 1. *The real balances of a representative buyer are denoted as $z(\mathbf{w}_t) \equiv \boldsymbol{\phi}_t \mathbf{w}_t$. Define $q^* = \{q_t : u'(q_t) = 1\}$, and z^* as the real balances of the portfolio (m_t, a_t) such that $\phi_t m_t + g a_t = q^*$. Also, $\mathbf{p}^*$ is the pairs of (m_t, a_t) in z^*. Then, the terms of trade are given by*

$$q^1(\mathbf{w}_t) = \begin{cases} q^*, & \text{if } z(\mathbf{w_t}) \geq z^*, \\ z(\widetilde{\mathbf{w}}_t), & \text{if } z(\mathbf{w}_t) < z^*. \end{cases} \quad \boldsymbol{p}(\mathbf{w}_t) = \begin{cases} \boldsymbol{p}^*, & \text{if } z(\mathbf{w}_t) \geq z^*, \\ \widetilde{\mathbf{w}}_t, & \text{if } z(\mathbf{w}_t) < z^*. \end{cases} \tag{7}$$

Proof. See the appendix □

Similarly, in the DM2, a buyer makes a TIOLI offer to a seller as in the DM1. The buyer maximizes his/her surplus subject to the credit limit constraint, unlike the effective budget constraint in the DM1. The buyer uses his/her bond holdings as collateral to obtain credit from the seller. The credit limit is determined by a fraction $h \in (0, 1)$ of the real value of the bond holdings, where h can be interpreted as an LTV ratio. Then, the bargaining problem is described as follows.

$$\max_{q_t^2,\ell_t} \left\{ u(q_t^2) + W^B(\mathbf{w}_t, \ell_t) - W^B(\mathbf{w}_t, 0) \right\} \tag{8}$$
$$\text{s.t.} - q_t^2 + W^S(\mathbf{0}, \ell_t) - W^S(\mathbf{0}, 0) = 0,$$

and the credit limit constraint $\ell_t \leq ha_t$. Substituting (2) and (3) into (8) yields the following expression.

$$\max_{q_t^2,\ell_t} \left\{ u(q_t^2) - \ell_t \right\}$$
$$-q_t^2 + \ell_t = 0,$$

and $\ell_t \leq ha_t$. The solution to the bargaining problem is described by the following lemma.

Lemma 2. *Define the credit limit which a buyer with bond holdings a_t can borrow as $z^a(\mathbf{w}_t) \equiv ha_t$. Also, define $z^{a*} \equiv q^*$. The terms of trade are given by*

$$q^2(\mathbf{w}_t) = \begin{cases} q^*, & \text{if } z^a(\mathbf{w}_t) \geq z^{a*}, \\ z^a(\mathbf{w}_t), & \text{if } z^a(\mathbf{w}_t) < z^{a*}, \end{cases} \qquad \boldsymbol{\ell}(\mathbf{w_t}) = \begin{cases} z^{a*}, & \text{if } z^a(\mathbf{w}_t) \geq z^{a*}, \\ z^a(\mathbf{w}_t), & \text{if } z^a(\mathbf{w}_t) < z^{a*}. \end{cases} \tag{9}$$

Proof. See the appendix □

Lemma 1 and 2 demonstrate that the real values of money and bonds that can be used as an MOE or as collateral, respectively, determine the quantities of goods exchanged in the DM1 and DM2. For example, if the real balances or obtained credits are enough to purchase the optimal consumption level q^*, i.e., if $z(\mathbf{w}_t) \geq z^*$ or if $z^a(\mathbf{w}_t) \geq z^{a*}$, then the optimal level q^* will be produced and exchanged for the corresponding payment, z^*. On the other hand, if they are not enough in the same sense, buyers will hand over all of their real balances, or borrow up to the credit limit, in order to purchase as many DM goods as possible. Sellers will produce the quantity that their participation constraint implies.

2.4 Buyers' Optimal Choices

Now, I describe the objective function which a buyer maximizes by choosing money and bonds (m_{t+1}, a_{t+1}) which he/she brings to the DM next period. Substituting (4) into the inside of the maximization operator in (1) and using the linearity of the value functions yield the following objective function J.

$$J = -\boldsymbol{\phi}_t'\mathbf{w}_{t+1} + \beta\Big\{\theta\big[u(q_{t+1}^1) + \boldsymbol{\phi}_{t+1}(\mathbf{w}_{t+1} - \boldsymbol{p}_{t+1})\big] + (1-\theta)\big[u(q_{t+1}^2) + \boldsymbol{\phi}_{t+1}\mathbf{w}_{t+1} - \ell_{t+1}\big]\Big\}$$

The first term stands for the total cost of buying money and bonds in the CM. The terms in the curly bracket show the benefits which a buyer can obtain from transactions in the DM subject to their portfolio. Then, the Euler equations are given by

$$\phi_t = \beta\left[\theta u'\left(min\{\boldsymbol{\phi}_{t+1}\widetilde{\mathbf{w}}_{t+1}, q^*\}\right) + (1-\theta)\right]\phi_{t+1}, \tag{10}$$

$$\psi_t = \beta\Big\{\theta\big[(1-g) + gu'\left(min\{\boldsymbol{\phi}_{t+1}\widetilde{\mathbf{w}}_{t+1}, q^*\}\right)\big] + (1-\theta)\big[(1-h) + hu'\left(min\{ha_{t+1}, q^*\}\right)\big]\Big\}. \tag{11}$$

The left-hand side on each Euler equation shows the marginal cost of buying a unit of money or bond, which is the price that a buyer pays for each unit in the CM. On the other hand, the right-hand side is the marginal benefit obtained by using money or a bond as a means of payment in the DM1 and DM2.

Figure 2 shows the decreasing money demand against the cost of holding money captured by $\phi_t/(\phi_{t+1}\beta)$, which comes from Equation (10).[11] Similarly, inserting Equation (10) into (11) shows the inverse bond demand curve against its price, as in Figure 3. This negative relationship makes sense because the bond price implies the cost of purchasing bonds, given the fixed dividend in the forthcoming CM. Also, the bond demand curve is affected by changes in the cost of holding money. For instance, the curve shifts out (or in) as the money holding cost increases (or decreases). This shift is intuitively straightforward to understand. If the money holding cost becomes higher, agents become less willing to hold money, consequently leading to a decrease in the money demand. However, since bonds can also to some extent play a role in relaxing the liquidity constraint in the DM as money does, the demand for bonds as a substitute for money will increase. Lastly, both demand curves are flat in the regions where $m \geq m^*$ and $ha_t \geq q^*$, respectively. One extra unit of money or bonds is no longer useful in the DM transactions because buyers already hold enough money or bonds to purchase q^* in these territories.

[11] $\phi_t/(\phi_{t+1}\beta)$ is equal to the nominal interest rate in the equilibrium, and so represents the opportunity cost of holding money.

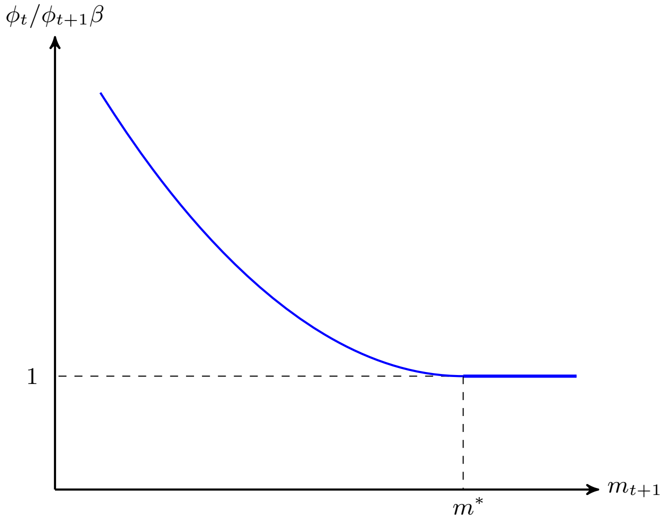

Figure 2: Money Demand

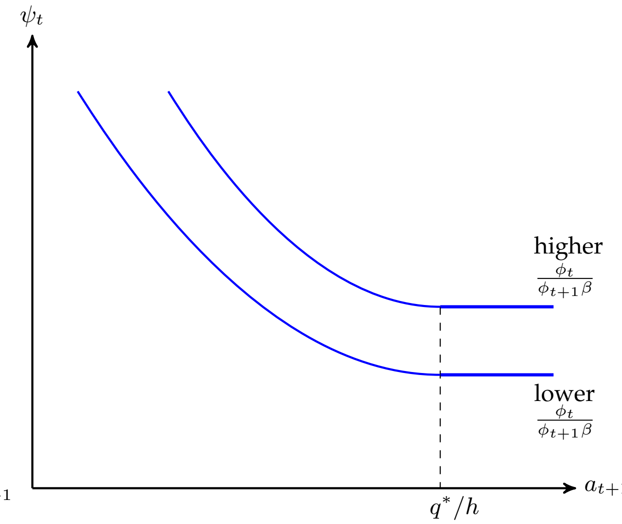

Figure 3: Asset Demand

2.5 Equilibrium and Characterization

I focus on symmetric and stationary equilibria, in which each type of agent follows identical strategies, and both real money and bond balances are constant over time. It implies that $\phi_t M_t = \phi_{t+1} M_{t+1}$ and $A_t = A$. Then, the money growth rate is equal to the inflation rate in the CM, i.e., $1+\mu = \phi_t/\phi_{t+1} = 1+\pi$.

Definition 1. A steady state equilibrium is a list of real balances of buyers, $\tilde{z}_t = \phi_t M_t + gA$, and bond holdings $\tilde{z}^a = hA$, money and bond prices $\boldsymbol{\phi}'_t$, the bilateral terms of trade in $DM1$: $q^1(\mathbf{w}_t)$ and $\boldsymbol{p}(\mathbf{w}_t)$ which are given by Lemma 1, and the bilateral terms of trade in $DM2$: $q^2(\mathbf{w}_t)$ and $\ell(\mathbf{w}_t)$ which are given by Lemma 2 such that:

(i) the decision rule of a representative buyer solves the individual optimization problem (1), taking prices $\boldsymbol{\phi}'_t$ and $\phi_t/\phi_{t+1} = 1+\mu = 1+\pi$ as given;

(ii) the terms of trade in the DM satisfy (7) and (9);

(iii) prices are such that the CM clears, i.e., $\mathbf{w}_{t+1} = [\mu M_t, A]$ for buyers.

Then, the following lemma summarizes the equilibrium objects.

Lemma 3. *There exists a unique steady state equilibrium with four different cases.*

(i) If $\tilde{z}_t \geq z^*$ and $\tilde{z}^a \geq z^*$, then, $q_t^1 = q_t^2 = q^*$, $\phi_t = (z^* - gA)/M_t$, and $\psi_t = \beta$;

(ii) If $\tilde{z}_t \geq z^*$ and $\tilde{z}^a < z^*$, then, $q_t^1 = q^*$, $q_t^2 = \tilde{z}_t^a$, $\phi_t = (z^* - gA)/M_t$, and $\psi_t = \beta\{\theta + (1-\theta)\big[(1-h) + hu'(q_t^2)\big]\}$;

(iii) If $\tilde{z}_t < z^*$ and $\tilde{z}^a \geq z^*$, then, $q_t^1 = \tilde{z}_t$, $q_t^2 = q^*$, $\phi_t = (q_t^1 - gA)/M_t$, and $\psi_t = \beta\{\theta[(1-g) + gu'(q_t^1)] + (1-\theta)\}$;

(iv) If $\tilde{z}_t < z^*$ and $\tilde{z}^a < z^*$, then, $q_t^1 = \tilde{z}_t$, $q_t^2 = \tilde{z}_t^a$, $\phi_t = (q_t^1 - gA)/M_t$, and $\psi_t = \beta\{\theta[(1-g) + gu'(q_t^1)] + (1-\theta)[(1-h) + hu'(q_t^2)]\}$.

Proof. See the appendix. □

The definition of equilibrium is quite standard and straightforward. The fact that the real money balances and the bond supply are constant over time in the steady state implies that both $\tilde{z}_t$ and $\tilde{z}^a$ are constant. Given the market clearing condition, $\tilde{z}_t$ and $\tilde{z}^a$ determine the quantities and the real balances of money and bonds exchanged in the DM, following Lemma 1 and 2, and prices ϕ_t and ψ_t. However, not all $\mu \in (\beta - 1, \infty)$ are consistent with monetary equilibria. The next corollary describes the range of money growth rates for the monetary equilibria when $gA < q^*$.

Corollary 1. *A monetary equilibrium is supported on the range of $(\beta - 1, \bar{\mu})$, where $\bar{\mu} \equiv \{\mu : \mu = \beta[1 + \theta[u'(gA) - 1]] - 1\}$ when $gA < q^*$.*

Proof. See the appendix. □

As μ increases, buyers reduce their money balances because higher μ implies the higher opportunity cost of holding money. Once the money growth rate exceeds the upper bound, $\bar{\mu}$, they do not hold money for transactions in the DM anymore; because doing so leads to less consumption than they pay for obtaining money. Such an upper bound $\bar{\mu}$ falls down in bond supply A. It implies that, given the level of money supply, agents are less patient with higher inflation, so they are less willing to hold money as the supply of bonds increases.[12] On the other hand, if we allow for the case where $\mu \to \beta - 1$, which is the Friedman rule, it implies $\tilde{z}_t \geq q^*$, and so will be the lower bound for a monetary equilibrium.[13] In this case, the marginal change of money supply never affects buyers' consumption in the DM.

The Euler equations, (10) and (11), for the optimal money and bond holdings with the above defini-

[12] If $gA = q^*$, $\bar{\mu} = \beta - 1$. In this case, money does not facilitate transactions in the DM, and therefore there only exist the non-monetary equilibria.

[13] The zero net nominal interest rate ($i = 0$) implies that money growth equals the Friedman rule, i.e., $\mu = \beta - 1$ and thereby the money growth rate does not affect the bond demand and thus price any more because agents already hold enough money to purchase the first best consumption q^* in the DM1.

tion objects can be re-expressed as follows.

$$\phi_t = \beta\Big\{1 + \theta\left[u'\left(min\{\tilde{z}_{t+1}, q^*\}\right) - 1\right]\Big\}\phi_{t+1} \tag{12}$$

$$\psi_t = \beta\Big\{1 + \theta \cdot g\left[u'\left(min\{\tilde{z}_{t+1}, q^*\}\right) - 1\right] + (1-\theta)h\left[u'\left(min\{\tilde{z}^a, q^*\}\right) - 1\right]\Big\} \tag{13}$$

In Equation (13), the current bond price (ψ_t on the left hand side) is equal to the sum of its fundamental value (β) and the present value of the weighted average of its marginal benefits in the DM1 and DM2. These marginal benefits are reflected in its price as the *liquidity premium*. If the bond were not used in the DM transactions, i.e., $g = 0$ and $\theta = 1$, its price would be equal to fundamental value (β). Similarly, Equation (12) shows that the real price of money is also determined by its fundamental value and liquidity premium. The liquidity premia in money and bond prices are determined by the aggregate real values of money and bonds circulated within the economy.

Plugging (12) into (13) yields the following equations and allows us to examine how the equilibrium bond price responds to changes in money and bond supply when they are scarce, i.e., $\beta - 1 < \mu < \bar{\mu}$ and $\tilde{z}^a < q^*$.[14]

$$\psi = \beta\Big\{1 + g(\frac{1+\mu}{\beta} - 1) + (1-\theta)h\left[u'\left(\tilde{z}^a\right) - 1\right]\Big\} \tag{14}$$

$$= \beta\Big\{1 + gi + (1-\theta)h\left[u'\left(\tilde{z}^a\right) - 1\right]\Big\} \tag{15}$$

where $i \equiv (1+\mu)/\beta - 1$. Equation (15) is obtained by the Fisher equation, because $\mu = \pi$ in the stationary equilibrium and $1/\beta = 1 + r$. Here i represents a nominal interest rate of an illiquid real bond, which is strictly positive in monetary equilibria, and r stands for the real yield on a bond that is illiquid in the DM exchange in the sense that it is not accepted by sellers.[15] The price of a liquid bond (ψ) is higher than its fundamental value or that of an illiquid bond (β) when the bond supply is scarce, i.e., $\tilde{z}^a < q^*$. This implies that the rate of return on a liquid bond is lower than that on an illiquid bond: $\rho < 1/\beta$, if ρ denotes the real yield of a liquid bond $1/\psi$. This is important from the empirical perspective, because it implies that the yield difference between liquid and illiquid bonds can be used to measure the market

[14]We can think about the case where $\tilde{z}^a \geq q^*$. This is the case where the bond supply is plentiful in that it allows agents to purchase the first best quantity, q^*, in the DM2. The marginal increase in the bond supply does not allow buyers to additionally purchase goods in the DM2 anymore even though the probability of visiting the DM2, $1-\theta$, is positive. As a result, changing the bond supply does not affect trade volume in the DM2, but only in the DM1.

[15]Its real price (β), which is the inverse of the real interest rate $1/\beta$, is exactly equal to asset prices as derived from traditional asset pricing models, where assets are used only as a store of value and their supply does not affect their prices; the asset prices equal the present discount value of their future stream of consumption dividends.

price of the liquidity service provided by liquid bonds, i.e., the liquidity premium. Notice that since the inflation rate affects both liquid and illiquid bonds at the same time, the yield difference can be measured as the difference of either real or nominal yields.

Importantly, Equations (14) and (15) demonstrate that not only bond supply but also the money growth rate (or the nominal interest rate of illiquid bonds) influences the liquidity premium and the equilibrium bond price only if they are substitutes to some extent in the sense that bonds help to relax the liquidity constraint in the DM1 as money does: $0 < g < 1$. The demand for money and bonds is interconnected, because both of them are useful in exchange process.[16] More importantly, the price of the illiquid bond is fixed at β over time, whereas the price of the liquid bond varies with both money and bond supply. It follows that the yield difference between them, or the liquidity premium, is affected by both money and bond supply. Notice that both the money growth rate (μ) and the nominal interest rate (i) represent the opportunity cost of holding money. Equations (14) and (15) have the same economic implication: the price of a liquid bond is affected by the money holding cost.

The following proposition summarizes how the equilibrium bond price, or the liquidity premium, is affected by money and bond supply, and I will empirically test this in Section 3. I focus on the monetary equilibria, where $\mu \in (\beta - 1, \bar{\mu})$.

Proposition 1. *The bond price exceeds the fundamental value, i.e, $\psi > \beta$, as long as $\beta - 1 < \mu < \bar{\mu}$. Also,*

(i) the bond price (or the liquidity premium) is increasing in μ (i), given a fixed level of A: $\partial\psi/\partial\mu > 0$ ($\partial\psi/\partial i > 0$);

(ii) the bond price (or the liquidity premium) is decreasing in A: $\partial\psi/\partial A < 0$.

The proof is straightforward and so omitted. Higher μ implies the higher opportunity cost of holding money, which leads to a lower demand for money and a higher demand for bonds as a substitute for money, consequently ending up with a higher bond price. On the other hand, bond supply negatively affects the liquidity premium because the marginal effect of bond supply becomes less on relaxing the liquidity constraint in the DM exchange as it increases; the marginal benefit from holding bonds decreases.

Now, consider some extreme cases where money and bonds are perfect substitutes or not substitutes

[16] For example, if the bonds are not substitutes at all, i.e., $g = 0$, the demand for the bonds is not affected by the money supply or the nominal interest. In the empirical implementation, I check whether the data are consistent with two cases where $g = 0$, or where $0 < g < 1$. In the monetary search literature, Geromichalos, Licari, and Suarez-Lledo (2007) assume that money and real assets are perfect substitutes ($g = 1$) and Lester, Postlewaite, and Wright (2012) assume different recognizability among money and other assets to endogenize the illiquid parameter g.

at all. It helps us to understand intuitively how the parameters, g and θ, and the money growth rate can affect the bond price (or the liquidity premium). Moreover, this will allow us to better understand empirical tests and explain which case can be well supported by the U.S. data in Section 3. As mentioned in subsection 2.1, g is an illiquid parameter, implying how bonds are more liquid than money in DM transactions. Meanwhile, the parameter $(1-\theta)$ can be interpreted as how well the collateralized credit market functions where money may not work; or how strong the demand for liquid bonds is in repurchase agreements. More collateralized transactions in the credit market can lead to higher $1-\theta$.

Consider the four different cases as follows, depending on the combinations of g and θ. All the results are derived from Equations (14) and (15).

(*Case 1: Perfectly illiquid bonds*) Bonds are totally illiquid in the sense that they are useless in the DM exchange, i.e., $g \to 0$ and $\theta \to 1$. Here the bonds function only as a store of value. Hence, the real bond price ψ is equal to the fundamental value, β, i.e., the present value of the dividend (a unit of CM goods) that the bonds deliver next period. The price does not carry the liquidity premium at all. As a result, it is not affected by money supply at all: $\partial\psi/\partial\mu = 0$.

(*Case 2: Perfect substitutes for money*) Bonds are perfect substitutes for money and the DM2 does not exist, i.e., $g \to 1$ and $\theta \to 1$. The bond prices are equal to $1+\mu$. Then its nominal yield ρ is given by

$$1+\rho = (1+\pi)\frac{1}{\psi} = \phi_t/\phi_{t+1} \times \phi_{t+1}/\phi_t = 1.$$

The gross nominal interest rate $(1+\rho)$ equals 1 and the net nominal interest rate equals *zero*. In this case, since the bonds are identical with money in being able to facilitate transactions in the DM and additionally deliver dividends, their real prices are higher than the fundamental value: $\psi = \beta\big[1 + g(\frac{1+\mu}{\beta} - 1)\big] > \beta$. Moreover, a high (a low) money growth rate causes high (low) real prices, or liquidity premia: $\partial\psi/\partial\mu > 0$.

(*Case 3: Liquid bonds but not substitutes for money*) Bonds are liquid in the DM, but are not substitutes for money at all, i.e., $g \to 0$ and $0 < \theta < 1$. Here the bonds are perfectly illiquid in the DM1, but liquid in the DM2; so the two decentralized markets are totally separated. The bond price is higher than its fundamental value due to its usefulness in the DM2: $\psi = \beta\big\{1+(1-\theta)h\left[u'\left(\tilde{z}^a\right)-1\right]\big\} > \beta$. However, the supply of each does not affect the other, so that liquidity premia are affected only by the bond supply, not by the money growth rate: $\partial\psi/\partial\mu = 0$.

(*Case 4: Liquid bonds and perfect substitutes*) Bonds are liquid in the DM2 and also perfect substitutes for money in the DM1, i.e., $g \to 1$ and $0 < \theta < 1$. Bonds carry extra values in exchange process in the DM2 in addition to the DM1. Then, Equation (14) yields the net nominal interest rate as follows.

$$\rho = \frac{-\beta(1-\theta)h\left[u'(\tilde{z}^a)) - 1)\right]}{(1+\mu) + \beta(1-\theta)h\left[u'(\tilde{z}^a)) - 1)\right]} \tag{16}$$

$$= \frac{(1-\theta)h\left[u'(\tilde{z}^a)) - 1)\right]}{i + (1-\theta)h\left[u'(\tilde{z}^a)) - 1)\right]} < 0 \tag{17}$$

The numerators in Equations (16) and (17) are negative only if bond supply is scarce, i.e., $u'(\tilde{z}^a) > 1$. The nominal rate of return on a liquid bond is negative, irrespective of money and bond supply, or the nominal rate of return on an illiquid bond i. It implies that lenders are willing to pay interests to buyers, even though it is lenders who take the default risk on lending, because the bonds provide extra liquidity services in transactions. As a result, the liquidity premium is affected by money supply: $\partial\psi/\partial\mu > 0$. I will discuss more details about whether the liquidity properties of bonds can cause negative yields in reality, and under what conditions negative yields emerge in a generic case, unlike this extreme case, in Section 4.

3 Data and Empirical Results

I empirically test the predictions in Proposition 1 with the U.S. data: whether money growth rate or nominal interest rates have a positive effect on liquidity premia; and whether bond supply has a negative effect on liquidity premia.[17] Since the negative effect of bond supply on liquidity premia is well documented in Krishnamurthy and Vissing-Jorgensen (2012), the former test result is mainly tested and demonstrated. Notice that liquidity premia are used rather than real bond prices in this section, because what the theory presents is that changes in their liquidity premia cause those in liquid bond prices. Moreover, there seems to be no consensus on how to measure the real prices of liquid and default-free bonds; but we can find the empirical data widely used to measure liquidity premia in the literature. I will explain more details about such data in the following subsection.

[17] The theoretical results present that those relationships hold only if the bond supply is scarce in the sense that q^* cannot be achieved in the DM. We can find several papers in the monetary search literature which reflect asset scarcity through higher asset prices than the fundamental values and their fluctuations. The examples include Williamson (2016), Lagos (2011) and Hu and Rocheteau (2012).

3.1 Data

The primary variables used in empirical tests include liquidity premia, money growth rate, nominal interest rates, and bond supply. First, I use the yield spreads between two bonds (or financial assets) with different liquidity but the same (or at least similar) maturity and default risk in order to measure the liquidity premia of liquid bonds. These spreads are not only easily computed, but also conceptually consistent with the liquidity premia meant by the theory. According to the theory, the nominal yields of illiquid and liquid bonds are expressed as follows.

$$1 + i = (1 + \pi)(1 + r) = (1 + \pi)\frac{1}{\beta} \tag{18}$$

$$1 + \rho = (1 + \pi)\frac{1}{\psi} = (1 + \pi)\frac{1}{\beta}\{1 + g(\frac{1 + \mu}{\beta} - 1) + (1 - \theta)h\left[u'\left(\tilde{z}^a\right) - 1\right]\}^{-1} \tag{19}$$

The difference between liquid and illiquid bonds is found in the terms in the curly bracket in Equation (19). It implies that the yield spread responds to changes in money growth rate and bond supply. More precisely, their spread, approximately measured by the log difference between (18) and (19), can be expressed as follows.

$$\begin{aligned} spread &\approx \left[1 + g(\frac{1 + \mu}{\beta} - 1)\right] + (1 - \theta)h\left[u'\left(\tilde{z}^a\right) - 1\right] \\ &= 1 + gi + (1 - \theta)h\left[u'\left(\tilde{z}^a\right) - 1\right] \end{aligned}$$

Consequently, what is empirically tested is whether money growth rates and nominal interest rates have positive effects on the spread above.

Then what types of bonds (or financial assets) can represent liquid and illiquid bonds in reality? The fact that bonds are liquid implies that they should be useful in exchange process, whereas illiquid bonds are not. Specifically, liquid bonds should be easily accepted directly as a medium of exchange, sold for more liquid assets such as cash in secondary asset markets, or used as collateral for credit (or loans) in the credit markets such as the Repurchase Agreement market (or Repo in short) and the collateralized federal funds market. In light of these features, I take three-month Treasury Bills (henceforth T-Bills) as the liquid bond, whereas three-month bankers' acceptance (henceforth BA), three-month general collateral Repos (henceforth GC Repos), and three-month Aa-rated commercial papers (henceforth CPs) as the illiquid assets. As is well-known, the three-month T-Bills are liquid in the sense mentioned above, but the CPs, the BA, and the GC Repos are not.[18] Additionally, they have the same maturity, three months, and the illiquid assets have the similar safety to the T-Bills compared to other financial assets.[19] In particular, the GC Repos are as safe as the T-Bills because the GC Repos use a portfolio of U.S. securities as collateral.[20] Hence, the yield spread between the Treasury Bills and each of the illiquid financial assets is an accurate measure to reflect the difference in liquidity meant by the theory. I obtain the data of these yield spreads from Krishnamurthy and Vissing-Jorgensen (2012) and Nagel (2014).[21]

Figure 4 shows the yield spreads from 1947 to 2007, i.e., the period after World War II and before the recent Great Recession. Following Nagel (2014), the three-month BA rate data represent the BA rates from 1947 to 1990 and the GC Repos rates from 1991 to 2007, because the GC repos rates are not available for the period before 1991. It is important to analyze the effects of the money growth rate and the bond supply for the long period with the annual data, because I examine the effects in the steady state. In the data, an unusual hike is found in 1974, which was caused by the 1974 stock market crash and accelerated by the oil crisis at the end of 1973. I attempt to control for this abnormal hike in the regressions later by using a dummy variable. Except for 1974, these two spreads show not a weak co-movement over the sample period in terms of frequency and amplitude. Last, as presented in Table 1, the average spreads of BA(GC Repos)/T-Bills and CPs/T-Bills during the sample period are 44.27 bps and 61.80 bps, respectively, and the standard deviation of both spreads is similar, about 40 bps.

Table 1: Summary Statistics: Yield Spreads

	Obs	Mean	Std. Dev.	Max	Min
BA(GC Repos)/T-Bills Spread	61	44.27	39.90	159.56	-5.32
CPs/T-Bills Spread	61	61.80	41.07	178.00	12.00

Notes: The spreads reported above are expressed in terms of basis points of annual rates. The spreads in 1974 is excluded. The BA(GC Repos)/T-Bills Spread is obtained from Nagel (2014) and the CPs/T-Bills Spread from Krishnamurthy and Vissing-Jorgensen (2012).

[18] An illiquid bond in the model implies a bond which cannot be used in exchange process, and so its holder should keep in his/her hand for the dividend in a form of consumption goods until maturity. Hence, an illiquid bond is inferior to a liquid bond only in terms of being liquidated or relaxing his/her liquidity constraint. Of course, an illiquid bond should be exactly or, at least, similarly as safe as a liquid bond to avoid the case where their yield difference includes the risk premium. When there is only a small difference in the default risk, it could be controlled in regressions by adding variables to explain it, even though it is not perfect. Similarly, the maturities should be matched for both bonds.

[19] One can be worried about the default risk of the Aa-rated CPs, but I like to emphasize that the default risk of the Aa-rated CPs is relatively very low by citing Krishnamurthy and Vissing-Jorgensen (2012)'s argument: "It is important to note that our evidence on the spread between commercial paper and Treasury bills is less likely to be affected by issues of omitted expected default or default risk premium controls than the evidence on the Aaa-Treasury spread. This is the case because historical default rates on the highest-rated commercial paper (A1/P1) are very low, with literally zero defaults during the period 1972-2000 for which Service (2010) provides data on commercial paper defaults. $\cdots\cdots$ Over a 3-month period, a highly rated firm may run into financial difficulties and be downgraded but is unlikely to enter bankruptcy."

[20] This is also why Nagel (2014) argues that the yield spread between the T-Bills and the GC Repos is a more accurate measure of a liquidity premium than any other measures frequently used in the literature.

[21] I annualized the monthly data from Nagel (2014) by averaging them with the same monthly weight.

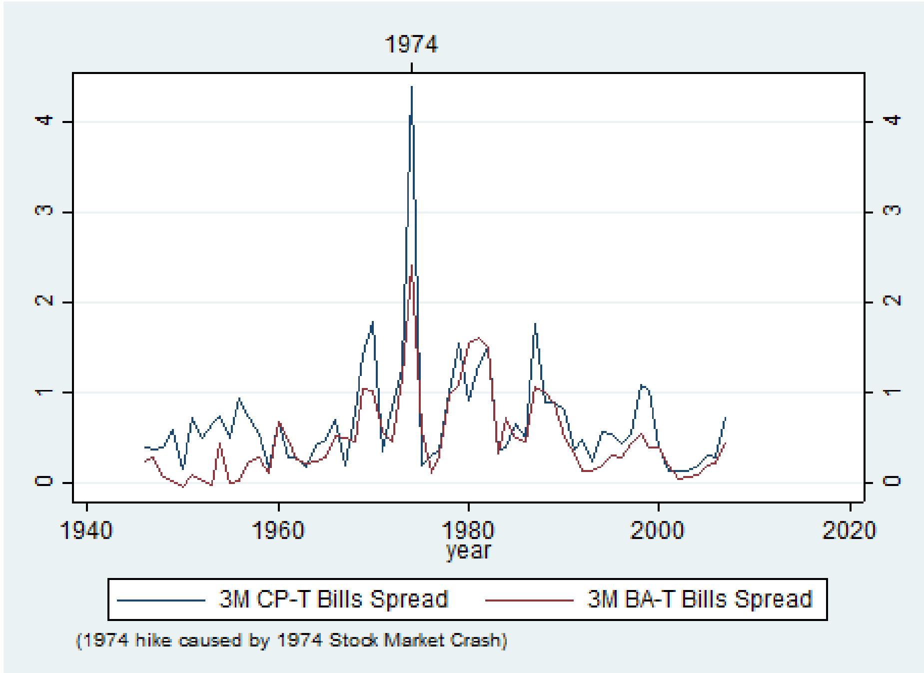

Figure 4: Yield Spreads

Next, let's consider which of monetary aggregates should be used to measure money supply. There are several monetary aggregates compiled by the Fed: Monetary Base, Narrow Money, M1, M2, and M3. In fact, the theory shows two criteria for which monetary aggregate is appropriate for the empirical tests. First, money should be perfectly liquid in exchange process, or at least it is a better MOE than bonds in the DM1. Hence, the monetary aggregate should not include any type of illiquid financial assets, such as savings deposits, money market deposit accounts, and small-denomination time deposits, precisely defined as time deposits in amounts of less than \$100,000.[22] This criterion excludes M2 and broader monetary aggregates, such as M3. Second, the money demand against the opportunity cost of holding money, which can be represented by nominal interest rates (or the inflation rate), should be stably downward sloping; when the opportunity cost of holding money rises up, the demand for money declines. Otherwise, the mechanism through which the liquidity substitution channel works could not be applied to explain the relationship between money supply and the liquidity premium: the higher opportunity cost does not lead to demand for liquid bonds as a substitute. It turns out that the demand on M1 against the nominal interest rate has not been stable even though M1 relatively includes liquid financial assets.[23] All things considered, I use Narrow Money to measure money supply in the theory.

[22]The scope of 'small-denomination time deposits' is found in http://www.federalreserve.gov/releases/h6/current/default.htm

[23]See Lucas Jr. and Nicolini (2015) for details about the instability of M1. They investigate why monetary aggregates become unstable over time.

Narrow Money is well suited to the theory in the sense that it is used as a perfect MOE in transactions. Also, it demonstrates a stable demand curve over the sample period, i.e., an unambiguously negative relationship with the federal fund rate during the period from 1955 to 2008. Last, I use the federal funds rate as the nominal interest rate on an illiquid bond in the theory. In reality, there are a variety of interest rates in financial markets and it is also difficult to find the yields of 100% illiquid bonds. However, as a well-known stylized fact, those interest rates present the strong co-movement among themselves. Also, the federal funds rate, as the policy rate of the Federal Reserve Bank, is comparable to money supply as a policy variable.

Figure 5 displays the ratio of Narrow Money to nominal GDP against its holding cost (i), which implies $L = M/PY$ in order to measure the real demand for money or real money balances proportional to Y implied by Equation (12).When there is not enough bond supply to achieve the first best outcome in the DM, it can be re-expressed to present the money demand in equilibrium as follows:

$$\frac{\phi_t}{\phi_{t+1}\beta} = 1 + i = 1 + \theta \left[u'(\tilde{z}) - 1 \right]$$

because the first equality holds in equilibrium.[24] Figure 5 shows that the real balances $\tilde{z}$ are negatively associated with the federal funds rate, which represents the opportunity cost of holding money.

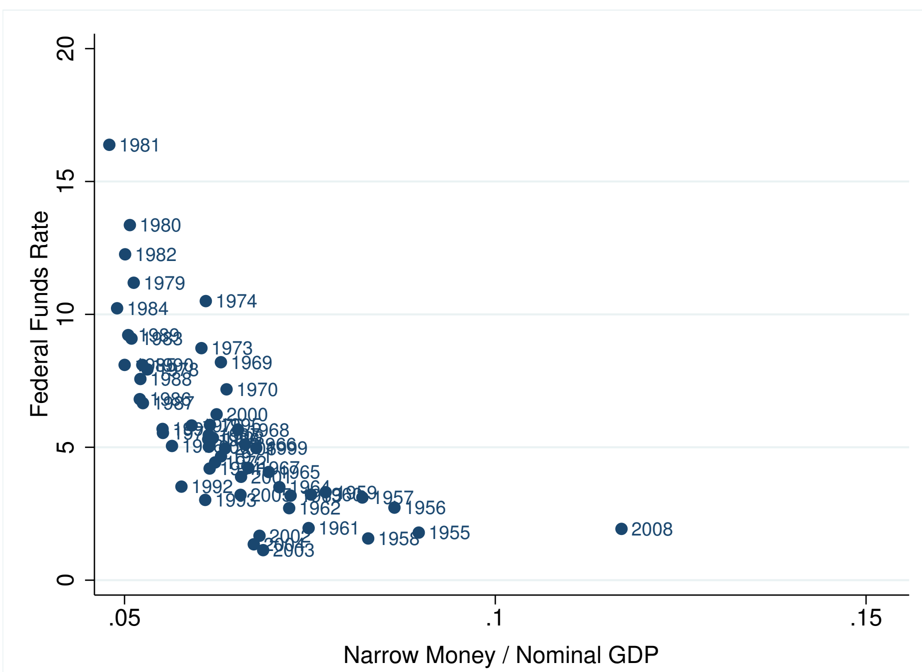

Figure 5: Money Demand

[24]Since $\tilde{z}_t = \tilde{z}_{t+1}$ in stationary equilibria, the time subscripts of i and $\tilde{z}$ are omitted.

For bond supply, I use the ratio of the outstanding stock of the T-Bills to nominal GDP for the liquid real bond supply. That ratio is measured as the market value of the T-Bills at the end of a fiscal year divided by the GDP of the same year. Also, a stock market volatility index obtained from Krishnamurthy and Vissing-Jorgensen (2012) is used as the control variables to account for the default risk premia of the illiquid bonds, even if the default risk of each of the illiquid assets would be small.[25]

3.2 Empirical Results

Importantly, since it is not the absolute level of money supply but its growth rate that affects liquidity premia as seen in Equation (12), I use the growth rate of Narrow Money (henceforth NM) rather than its level itself in the regressions. A one-time change in the money supply affects neither liquidity premia nor other real variables such as the real balances and quantities traded in the DM; but that in the growth rate does. Relative prices within the economy are adjusted instantly, responding to the one-time change in money supply, and so real variables stay unchanged. On the other hand, I use the absolute level of the T-Bills to nominal GDP ratio for the bond supply because it is not neutral, unlike money. Even a one-time change in bond supply affects the real variables.[26]

Table 2 presents the main empirical results. The dependent variables in Regressions (1) to (4) and (5) to (8) are BA(GC Repos)/T-Bills spread and CPs/T-Bills spread, respectively. All the coefficients of the NM growth rate and the federal funds rate turned out to be significantly non-zeros and positive.[27]

In Regressions (1) to (3), the coefficient of the money growth rate is significant and robust with the default-risk control variable, *Volatility*, and the dummy for year 1974. Regression (3) shows that a one percentage point increase in the money growth rate causes an increase of 3.846 bps in the BA(GC Repos)/T-Bills spread. At the same time, the T-Bill supply is negatively related to the spread, and its coefficient is -2.803. This regression result even with the money growth rate is consistent with the main finding of Krishnamurthy and Vissing-Jorgensen (2012) that the Treasury supply negatively affects the convenience yields, which is the market value for both liquidity and safety of the Treasury securities.

[25]Krishnamurthy and Vissing-Jorgensen (2012) argue that this measure has a high correlation with another default risk measure, such as the median expected default frequency credit measure from Moody's Analytics. See Krishnamurthy and Vissing-Jorgensen (2012) for more details about why this measure can be a proxy for default risk.

[26]This type of model in the literature delivers the similar effects by changing the supply of bond or money. In particular, see Rocheteau, Wright, and Xiao (2016) among others. They show that a one-time open market purchase or sale affects real variables within an economy by changing the supply of bonds, not by changing the money supply one time.

[27]The fact that the federal funds rate has a positive effect on the liquidity premium of T-Bills is presented in Nagel (2014). He focuses on the deviation from the steady state to analyze the short-run effect of the nominal interest rates, and uses the monthly spread data.

Regression (4) shows the positive effect of the federal funds rate on the BA(GC Repos)/T-Bills spread.[28] Like that of the money growth rate, its effect on the spread appears significant and positive, but the effect of the T-Bill supply is insignificant. Since the federal funds rate and the money growth rate both represent the opportunity cost of holding money, all these results confirm the substitute relationship between money and liquid T-Bills.[29]

Regressions (5) to (8) show results for regressions with the CPs/T-Bills spread as the dependent variable. The coefficients of money growth rate are also significant and robust with the default risk control variable, *Volatility*, and the dummy for year 1974. An increase of one percentage point in the money growth rate is associated with an increase of 3.206 bps in the Cps/T-Bills spread. The coefficient of the T-Bill supply is negative 3.780. It is not significant, but the coefficient appears negative even if reflecting its standard error. The effect of the federal funds rate on the CPs/T-Bills spread is also positive 8.379.

The coefficients of the money growth rate in Regression (3) and (4) are not so different in terms of their magnitude, and their average is around $+3.5$. The coefficient of the federal funds rate is around $+9.46$ on average. The average coefficients of the money growth rate are overall smaller than those of the federal funds rate. This difference seems to arise because nominal interest rates in financial markets show a strong co-movement, and the central bank conducts open market operations in order to adjust the federal funds rate to its target.

Next, the coefficients of $Volatility$ in Regressions (3), (4), (7) and (8) are insignificant and, moreover, in Regression (4) and (7) they are close to zero. This implies that BA(GC Repos) and CPs are as safe as the T-Bills, even if they do not exactly bear the same default-risk, and so confirms that the two spreads used as the dependent variables in the regressions are good measures of the liquidity premia. Also, $Dummy$ is added in Regressions (3), (4), (7), and (8) to control the stock market crash in 1974. Its coefficients turn out to be significant. In particular, the adjusted R-squared shows notable rises to 35.5% and 58.2%, respectively, in Regressions (3) and (7) with the $Dummy$, compared to Regressions (2) and (6).

Overall, the regression results strongly support the theoretical findings in Proposition 1 that liquidity premia increase in money growth rate and decrease in bond supply. It implies that the T-Bills play a role as substantive substitutes for money in terms of liquidity to some degree, i.e., $g > 0$ in Equations (14) and

[28]The federal funds rate is available back to 1955 on the website of Federal Reserve Economic Data.

[29]The one-to-one relationship between the money growth rate and nominal interest rates is empirically supported. Monnet and Weber (2001) show that the high positive correlation (0.87) between nominal interest rates and money growth rates over the period from 1961 to 1998 for a sample of 31 countries. Moreover, Berentsen, Menzio, and Wright (2011) found strong positive relationships between inflation and money growth, and between inflation and interest rates by using data from 1955 to 2005. According to the Fisher equation, this empirical evidence is consistent with the finding of Monnet and Weber (2001).

(15), and then the opportunity cost of holding money affects the demand for liquid T-Bills, consequently influencing their liquidity premia and prices. If they had not been the substitutes ($g = 0$), the coefficients of the money growth rate would have been close to zero or insignificant, which implies that the money growth rate would not have affected the liquidity premium of T-Bills at all.

Table 2: Impact on the Liquidity Premium of the three-month T-Bills

Dependent Vars	BA(GC Repos)/T-Bills spread				CPs/T-Bills spread			
	(1) 1947-2007	(2) 1947-2007	(3) 1947-2007	(4) 1955-2007	(5) 1947-2007	(6) 1947-2007	(7) 1947-2007	(8) 1955-2007
NM Growth	5.512**	4.820**	3.846**		5.574*	5.257*	3.206**	
	(2.167)	(1.942)	(1.483)		(3.016)	(2.858)	(1.426)	
T-Bills to GDP	-4.248*	-3.760	-2.803*	-3.120	-6.020	-5.796	-3.780	-2.711
	(2.498)	(2.261)	(1.610)	(1.978)	(4.866)	(4.757)	(2.707)	(2.792)
Volatility		2.231*	1.730	0.211		1.023	-0.0317	-0.981
		(1.172)	(1.069)	(0.546)		(1.583)	(1.251)	(0.941)
Dummy			1.715***	1.359***			3.611***	3.350***
			(0.134)	(0.0613)			(0.122)	(0.111)
Federal Funds Rate				10.54***				8.379***
				(0.810)				(1.574)
Constant	0.502***	0.203	0.223	0.118	0.835***	0.698**	0.739***	0.510**
	(0.175)	(0.195)	(0.179)	(0.186)	(0.261)	(0.305)	(0.224)	(0.247)
Observations	61	61	61	53	61	61	61	53
Adjusted R-squared	0.119	0.147	0.355	0.812	0.060	0.048	0.582	0.723

Notes: Coefficients are estimated by Newey-West estimators with lag(1) and its standard errors are presented in parenthesis. The dependent variables are the yield spreads between the illiquid assets and the T-Bills, which are measured in a percentage unit. Explanatory variables are the growth rate of Narrow Money and the ratio of the market value of T-Bills outstanding to nominal GDP. A control variable for the default risk on illiquid assets is $Volatility$, which is measured by the annualized standard deviation of weekly log stock returns on the $S\&P$ 500 index (Source: KVJ 2012). *** p<0.01, ** p<0.05, * p<0.1.

4 Discussion of Negative Interest Rates

Negative Treasury yields have been observed in some advanced countries such as Switzerland, the United States, Japan, and Germany for recent years, as described in Figure 6. In particular, since the European Central Bank (henceforth ECB) lowered its target rate to below zero on June 11th, 2014, several European countries have been experiencing negative yields on their government bonds, including both short- and long-term bonds.[30] Since the negative policy rate implies imposing a negative interest rate on the excess reserves deposited in central banks, and government bonds can be an alternative as a safe and liquid store of value for investors who hold a large denomination of excess money in their hands, this negative yield on government bonds seems to be reasonable. This is the case for most of the

[30]The ECB set the rate on the deposit facility from 0.00% to -0.10%, which banks may use to make overnight deposits with the Eurosystem on June 11th, 2014. The deposit facility rate remains at -0.40% as of November 10th, 2017.

countries that have been observing negative yields on their governments bonds in recent years. However, there are two countries where the negative Treasury yields were observed, even during the period when their monetary policy rates were positive: Switzerland and the United States. In Switzerland, the yields of almost all the government bonds were negative for the several years since 2008. Also, in the United States, the secondary market yields of three-month Treasury bills were negative for several days in September 2015, even though it was not for a long time. Their policy rates were slightly positive during those periods. Why do investors buy the government bonds with negative nominal yields?

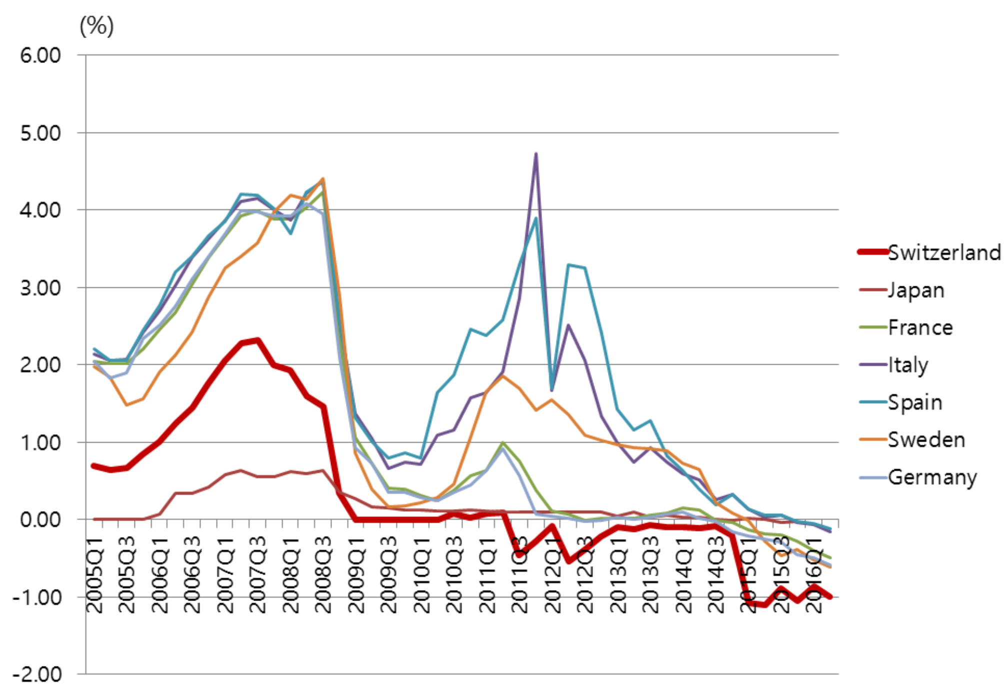

Figure 6: Negative Yields on Treasury Bills
Source: International Financial Statistics (IFS), www.investing.com

Specifically, a negative bond yield implies that a bond buyer (or a lender) pays a bond issuer (or a borrower) interest on her/his lending. Intuitively, it may not make sense because it is the lender who takes advantage of the loan for his/her own sake. Moreover, there is an alternative: money. Money is a liability issued by central banks, and therefore money is unambiguously as safe as government bonds. The existence of money implies that the lender could hold cash, as a perfect substitute for bonds whose interest rate is 0% and at least non-negative. Or if the lender is a financial institution, it can hold money as a reserve deposit at the central bank because it pays a positive interest. I use the theoretical findings and the data on Switzerland in order to offer a rationale for this puzzling observation. Precisely, I address the following questions: whether the liquidity properties of government bonds can cause such negative nominal yields unlike other government bonds, and if so, under what conditions that can happen.

From Equation (19), the nominal yield of a liquid bond is given by

$$\begin{aligned}\rho =& \frac{(1-g)\left[\frac{1+\mu}{\beta}-1\right]-(1-\theta)h\left[u'(z^a)-1\right]}{(1-g)+g\frac{1+\mu}{\beta}+(1-\theta)h\left[u'(z^a)-1\right]} \\ =& \frac{(1-g)i-(1-\theta)h\left[u'(z^a)-1\right]}{1+gi+(1-\theta)h\left[u'(z^a)-1\right]}.\end{aligned} \tag{20}$$

The following proposition summarizes under what conditions a bond's liquidity can cause negative yields.

Proposition 2. *The yield on a liquid bond falls below zero under the following conditions, and it is caused by bond liquidity.*

(i) If $1-\theta=0$, the yield on a liquid bond is negative, only when $i<0$.

(ii) If $1-\theta\neq 0$, the yield on a liquid bond can be negative, even when $i\geq 0$,

- *a) if $i=0$ and $u'(z^a)>1$, i.e., the nominal yield on an illiquid bond is zero and the liquidity premium is positive, or*
- *b) if $i>0$ and $i<(1-\theta)h\left[u'(z^a)-1\right]/(1-g)$., i.e., a certain portion of the marginal liquidity benefit provided by a liquid bond is greater than the marginal cost of holding money.*

The proof is straightforward from Equation (20) and thereby is omitted here. The proposition presents the following interesting results. First of all, if $1-\theta=0$, i.e., there do not exist financial markets where liquid bonds are used only in facilitating transactions, called the DM2 in the model, the yields on liquid bonds (or government bonds) can not be negative when i is positive or zero. This holds even if bonds are perfect substitutes for money, i.e., $g=1$. Consider i as a monetary policy rate. Then, negative Treasury yields take place only if the monetary policy rate, i, falls down to the negative territory. This can be applied to some European countries, such as Germany and France, where negative Treasury yields were observed after the ECB decreased its policy rate to a negative rate. However, Switzerland and the U.S. cannot be explained this way, because the monetary policy target range of Switzerland was positive until December 2014,[31] and the federal funds rate has never been negative so far. On the other hand, if $1-\theta\neq 0$, i.e., there exists the DM2, negative yields can emerge due to the liquidity properties, even when i is positive. The condition, $i<(1-\theta)h\left[u'(z^a)-1\right]/(1-g)$, implies that the bond liquidity value

[31] It is set at the range of 0 - 1.00% on 12/11/2008, 0 - 0.75% on 3/12/2009, 0 - 0.25% on 8/3/2011, -0.75 - 0.25% on 12/18/2014, and -1.25 - -0.25% on 1/15/2015.

generated in the DM1 and the DM2 outweighs the money holding cost. In particular, the low monetary policy rate increases the probability that the condition holds, and so the scarcity of the liquid bond supply can lead to high liquidity premia and negative nominal yields.[32]

Interestingly, the data which this theoretical prediction is consistent with are found in Switzerland, with a feature that its governments bonds are in strong demand among both domestic and foreign investors. As mentioned above, we can observe the negative Treasury yields even during the period when the Swiss monetary policy target was positive. Figure 7 shows huge changes in the government bond supply around 2007 and 2008. The ratio of the government bond supply relative to GDP was higher than 50%, but fell to around 30%. Moreover, the interest target range of the Swiss National Bank was at a low level, 0-1.00% at the end of 2008, and then continued to be slightly positive until the end of 2014. The negative yields on the government bonds emerged around 2010 for the first time. According to both theoretical and empirical results, all these data strongly suggest that the relative scarcity of liquid government bonds against money led to negative yields by increasing liquidity premia on government bonds when the policy rate remained close to zero.

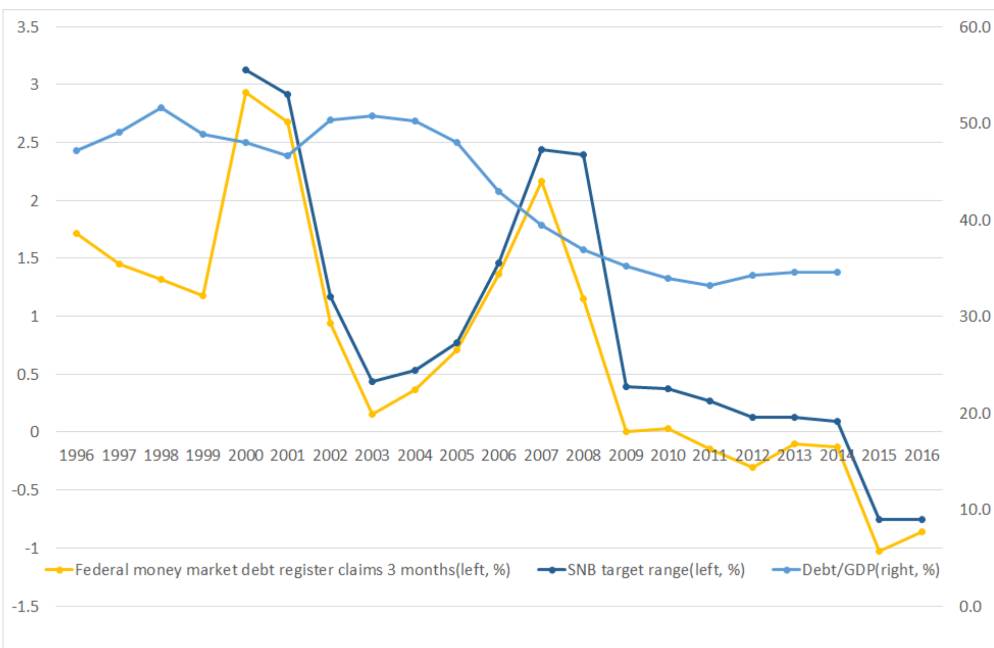

Figure 7: Interest Rates, Money and Gov't Bond Supply in Switzerland

Notes: *SNB target range* is the average of the median of the SNB target range in each period.
Source: Swiss National Bank

[32]The condition $1 - \theta \neq 0$ means that the DM2 exists, where bonds serve as collateral for credit. Although one might think that this market structure is impractical at first glance, our attempt is by no means the first one in the literature. Rocheteau, Wright, and Xiao (2016) similarly develop a monetary search model with transactions where nominal bonds are only used as a MOE. They set up this assumption on the grounds that government bonds, such as the Treasury Bills in the U.S.and Switzerland, are always in strong demand.

Another narrative example is found in the United States, if I abuse the theory a little bit. In particular, the theory is also consistent with the comments from the market participants during the period when the negative nominal yields on the Treasuries were observed. For example, according to *Bloomberg* (September 25, 2015), Kenneth Silliman, head of US short-term rates trading in New York at TD Securities unit, one of 22 primary dealers that trade with the Fed said,

"Yields on U.S. Treasury bills fell below zero as an influx of cash and pent-up appetite for safe assets led investors to accept negative returns after the Federal Reserve decided not to raise its short-term interest rate. ...... Investors will have additional funds totaling about $100 billion returned to them in the next month as the government cuts bill supply heading into negotiations with Congress about the statutory debt limit."[33] [34]

In brief, he mentioned that the main factors that drove down nominal Treasury yields to below zero were 'a cut in bill supply' and the low short-term interest rate. Notice that the policy rate (or rate range) of the Fed has been hovering around zero for more than seven years since 2008. When negative bond yields occurred, the Treasury Bill supply was expected to decrease to be lower than the past. Moreover, there was a strong demand for the government bonds among investors in financial markets. Both factors worked together in the direction to raise up the liquidity premium of the Treasury Bills, and so nominal bond yields seems to fall down to the negative territory even during the short time period. Consequently, the observed example in U.S. also suggest that the low money holding cost and the scarcity of liquid bonds caused negative Treasury yields by raising liquidity premia.

5 Conclusion

This paper explores the effect of monetary policy on liquidity premia counted in the prices of liquid financial assets such as government bonds. The theoretical model built in the paper delivers elaborate predictions about under what conditions and through what mechanisms money supply can affect liquidity premia. Specifically, the money growth rate, not its level, has a positive effect on liquidity premia because the high money growth rate implies the high opportunity cost of holding money, thereby leading to a high demand for liquid bonds as substitutes for money to some degree. In the meanwhile, the

[33]Source: http://www.bloomberg.com/news/articles/2015-09-17/treasury-bill-yields-turn-negative-as-fed-leaves-rates-unchanged

[34]See the following comment in *The Wall Street Journal* (Sept 23, 2014) for another narrative example: "Short-term debt trading at negative yields was essentially unheard of before the 2008 financial crisis. But since then, the condition has cropped up at times of market stress, reflecting extraordinarily expansive central-bank policy and anemic growth in much of the world. Yields on some U.S. bills traded below zero at the end of each of the past three years amid strong demand for liquid assets, according to analysts." Source: http://www.wsj.com/articles/treasury-bill-yield-tips-into-negative-territory-1411516748

level of liquid bond supply negatively affects liquidity premia by changing the relative scarcity of bond supply to money supply. The empirical analysis with the U.S. data provides strong evidence for the theoretical predictions: the money growth rate and the nominal interest rate are positively correlated with liquidity premia measured by the spreads between liquid and illiquid financial assets with the same, or at least similar, safety and maturity. Last but not least, the paper explains how the liquidity properties of government bonds are associated with negative nominal yields as observed in Switzerland and the U.S. The theory and the relevant empirical evidence suggest that such negative yields should be caused by high liquidity premia due to the scarcity of the liquid bond supply, when the money holding cost is low.

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A Appendix

Proof. Proof of Lemmas 1 and 2.

First, consider Lemma 1. Substituting $\boldsymbol{\phi}_t \boldsymbol{p}_t$ into the objective function in Equation (6) re-express the bargaining problem as

$$\max_{q_t^1} \left\{ u(q_t^1) - q_t^1 \right\}$$

subject to $q_t^1 = \boldsymbol{\phi}_t \boldsymbol{p}_t$, and $\boldsymbol{p}_t \leq \widetilde{\mathbf{w}}_t$. If $\boldsymbol{\phi}_t \widetilde{\mathbf{w}}_t \geq q^*$, the optimal choice of q_t^1 will be the first best quantity q^*, i.e., $q_t^1 = q^*$. Then, $\boldsymbol{p}_t = (p_t^m, p_t^a)$ such that $\phi_t p_t^m + g p_t^a = q^*$. However, if $\boldsymbol{\phi}_t \widetilde{\mathbf{w}}_t < q^*$, the effective budget constraint is binding. Accordingly, the buyer will give up all her real balances in order to purchase as many as possible. Then, the optimal choice of q_t^1 will be the same as her real balances $\boldsymbol{\phi}_t \widetilde{\mathbf{w}}_t$. Also, $\boldsymbol{p}_t = (m_t, a_t)$. When it comes to Lemma 2, the same steps above can be taken for proof. Since it is straightforward, it is omitted. □

Proof. Proof of Lemma 3

First, consider whether the real balances are used as a direct medium of exchange or are as collateral to borrow enough credit to obtain the optimal quantity q^* in each of the two DM markets. If $\tilde{z}_t \geq q^*$ or $\tilde{z}_t^a \geq q^*$, $q_t^1 = q^*$ or $q_t^2 = q^*$; otherwise, $q_t^1 = \tilde{z}_t$ or $q_t^2 = \tilde{z}_t^a$ by lemmas 1 or 2. Then, plugging these results into the first order conditions (10) and (11) for the maximum of the objective function will yield the equilibrium prices ϕ_t and ψ_t. Also, the marginal utility function u' is monotonically decreasing in its argument, so the equilibrium is uniquely determined. □

Proof. Proof of Corollary 1

The quantity that buyers can consume with their money holdings in the DM is determined by Equation (12) when the real balances are not enough to obtain q^*.

$$\frac{1+\mu}{\beta} = 1 + \theta \left[u'(\phi M + gA) - 1 \right]$$

Then, if we plug zero into ϕM and solve this equation for μ, we can obtain the upper bound of μ for the monetary equilibria as follows.

$$\bar{\mu} \equiv \{\mu : \mu = \beta[1 + \theta[u'(gA) - 1]]\} - 1$$

□